

Modeling the Corruption Dynamics with Awareness Impact: A Fractional Approach

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Abstract. Corruption is a pervasive issue that undermines the socio-economic development of nations and weakens effective governance. This study proposes a mathematical model to analyze and simulate corruption dynamics, focusing on the impact of the awareness parameter. We examine the patterns of changing transmission of corruption that evolve through the Caputo fractional derivative. The model has been well-posed from both epidemiologically and mathematical perspectives. The basic reproduction number $\mathcal{R}_0$ is computed to forecast and evaluate the persistence of corruption. The stability analysis of equilibrium points have been examined using the Jacobian matrix and Lyapunov function methods. We analyze the model dynamics using numerical results for various parameters and fractional orders. Sensitivity analysis is carried out to identify the key factors influencing the dynamics of corruption. Numerical simulations confirm the analytical results.

1. INTRODUCTION

Corruption is one of the pressing global issues, serving as a major cause of poverty and a significant barrier to its reduction and eradication. It undermines the economic development of businesses and societies, hindering progress and growth at all levels [1]. Since corruption affects social life and the rule of law, it is viewed as a significant concern for moral values, justice, and sustainable economic progress [2]. As explored in [3], corruption and sustainable development goals correlate negatively. Corruption is estimated to be an aggravating factor, exacerbating income disparities among various socioeconomic classes within society [4,5]. Corruption may

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be perceived as an inherent disease to public authority and indicative of poor governance [6]. The spread of corruption resembles the transmission of diseases, where a non-corrupt individual is highly likely to become corrupt if the prevalence of corrupt individuals within their social environment surpasses a specific limit. In cases of mean-field dependence, an individual may become corrupt simply due to the high prevalence of corruption within society [7]. Implementing strong measures to combat corruption is challenging due to its secretive and illicit nature. However, it is possible to raise awareness and promote a cultural shift against corruption within a society or nation [8].

In recent decades, mathematical modeling has been utilized more frequently to reduce the effects of social challenges because it is a valuable tool for forecasting and handling these issues. A variety of epidemiological models are used to examine how corruption behaves as a disease, aiming to determine effective strategies for managing its spread. For example, Hathroubi and Trabelsi [9] explained the dynamics of corruption using the SCH (Susceptible, corrupt and honest individual) model. A threshold for epidemiological corruption was established, taking into account an estimate of the honest population. The research carefully and thoroughly analyzed the evaluation and forecasting of corruption spread, identifying its key elements [10–18].

Eguda and colleagues [19] formulated and examined the corruption dynamics with immune individuals. Using an immunity clause, Gweryina et al. [20] developed and examined an epidemiological model to address the corruption issue in Nigeria. Their simulation outcome suggests that fighting corruption in a system with an immunity clause will be difficult if it continues to exist. Athithan et al. [21] developed a model for corruption control and conducted an analysis using differential equations principles. Danford et al. [22] extended the work [21] by introducing immune individuals and examining preventive measures that integrate mass education with religious belief. Nathan and Jakob [23] developed and analyzed a mathematical model considering susceptible, corrupt, and politically corrupt compartments to describe the dynamics of corruption in Kenya. In their study, prevention, and disengagement strategies are incorporated into the model through specific parameters. Akanni et al. [24], suggested a model to investigate the corruption transmission, considering how society influences individuals with strong moral principles. Fantaye et al. [25] proposed a model for the spread of corruption, considering the impact of social factors on honest individuals with optimal control strategies. Keno and Legesse [26] examined the dynamics of corruption transmission, emphasizing control strategies like individual deterrence and the establishment of strong anti-corruption laws. Control of corruption spread considering susceptible, corrupt, recovered, and honest individuals was discussed [27].

In recent years, fractional calculus, which extends integrals and derivatives to any positive real order, has emerged as a significant area within mathematical modeling. It has drawn considerable attention from researchers. Its unique characteristics have attracted great interest in mathematics and related fields [28–30]. The nonlocal nature of these differential operators distinguishes them from classical differential operators. Various forms of fractional derivatives including the

Riemann, Liouville, and Caputo are commonly used in practical applications. The rate of change at a given point in a system with non-local interactions may depend on values at other regions. Standard differential equations, however, typically only account for local information at a single point. Fractional calculus allows for modeling these non-local interactions, capturing the influence of distant points within a system and providing a more comprehensive description of the system dynamics. The Caputo fractional approach has become more popular during the last decade. Researchers can enhance the accuracy and reliability of their models by incorporating memory effects using the Caputo fractional differential operator. For instance, Baleanu et al. [31] investigated the application of real-world pandemic data to enhance the numerical efficiency of the Caputo fractional derivative. Vellappandi et al. [32] studied the role of spraying and roguing on the effective control of the spread of mosaic with Caputo derivative. Calatayud et al. [33] provide a comprehensive analysis of the pros and cons of the Caputo approach.

As discussed, numerous studies have formulated mathematical models to explore the spread of corruption from different perspectives. However, many of these models have not considered the saturated incidence rate. Moreover, in our proposed model, we have considered the impact of the awareness parameter on the transmission and control of corruption. The concept of saturation incidence rate $\frac{\beta SI}{1+aI}$ was proposed by Capasso and Serio [34] in 1978. We considered the saturated incidence rate of type $\frac{\beta SI}{1+aS}$ proposed by Anderson and May [35]. A mathematical model for the drinking problem with a saturated incidence rate of $\frac{\beta SI}{1+aS}$ is discussed in [36]. Yue and Zhu [37], present a model with impact of media reports on rumor propagation using a Caputo derivative. Agrawal et al. [38] conducted a study on a SIR model characterized by a nonlinear incidence rate expressed as $\frac{\beta I}{1+a_1S+a_2I^2}$.

Based on the preceding discussion, our focus is to analyze the corruption model using the Caputo fractional operator. Many studies have demonstrated that the Caputo derivative can accurately describe the progression of complex phenomena. This motivates the analysis presented in this work. In the modeling of epidemic spread, non-local factors such as individual past interactions, past events, and behaviors become particularly valuable for accurately capturing the true dynamics of transmission. The Caputo fractional derivative provides a more accurate picture of complex systems represented by correlations that span time and exhibit power-law behavior.

This paper is organized as follows: Section 2 outlines the essential preliminary definitions. Section 3, introduces the fractional-order corruption model. Section 4 is devoted to the theoretical analysis of the model. Section 5 focuses on examining numerical simulations. Finally, Section 6 provides the conclusion.

2. PRELIMINARIES

Here, we present definitions of fractional derivatives and a related theorem [39].

Definition 2.1. *The Caputo derivative of a function $f(t) \in C^1([a, b])$ of fractional order $0 < \omega < 1$ is defined as:*

$$D_t^\omega f(t) = \frac{1}{\Gamma(1-\omega)} \int_a^t f'(s)(t-s)^{-\omega} ds, \quad (2.1)$$

where $\Gamma(\cdot)$ represents the Gamma function.

Definition 2.2. The corresponding integral formulation of order $\omega \geq 0$ is defined as:

$$\mathcal{I}_t^\omega f(t) = \frac{1}{\Gamma(\omega)} \int_a^t \frac{f(s)}{(t-s)^{1-\omega}} ds. \quad (2.2)$$

Theorem 2.1. The Laplace transform for Caputo derivatives is as follows:

$$\mathcal{L}\{D^\omega f(t)\} = s^\omega F(s) - \sum_{\ell=0}^{n-1} s^{\omega-\ell-1} f^{(\ell)}(0), \quad n-1 < \omega \leq n, \quad n \in \mathbb{N}, \quad (2.3)$$

where $F(s) = \mathcal{L}\{f(t)\}$.

3. MATHEMATICAL MODEL

The mathematical model of corruption transmission will be discussed in this section, focusing on the impact of public awareness. The saturation incidence reflects the inhibitory effect due to protection measures taken by the general public. The model is divided into five population compartments: Susceptible class $S(t)$, Exposed class $E(t)$, Corrupted class $C(t)$, Jailed class $J(t)$, and Recovered class $R(t)$. Consequently, the total population is represented as $N(t) = S(t) + E(t) + C(t) + J(t) + R(t)$. Susceptible class $S(t)$ are those who are not engaged in any type of corruption and positively contribute to the country economic development. Exposed individuals $E(t)$ are people who have already been exposed to corruption, Corrupt individuals $C(t)$ are those who are often involved in corrupt practices and can influence susceptible individuals to engage in corruption. Jailed individuals $J(t)$ are those who have been sentenced for corrupt practices and imprisoned due to their involvement in corrupt activities for a particular time frame as per the law. During this time, they are unable to engage in corrupt activities or influence others. Recovered people $R(t)$ are those who were previously involved in corrupt practices but have stopped doing any corrupt activities, and the assumption is that they will not be involved in any corrupt practices in the future.

The mathematical model of corruption in the Caputo sense is developed based on the propagation rules shown in Fig. 1 as follows:

$$\begin{aligned} D^\omega S(t) &= \Pi - \frac{\beta_1 SC}{1+aS} - \mu S, \\ D^\omega E(t) &= \frac{\beta_1 SC}{1+aS} - (\mu + \eta + \theta)E, \\ D^\omega C(t) &= \eta E - (\mu + \gamma + \tau)C, \\ D^\omega J(t) &= \gamma C - (\mu + \alpha)J, \\ D^\omega R(t) &= \theta E + \tau C + \alpha J - \mu R, \end{aligned} \quad (3.1)$$

with initial conditions

$$S(0) = S_0 \geq 0, E(0) = E_0, C(0) = C_0 \geq 0, J(0) = J_0 \geq 0, R(0) = R_0 \geq 0. \quad (3.2)$$

In above model, Π represents a positive recruitment rate within the susceptible class, while

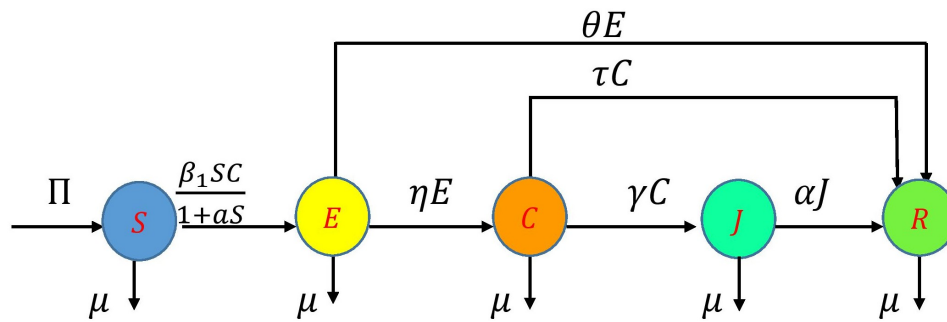


FIGURE 1. Flow chart of the model (3.1)

μ denotes the natural death rate applicable to all classes. After the interaction with corrupted people, susceptible people can move to the exposed class at a rate of β_1 . The parameter a is the awareness parameter, which measures the inhibitory impact due to awareness campaigns or preventive measures that reduce corruption. The exposed individuals will join the corrupt class at a rate of η due to their corrupt activities, while those who abandon corruption will transition to the recovered class at a rate of θ . The corrupt people join the jailed class at a rate of γ , and transit to the recovered class by leaving corruptive activities at a rate of τ . However, the jailed individuals enter the recovered class at a rate of α due to anti-corruption therapy in prison, or the completion of their conviction period. For explanations of the model parameters and their values, see Table 1

TABLE 1. Parameters description

Parameters	Description	Values	Source
Π	Recruitment rate	0.7	Assumed
μ	Natural death rate	0.016	[7]
β_1	Effective contact rate	0.234	Assumed
η	Transfer rate of exposed people into the corrupt compartment	0.02	[7]
θ	Percentage of exposed people that join recovered group	0.01	Assumed
γ	Rate at which corrupt individuals are captured and put in jail	0.007	[45]
α	Rate at which people in jail are released	0.0159	[15]
a	Awareness effect parameter	0.9	Assumed
τ	Transfer rate from the corrupt compartment to recovered class	0.003	Assumed

4. ANALYSIS OF THE MODEL

4.1. Non-negativity and boundedness. The following part has established the criteria to assess the feasibility of the solution to the system (3.1). Let $\mathbb{R}_+$ denote the non-negative real numbers and $\Omega = \{(S, E, C, J, R) \in \mathbb{R}_+^5 : N = S + E + C + J + R \leq \frac{\Pi}{\mu}\}$.

Theorem 4.1. *The solution of the system (3.1) subject to the conditions (3.2) is non-negative within the closed set Ω .*

Proof. First, we show that the solutions of (3.1) are positively invariant in $\mathbb{R}_+^5$. It follows from (3.1) that

$$\begin{aligned} D^\omega S(t)|_{S=0} &= \Pi \geq 0, \\ D^\omega E(t)|_{E=0} &= \frac{\beta_1 SC}{1+aS} \geq 0, \\ D^\omega C(t)|_{C=0} &= \eta E \geq 0 \\ D^\omega J(t)|_{J=0} &= \gamma C \geq 0 \\ D^\omega R(t)|_{R=0} &= \theta E + \tau C + \alpha J \geq 0. \end{aligned} \quad (4.1)$$

We conclude that every solution to the model (3.1) is positive for all $t \geq 0$.

Next, we will demonstrate the boundedness of model (3.1). The overall population is defined as $N(t) = S(t) + E(t) + C(t) + J(t) + R(t)$, then

$$\begin{aligned} D_t^\omega N(t) &= D^\omega S(t) + D^\omega E(t) + D^\omega C(t) + D^\omega J(t) + D^\omega R(t) \\ &= \Pi - \mu(S + E + C + J + R), \\ &= \Pi - \mu N. \end{aligned} \quad (4.2)$$

By employing the Laplace transform, we obtain

$$N(t) = N_0 \mathcal{E}_{\omega,1}(-\mu t^\omega) + \Pi \mathcal{E}_{\omega,\omega+1}(-\mu t^\omega), \quad (4.3)$$

where $\mathcal{E}_{\omega,1}$ is the Mittag-Leffler function. From the properties of the Mittag-Leffler function [40], we get that

$$N(t) = \left(N_0 - \frac{\Pi}{\mu}\right) \mathcal{E}_{\omega,1}(-\mu t^\omega) + \frac{\Pi}{\mu}. \quad (4.4)$$

Then, taking the $\limsup$ as $t \rightarrow \infty$, implies that $N(t) \leq \frac{\Pi}{\mu}$ and for all $t > 0$. Thus the solution is bounded in Ω and this completes the proof.

We will now show that the system of equations (3.1) has a unique solution.

Lemma 4.1. [41] *Consider the system*

$$D_t^\omega u(t) = f(t, u), \quad u(t_0) = u_0, \quad (4.5)$$

where $\omega \in (0, 1]$, $f : [t_0, \infty) \times \Omega \rightarrow \mathbb{R}^n$, $\Omega \subset \mathbb{R}^n$, and $t_0 \geq 0$. If $f(t, u)$ satisfies the local Lipschitz condition for u , meaning that there exists a constant $L > 0$ such that $\|f(t, u') - f(t, u'')\| \leq L\|u' - u''\|$ for all $u', u'' \in \Omega$, where L is Lipschitz constant, then there is a unique solution to Eqn. (4.5) on the interval $[t_0, \infty)$.

Theorem 4.2. *A unique solution to the fractional order system (3.1) exists for every non-negative initial condition.*

Proof. Consider a mapping $V(X) = (V_1(X), V_2(X), V_3(X), V_4(X), V_5(X))$, where

$$\begin{aligned} V_1(X) &= \Pi - \frac{\beta_1 SC}{1+aS} - \mu S, \\ V_2(X) &= \frac{\beta_1 SC}{1+aS} - (\mu + \eta + \theta)E, \\ V_3(X) &= \eta E - (\mu + \gamma + \tau)C, \\ V_4(X) &= \gamma C - (\mu + \alpha)J, \\ V_5(X) &= \theta E + \tau C + \alpha J - \mu R. \end{aligned} \quad (4.6)$$

Since the vector field V is in $C^1(\Omega)$, it is locally Lipschitz. Therefore, the result in Theorem 4.2 is trivial (see [30]). Hence, the system (3.1) has a unique solution with the help of Lemma 4.1.

4.2. Model equilibria. To find the critical points of the corruption model (3.1) we put the right hand side of the system (3.1) to zero and we obtain two equilibrium points:

- The corruption-free equilibrium (CFE) point, $\mathbb{E}^0 = (S^0, E^0, C^0, J^0, R^0) = (\frac{\Pi}{\mu}, 0, 0, 0, 0)$.
- The endemic equilibrium (EE) point, $\mathbb{E}^* = (S^*, E^*, C^*, J^*, R^*)$,

where

$$\begin{aligned} S^* &= \frac{(\mu + \gamma)(\mu + \eta + \theta)}{\eta\beta_1 - a(\mu + \gamma)(\mu + \eta + \theta)}, \\ E^* &= \frac{(\mu + \gamma + \tau)(\mu + a\Pi)(\mathcal{R}_0 - 1)}{(\eta\beta_1 - a(\mu + \gamma + \tau)(\mu + \eta + \theta))}, \\ C^* &= \frac{\eta E^*}{(\mu + \gamma + \tau)}, \\ J^* &= \frac{\gamma \eta E^*}{(\mu + \alpha)(\mu + \gamma + \tau)}, \\ R^* &= \frac{(\tau\eta(\mu + \alpha) + \eta\gamma\alpha + \theta(\mu + \gamma + \tau)(\mu + \alpha))E^*}{\mu(\mu + \alpha)(\mu + \gamma + \tau)}. \end{aligned}$$

We used the next-generation matrix approach [14] to obtain the basic reproduction number $\mathcal{R}_0$, for the proposed system (3.1). To compute $\mathcal{R}_0$, consider the corrupt vector $X = (E, C, J)^T$, the general transition matrices are defined as:

$$F = \begin{bmatrix} \frac{\beta_1 SC}{1+aS} \\ 0 \\ 0 \end{bmatrix}, \quad V = \begin{bmatrix} (\mu + \eta + \theta)E \\ -\eta E + (\mu + \gamma + \tau)C \\ -\gamma C + (\mu + \alpha)J \end{bmatrix}. \quad (4.7)$$

The Jacobian matrices of F and V at $\mathbb{E}^0$ calculated as follows:

$$\mathcal{F} = \begin{bmatrix} 0 & \frac{\beta_1 S^0}{1+aS^0} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} \mu + \eta + \theta & 0 & 0 \\ -\eta & \mu + \gamma + \tau & 0 \\ 0 & -\gamma & \mu + \alpha \end{bmatrix}.$$

So, $\mathcal{R}_0$ is determined by calculating $\rho(\mathcal{FV}^{-1})$, where $\rho(Y)$ denotes the spectral radius of matrix Y . Therefore,

$$\mathcal{R}_0 = \frac{\eta\Pi\beta_1}{(\gamma + \mu + \tau)(\eta + \theta + \mu)(\mu + a\Pi)}.$$

The basic reproduction number $\mathcal{R}_0$ is a metric used to evaluate the transmission potential of corruption over time [17,27]. It represents the average number of additional individuals who adopt corrupt practices in a susceptible population due to the influence of a single corrupt individual. It provides a framework for analyzing and mitigating the spread of corruption which treated as social contagion within the systems. If $\mathcal{R}_0 < 1$, corruption will not spread and will eventually die out. Conversely, if $\mathcal{R}_0 > 1$, corruption will spread and persist. The higher value of $\mathcal{R}_0$, indicating a faster spread of corruption. When $\mathcal{R}_0 = 1$, the threshold condition arises, meaning corruption remains steady over time.

4.3. Stability analysis. We analyze the local and global stability of the equilibrium points $\mathbb{E}^0$, and $\mathbb{E}^*$ of the model (3.1). (see e.g. [37])

4.3.1. Local stability.

Theorem 4.3. *The CFE point $\mathbb{E}^0$ is locally asymptotically stable (LAS) if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.*

Proof. Evaluating the Jacobian matrix to linearize model (3.1) around the CFE point $\mathbb{E}^0$ results in:

$$J_{(S^0, E^0, C^0, I^0, R^0)} = \begin{bmatrix} -\mu & 0 & -\frac{\Pi\beta_1}{\mu+a\Pi} & 0 & 0 \\ 0 & -(\eta + \theta + \mu) & \frac{\Pi\beta_1}{\mu+a\Pi} & 0 & 0 \\ 0 & \eta & -(\gamma + \mu + \tau) & 0 & 0 \\ 0 & 0 & \gamma & -(\alpha + \mu) & 0 \\ 0 & \theta & \tau & \alpha & -\mu \end{bmatrix}.$$

From the above Jacobian matrix, we observe that three eigenvalues are negative $(-\mu, -\mu, -(\alpha + \mu))$ and the remaining eigenvalues can be calculated using the characteristic equation given below:

$$\lambda^2 + a_1\lambda + a_2 = 0, \quad (4.8)$$

where,

$$a_1 = (\eta + \theta + \gamma + 2\mu + \tau),$$

$$a_2 = (\mu + \eta + \theta)(\mu + \gamma + \tau)(1 - \mathcal{R}_0).$$

Clearly, $a_2 > 0$, whenever $\mathcal{R}_0 < 1$. Since $a_1 > 0$ and $a_2 > 0$, thus by the Routh-Hurwitz criteria, the CFE point $\mathbb{E}^0$ is LAS. Conversely, if $\mathcal{R}_0 > 1$, then $a_2 < 0$, implying the existence of a positive eigenvalue; hence, $\mathbb{E}^0$ is unstable.

Theorem 4.4. *The EE point $\mathbb{E}^*$ is LAS if $\mathcal{R}_0 > 1$ and unstable otherwise.*

Proof. The Jacobian of the system (3.1) around $\mathbb{E}^*$ is

$$J_{(S^*, E^*, C^*, I^*, R^*)} = \begin{bmatrix} -\mu + \frac{a\beta_1 C^* S^*}{(aS^*+1)^2} - \frac{\beta_1 C^*}{aS^*+1} & 0 & -\frac{\beta_1 S^*}{(1+aS^*)} & 0 & 0 \\ -\frac{a\beta_1 C^* S^*}{(aS^*+1)^2} + \frac{\beta_1 C^*}{aS^*+1} & -(\eta + \theta + \mu) & \frac{\beta_1 S^*}{1+aS^*} & 0 & 0 \\ 0 & \eta & -(\gamma + \mu + \tau) & 0 & 0 \\ 0 & 0 & \gamma & -(\alpha + \mu) & 0 \\ 0 & \theta & \tau & \alpha & -\mu \end{bmatrix}.$$

Based on this Jacobian matrix, we have two negative eigenvalues $-\mu$ and $-(\alpha + \mu)$. The remaining eigenvalues can be obtained by following characteristic equation:

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \quad (4.9)$$

in which

$$\begin{aligned} b_1 &= \frac{(\mu + k_1 + k_2)k_1k_2\beta_1\eta^2 + \eta(\beta_1\eta - ak_1k_2)(k_1k_2k_3(\mathcal{R}_0 - 1))}{\eta^2\beta_1k_1k_2}, \\ b_2 &= \frac{\eta(k_1 + k_2)[k_1k_2\beta_1\eta^2 + (\beta_1\eta - ak_1k_2)(k_1k_2k_3(\mathcal{R}_0 - 1))]}{\eta^2\beta_1k_1k_2}, \\ b_3 &= \frac{\eta(\beta_1\eta - ak_1k_2)(k_1k_2k_3(\mathcal{R}_0 - 1))}{\eta^2\beta_1}, \end{aligned}$$

with $k_1 = \mu + \eta + \theta$, $k_2 = \mu + \gamma + \tau$ and $k_3 = \Pi a + \mu$. If $\mathcal{R}_0 > 1$, then all coefficients b_i are positive for $i = 1, 2, 3$ and the condition $b_1b_2 > b_3$ is satisfied. Using Routh-Hurwitz criteria, the endemic equilibrium point $\mathbb{E}^*$ is LAS.

4.3.2. Global stability.

Theorem 4.5. The critical point $\mathbb{E}^0$ of system (3.1) is globally asymptotically stable (GAS) if $\mathcal{R}_0 \leq 1$.

Proof. We take the Lyapunov function

$$\mathfrak{Q}_g = \frac{S_0}{1+aS_0} \left(\frac{S}{S_0} - 1 - \ln \frac{S}{S_0} \right) + E + \frac{(\mu + \eta + \theta)}{\eta} C. \quad (4.10)$$

Applying the Caputo derivative D^ω on $\mathfrak{Q}_g$. Then from the system (3.1), we have

$$\begin{aligned} D^\omega \mathfrak{Q}_g &\leq \frac{1}{1+aS_0} \left(1 - \frac{S_0}{S} \right) \left(\Pi - \frac{\beta_1 SC}{1+aS} - \mu S \right) + \left[\frac{\beta_1 SC}{1+aS} - (\mu + \eta + \theta) E \right] \\ &\quad + \frac{\mu + \eta + \theta}{\eta} [\eta E - (\mu + \gamma + \tau) C]. \end{aligned} \quad (4.11)$$

After simplification we get

$$\begin{aligned} D^\omega \mathfrak{Q}_g &\leq \frac{1}{1+aS_0} \left(1 - \frac{S_0}{S} \right) \left(\Pi - \mu S \right) - \frac{\beta_1 SC}{(1+aS_0)(1+aS)} + \frac{\beta_1 S_0 C}{(1+aS_0)(1+aS)} \\ &\quad + \frac{\beta_1 SC}{(1+aS)} - \frac{(\mu + \eta + \theta)(\mu + \gamma + \tau)}{\eta} C, \end{aligned} \quad (4.12)$$

and

$$D^\omega \mathfrak{Q}_g \leq \frac{-\mu(S - S_0)^2}{S(1 + aS_0)} + \frac{(\mu + \eta + \theta)(\mu + \gamma + \tau)}{\eta} (\mathcal{R}_0 - 1)C. \quad (4.13)$$

Now if $\mathcal{R}_0 \leq 1$, then $D^\omega \mathfrak{Q}_g(t) \leq 0$, for all $S, C > 0$. If $S = S_0$ and $C = 0$ this implies that $D^\omega \mathfrak{Q}_g(t) = 0$. This holds that $\dot{C} = 0$. We can observe from model (3.1) as t tends to ∞ , we obtain

$$E \rightarrow 0, J \rightarrow 0, R \rightarrow 0. \quad (4.14)$$

Thus, by LaSalle's invariance principle [42, 43], the equilibrium $\mathbb{E}^0$ is GAS.

Theorem 4.6. *The equilibrium point $\mathbb{E}^*$ of the model (3.1) is GAS if $\mathcal{R}_0 > 1$.*

Proof. To prove the globally stability of equilibrium point $\mathbb{E}^*$, we consider a Lyapunov function $\mathfrak{Q} : \Omega \rightarrow \mathbb{R}$ defined by

$$\begin{aligned} \mathfrak{Q} = & \frac{S - S^* - S^* \ln(S/S^*)}{1 + aS^*} + E - E^* - E^* \ln(E/E^*) \\ & + \frac{b}{\eta} (C - C^* - C^* \ln(C/C^*)), \end{aligned} \quad (4.15)$$

where $b = \mu + \eta + \theta$. This function is nonnegative for $S, E, C > 0$ and vanishes only when $S = S^*$, $E = E^*$, and $C = C^*$. Applying the Caputo logarithmic Lyapunov inequality and substituting the equations for S, E , and C gives

$$\begin{aligned} D^\omega \mathfrak{Q} \leq & \frac{1}{1 + aS^*} \left(1 - \frac{S^*}{S}\right) \left(\Pi - \frac{\beta_1 SC}{1 + aS} - \mu S\right) \\ & + \left(1 - \frac{E^*}{E}\right) \left(\frac{\beta_1 SC}{1 + aS} - bE\right) \\ & + \frac{b}{\eta} \left(1 - \frac{C^*}{C}\right) (\eta E - (\mu + \gamma + \tau)C). \end{aligned} \quad (4.16)$$

At the endemic equilibrium,

$$\Pi = \mu S^* + \frac{\beta_1 S^* C^*}{1 + aS^*}, \quad \frac{\beta_1 S^* C^*}{1 + aS^*} = bE^*, \quad (\mu + \gamma + \tau)C^* = \eta E^*. \quad (4.17)$$

We first substitute the expression for Π and combine $\mu S^* - \mu S = \mu(S^* - S)$:

$$\begin{aligned} D^\omega \mathfrak{Q} \leq & \frac{1}{1 + aS^*} \left(1 - \frac{S^*}{S}\right) \left[\mu(S^* - S) + \frac{\beta_1 S^* C^*}{1 + aS^*} - \frac{\beta_1 SC}{1 + aS}\right] \\ & + \left(1 - \frac{E^*}{E}\right) \left(\frac{\beta_1 SC}{1 + aS} - bE\right) \\ & + \frac{b}{\eta} \left(1 - \frac{C^*}{C}\right) (\eta E - (\mu + \gamma + \tau)C). \end{aligned} \quad (4.18)$$

The term containing $\mu(S^* - S)$ is

$$\frac{1}{1 + aS^*} \left(1 - \frac{S^*}{S}\right) \mu(S^* - S) = -\frac{\mu(S - S^*)^2}{S(1 + aS^*)}.$$

Using the remaining identities in (4.17), we therefore obtain

$$\begin{aligned}
 D^\omega \mathfrak{Q} \leq & -\frac{\mu(S-S^*)^2}{S(1+aS^*)} \\
 & + \frac{1}{1+aS^*} \left(1 - \frac{S^*}{S}\right) \left(bE^* - \frac{\beta_1 SC}{1+aS}\right) \\
 & + \left(1 - \frac{E^*}{E}\right) \left(\frac{\beta_1 SC}{1+aS} - bE\right) \\
 & + b \left(1 - \frac{C^*}{C}\right) \left(E - E^* \frac{C}{C^*}\right).
 \end{aligned} \tag{4.19}$$

From $\beta_1 S^* C^* / (1 + aS^*) = bE^*$, it follows that

$$\frac{\beta_1 SC}{1+aS} = bE^* \frac{SC(1+aS^*)}{S^* C^* (1+aS)}.$$

Substituting this expression into (4.19) gives

$$\begin{aligned}
 D^\omega \mathfrak{Q} \leq & -\frac{\mu(S-S^*)^2}{S(1+aS^*)} \\
 & + \frac{bE^*}{1+aS^*} \left(1 - \frac{S^*}{S}\right) \left[1 - \frac{SC(1+aS^*)}{S^* C^* (1+aS)}\right] \\
 & + bE^* \left(1 - \frac{E^*}{E}\right) \left[\frac{SC(1+aS^*)}{S^* C^* (1+aS)} - \frac{E}{E^*}\right] \\
 & + bE^* \left(1 - \frac{C^*}{C}\right) \left[\frac{E}{E^*} - \frac{C}{C^*}\right].
 \end{aligned} \tag{4.20}$$

For the first product, use

$$\frac{1}{1+aS^*} \left(1 - \frac{S^*}{S}\right) = 1 - \frac{S^*(1+aS)}{S(1+aS^*)}.$$

Expanding the three products in (4.20) yields

$$\begin{aligned}
 D^\omega \mathfrak{Q} \leq & -\frac{\mu(S-S^*)^2}{S(1+aS^*)} \\
 & + bE^* \left[1 - \frac{S^*(1+aS)}{S(1+aS^*)} - \frac{SC(1+aS^*)}{S^* C^* (1+aS)} + \frac{C}{C^*}\right] \\
 & + bE^* \left[\frac{SC(1+aS^*)}{S^* C^* (1+aS)} - \frac{SC(1+aS^*)E^*}{S^* C^* (1+aS)E} - \frac{E}{E^*} + 1\right] \\
 & + bE^* \left[\frac{E}{E^*} - \frac{EC^*}{E^* C} - \frac{C}{C^*} + 1\right].
 \end{aligned} \tag{4.21}$$

The terms containing $SC(1+aS^*)/[S^* C^* (1+aS)]$, E/E^* , and C/C^* cancel between the three square brackets. The three constant terms add to three. Hence

$$\begin{aligned}
 D^\omega \mathfrak{Q} \leq & -\frac{\mu(S-S^*)^2}{S(1+aS^*)} \\
 & + bE^* \left[3 - \frac{S^*(1+aS)}{S(1+aS^*)} - \frac{SC(1+aS^*)E^*}{S^* C^* (1+aS)E} - \frac{EC^*}{E^* C}\right].
 \end{aligned} \tag{4.22}$$

The three positive fractions subtracted from 3 have product

$$\frac{S^*(1+aS)}{S(1+aS^*)} \cdot \frac{SC(1+aS^*)E^*}{S^*C^*(1+aS)E} \cdot \frac{EC^*}{E^*C} = 1.$$

By the arithmetic–geometric mean inequality, their sum is at least 3. Thus $D^\omega \mathfrak{Q} \leq 0$. Equality in the bound requires $S = S^*$ and equality in the arithmetic–geometric mean inequality. These conditions initially give $E/E^* = C/C^*$. For a solution to remain in this equality set, the S equation and its equilibrium identity require $C = C^*$, which then gives $E = E^*$. The equations for J and R determine their equilibrium values on the largest invariant set. Therefore the largest invariant subset of the equality set is $\{\mathbb{E}^*\}$. By LaSalle’s invariance principle [43, 44], the endemic equilibrium $\mathbf{E}^*$ is GAS in the positive feasible region.

4.4. Sensitivity analysis. Here, we examine the sensitivity indexes of $\mathcal{R}_0$ with respect to the model parameters in Table 1. For this purpose, we have calculated the normalized forward sensitivity index [45] using the formula described below:

$$Y_\nu^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \nu} \times \frac{\nu}{\mathcal{R}_0}, \quad (4.23)$$

where ν denotes the model basic parameter. This analysis helps identify key parameters, how they affect $\mathcal{R}_0$, and the information can be used to design strategies to slow down the spread of the epidemic.

From (4.23), the sensitivity indexes of $\mathcal{R}_0$ with respect to some of the parameters are given below:

$$\begin{aligned} Y_\Pi^{\mathcal{R}_0} &= \frac{\mu}{\mu+a\Pi} > 0, \\ Y_{\beta_1}^{\mathcal{R}_0} &= 1 > 0, \\ Y_\eta^{\mathcal{R}_0} &= \frac{\mu+\theta}{\eta+\mu+\theta} > 0, \\ Y_\theta^{\mathcal{R}_0} &= -\frac{\theta}{\eta+\mu+\theta} < 0, \\ Y_\gamma^{\mathcal{R}_0} &= -\frac{\gamma}{\gamma+\mu+\tau} < 0, \\ Y_\tau^{\mathcal{R}_0} &= -\frac{\tau}{\gamma+\mu+\tau} < 0, \\ Y_a^{\mathcal{R}_0} &= -\frac{a\Pi}{\mu+a\Pi} < 0. \end{aligned} \quad (4.24)$$

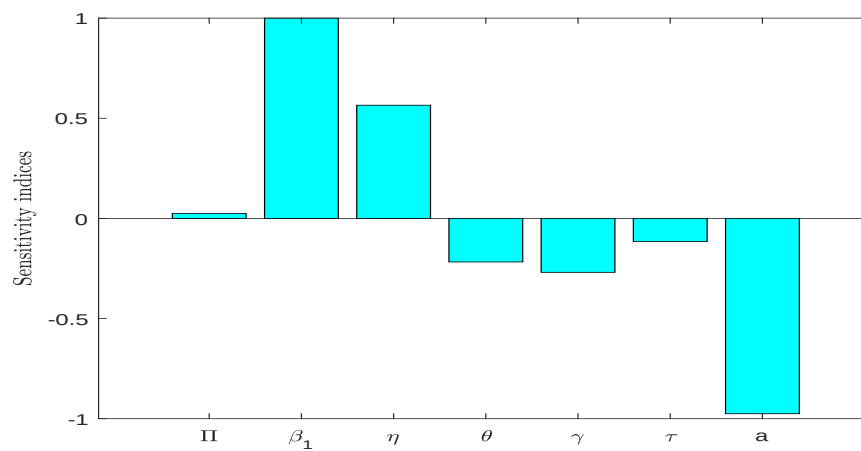
Table 2, displays the sensitivity indexes of $\mathcal{R}_0$, using the parameter values from Table 1. From the calculated indexes, it is clear that a 10% increase in transmission rate β_1 raises the value of $\mathcal{R}_0$ by 10%. Similarly, the 10% rise in the parameter η leads to an increase in $\mathcal{R}_0$ by 5.652%, while a 10% increase in awareness parameter a leads to a decline in $\mathcal{R}_0$ by 9.752%.

5. SIMULATION RESULTS

In this section, we present the numerical simulations and discuss the results for the corruption system (3.1). We employed the numerical technique based on the Lagrange polynomial interpolation as described in [46]. The results are graphically displayed considering the parameter values in Table 1 with the initial values $S(0) = 25, E(0) = 10, C(0) = 10, J(0) = 0$, and $R(0) = 5$. The computations were carried out using MATLAB R2022b software.

TABLE 2. The sensitivity index of parameters related to $\mathcal{R}_0$.

Parameters	Sensitivity index
Π	0.0248
β_1	+1
η	0.5652
θ	-0.2174
γ	-0.2692
τ	-0.1154
a	-0.9752

FIGURE 2. Sensitivity indexes of $\mathcal{R}_0$ to different parameters.

In Fig. 3, the impact of memory index (i.e., fractional order ω) is presented graphically. The decline in susceptible class is observed over time for all values of ω . The slopes corresponding to different fractional orders exhibit noticeable variations, reflecting the model's sensitivity to the fractional parameter. Furthermore, the system shows an initial increase in the exposed population class, which eventually declines after a certain period. A significant portion of the population is often encouraged to participate in corrupt activities during the first few days, as evidenced by the intense increase in corrupt people at higher fractional orders caused by an increase in the prevalence of poverty and desire in society. However, over time, the percentage of corrupt individuals declines as some exit this class through punishment, education, and religious instruction. The lower fractional orders allow for more effective containment of corruption dynamics. The graph shows that the number of people in jail is steadily increasing. This could be linked to the highly secretive nature of corruption, which allows corrupt individuals to remain undetected within the community. Lower fractional order values lead to a slower increase in the jail population. The use of available information is enhanced by lower fractional orders, allowing for a more thorough

application. In contrast, the higher recovery rates are related to higher fractional orders. There has been a significant increase in the number of people who have managed to recover from corruption, influenced by various factors such as education, religious convictions, and periods of incarceration.

We have demonstrated the behavior of the system by taking into account some key model parameters. Our simulations explore how different parameters affect the system dynamics. Fig. 4 illustrates a notable decline in the peaks of the exposed and corrupt curves when the implementation of awareness control strategies within the population is intensified. On the other hand, when such control measures do not exist, the number of corrupt individuals increases. This observation demonstrates the effectiveness of mass education and law enforcement strategies in reducing corrupt populations over a fixed period. The differing slope behaviors observed at various fractional orders highlight the benefits of using a fractional-order approach to accurately model the transmission of corruption. The corrupt population increases rapidly for the higher value of the parameter η , as illustrated in Fig. 5.

Fig. 6 illustrates how various factors influence the basic reproduction number. Fig. 6(a) presents the sensitivity of $\mathcal{R}_0$ to the exposure rate of corruption and the level of awareness among susceptible individuals. According to Table 2, β_1 has a positive impact on $\mathcal{R}_0$, while a has a negative impact. Consequently, it is clear from Fig. 6(a) that an increase in awareness in the population leads to a decrease in $\mathcal{R}_0$. Fig. 6(b) illustrates the reproduction number $\mathcal{R}_0$ sensitivity to changes in the awareness parameter a and the corruption rate η . It is evident that η has a quick impact on $\mathcal{R}_0$. The impact of a can be observed from the plot. Fig. 6(c) illustrates the $\mathcal{R}_0$ sensitivity against the corruption rate η and the probability of recovery from corruption τ . Fig. 6(d) shows the influence of transmission rate β_1 and the rate of recovery from corruption τ on $\mathcal{R}_0$. As shown, increasing β_1 and decreasing τ will produce greater $\mathcal{R}_0$. To minimize corruption transmission, lowering the transmission rate is important. A similar behavior and definition of the associated $\mathcal{R}_0$ in Fig. 6(e) can be interpreted based on the parameters used for the analysis.

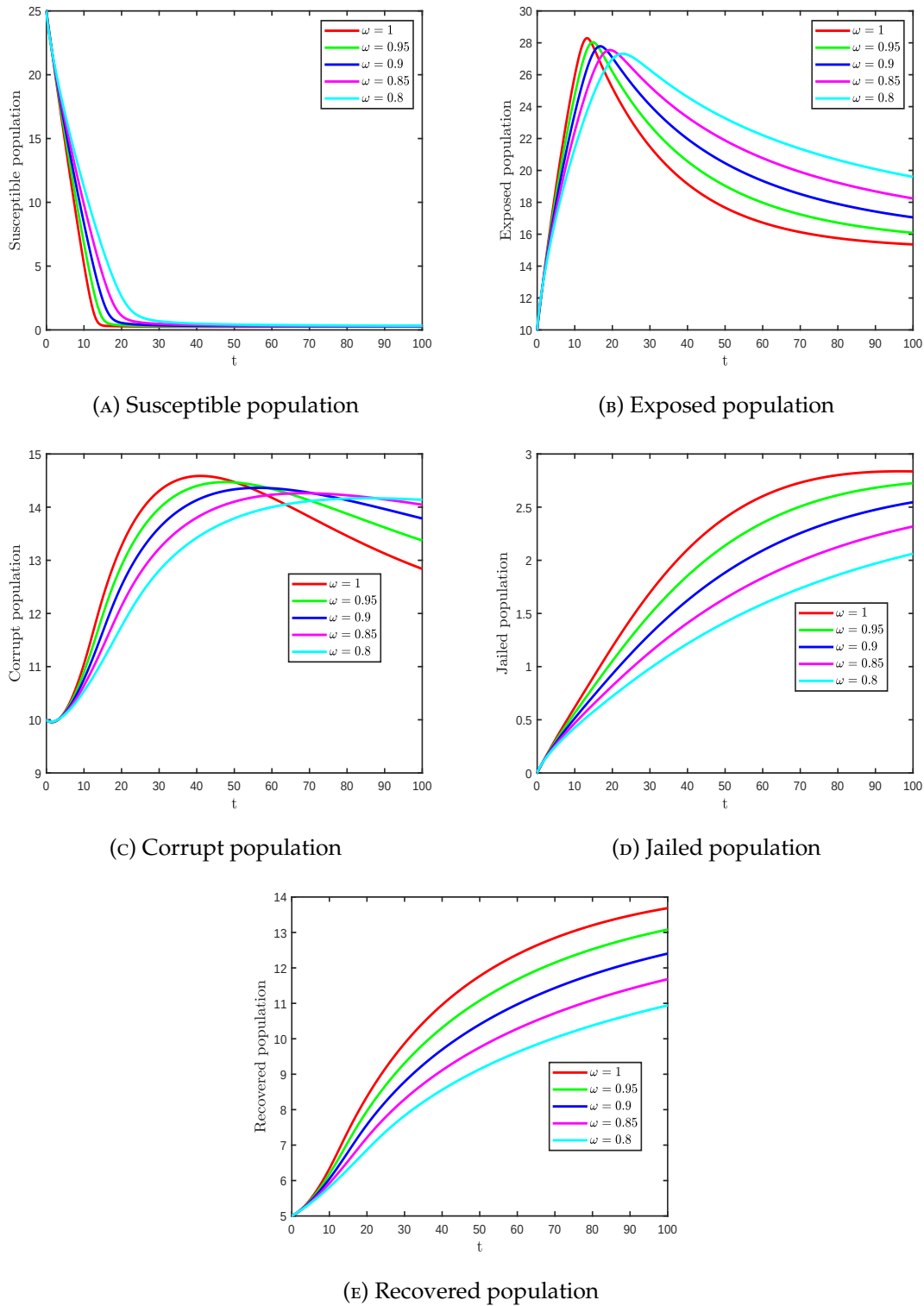
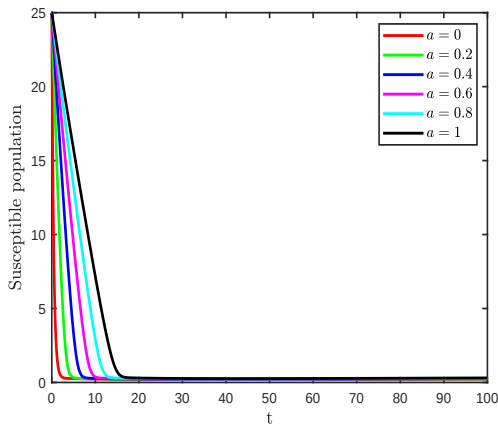
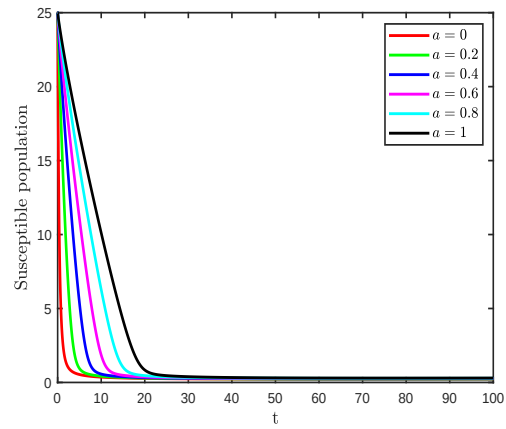
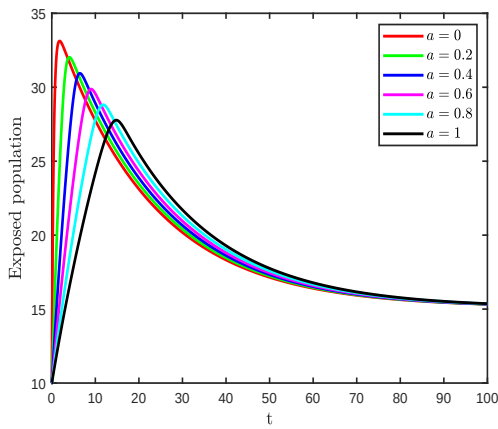
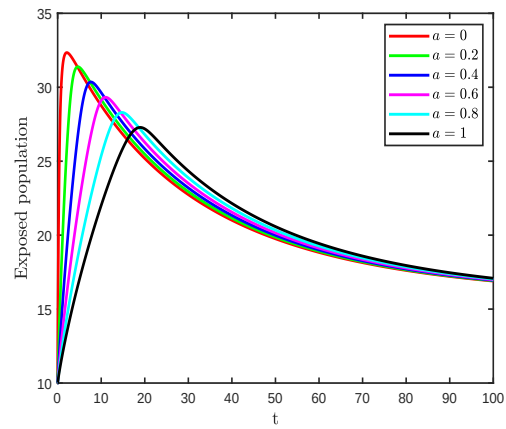
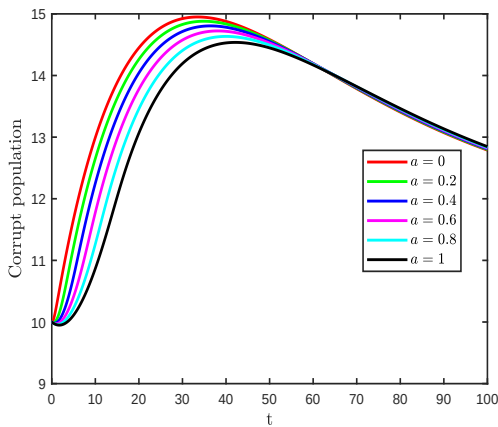
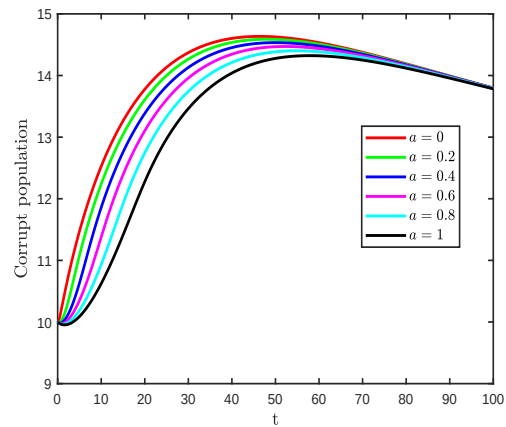
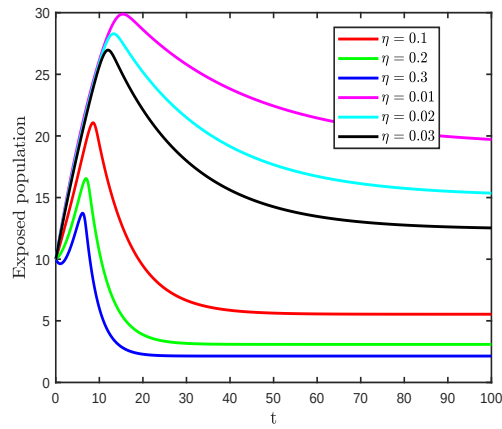
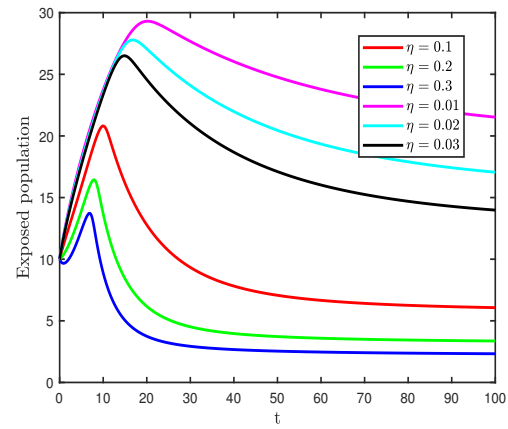
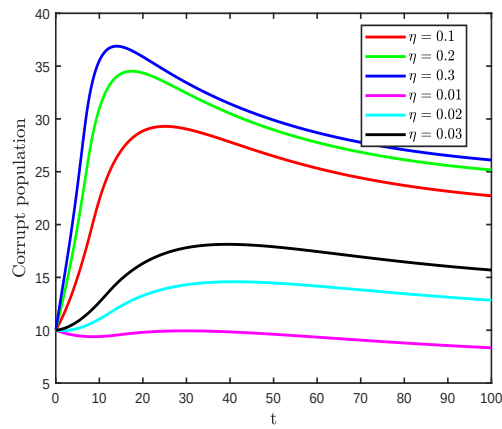
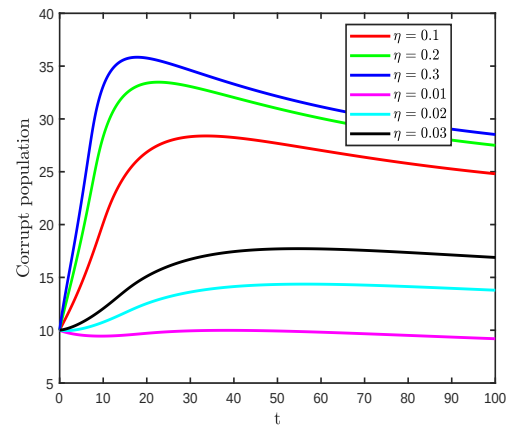


FIGURE 3. Simulation of the corruption model (3.1) using various fractional orders.

(A) Fractional order $\omega = 1$ (B) Fractional order $\omega = 0.9$ (C) Fractional order $\omega = 1$ (D) Fractional order $\omega = 0.9$ (E) Fractional order $\omega = 1$ (F) Fractional order $\omega = 0.9$ FIGURE 4. The impact of a on susceptible, exposed, and corrupt individuals.

(A) Fractional order $\omega = 1$ (B) Fractional order $\omega = 0.9$ (C) Fractional order $\omega = 1$ (D) Fractional order $\omega = 0.9$ FIGURE 5. The impact of η on exposed and corrupted individuals.

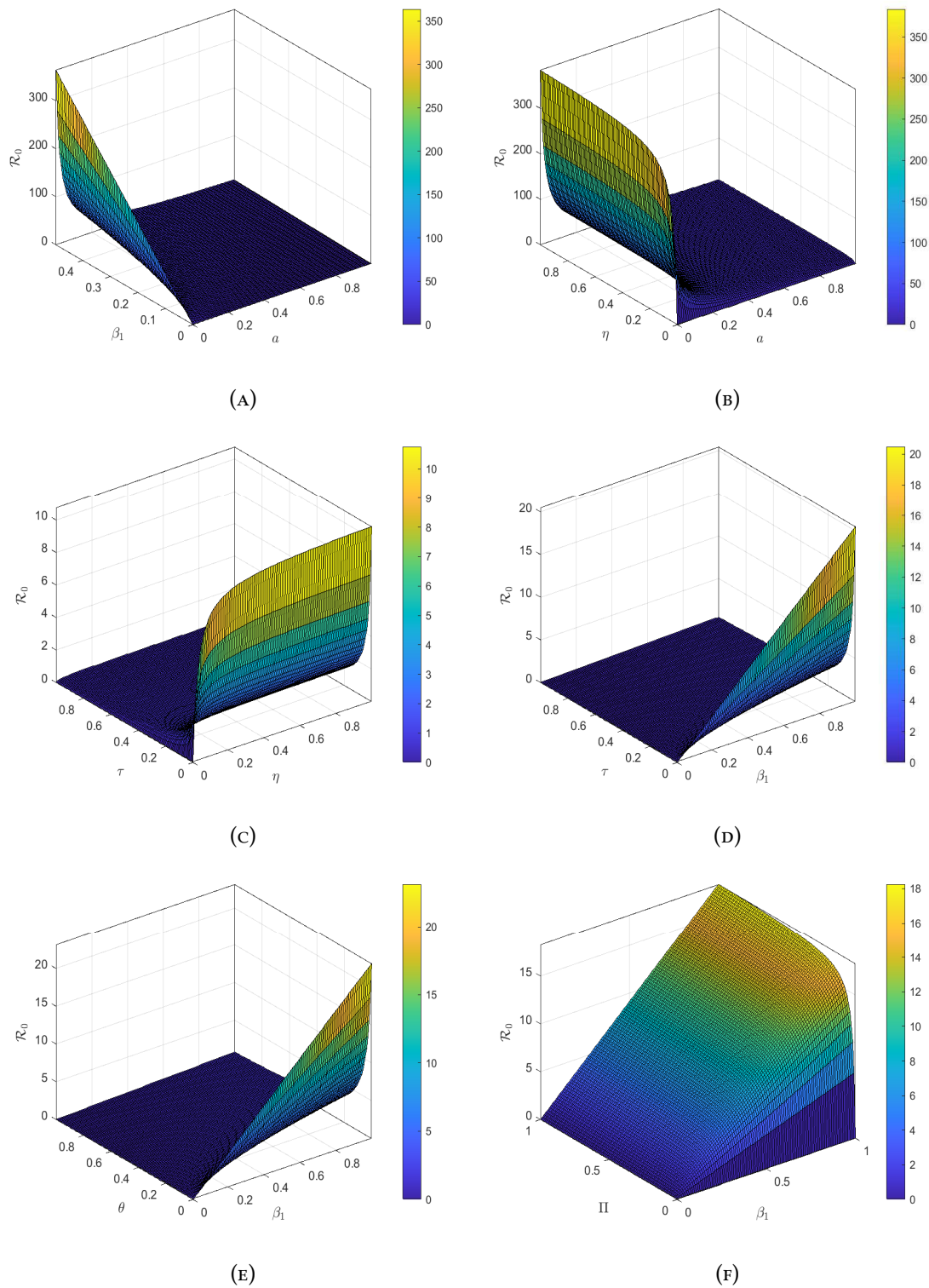


FIGURE 6. Variation of $\mathcal{R}_0$ with different parameters.

6. CONCLUSION

This study presents a deterministic mathematical model with the saturation incidence rate to understand the spread of corruption. We analyzed the corruption model through a fractional approach, utilizing fractional derivatives. We use the Caputo derivative due to its ability to integrate conventional initial conditions into the formulation of the problem. We calculated key criteria such as the positivity, boundedness, and the existence and uniqueness of solutions. The criteria for determining the stability of equilibrium points in the proposed model are examined. The Lyapunov functional method is utilized to demonstrate global stability. To examine the behavior of the proposed model, we present several numerical simulation showing how the dynamics change when varying model parameters and the fractional orders. Our analysis highlights the role of awareness as a preventive measure, demonstrating its significant impact on decreasing the occurrence of corruption within the population. We observe that as awareness in the population increases, the exposed population decreases, leading to a noticeable reduction in the corrupt population. This highlights the significant role of awareness in preventing corruption from growing within society. Significant changes occur with the introduction of fractional order, which demonstrates the value of the proposed mechanism in making more efficient use of available information. Additionally, the sensitivity analysis was conducted to evaluate the impact of parameters on model. In conclusion, the creation of a fractional-order framework allowed for a more rigorous analysis of corruption dynamics.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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