

## Upper and Lower $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -Continuity

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**Abstract.** This paper presents new concepts of continuous multifunctions defined between an ideal topological space and a bitopological space, namely upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions. Furthermore, several characterizations and some properties concerning upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions are discussed.

### 1. INTRODUCTION

In 1965, Lee [19] studied the concept of semiconnected functions. Kohli [17] introduced the notion of  $s$ -continuous functions and investigated some characterizations of semilocally connected spaces in terms of  $s$ -continuous functions. The concept of  $s$ -continuity as a generalization of continuity and semiconnectedness. Furthermore, Kohli [16] introduced the concepts of  $s$ -regular spaces and completely  $s$ -regular spaces and proved that  $s$ -regularity and complete  $s$ -regularity are preserved under certain  $s$ -continuous functions. On the other hand, the present author introduced and investigated the notions of  $(\tau_1, \tau_2)$ -continuous functions [3], almost  $(\tau_1, \tau_2)$ -continuous functions [2] and weakly  $(\tau_1, \tau_2)$ -continuous functions [1]. Lipski [20] extended the concept of  $s$ -continuous functions to the setting of multifunctions. Popa [24] introduced the concept of precontinuous multifunctions and showed that  $H$ -almost continuity and precontinuity are equivalent for multifunctions. Ewert and Lipski [10] introduced and investigated the concept of  $s$ -quasi-continuous multifunctions. Popa and Noiri [23] introduced a new class of multifunctions called  $s$ -precontinuous multifunctions is a generalization of  $s$ -continuous multifunctions and precontinuous multifunctions. Popa and Noiri [22] introduced and studied the notion of  $s$ - $\beta$ -continuous

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multifunctions. Furthermore, Popa and Noiri [21] introduced and investigated the concept of  $s$ - $m$ -continuous multifunctions as multifunctions defined on a set satisfying some minimal conditions. Pue-on et al. [26] presented two classes of continuous multifunctions defined between bitopological spaces, namely upper  $(\tau_1, \tau_2)$ -continuous multifunctions and lower  $(\tau_1, \tau_2)$ -continuous multifunctions. Klanarong et al. [15] introduced and investigated the notions of upper almost  $(\tau_1, \tau_2)$ -continuous multifunctions and lower almost  $(\tau_1, \tau_2)$ -continuous multifunctions. Thongmoon et al. [27] introduced and studied the concepts of upper weakly  $(\tau_1, \tau_2)$ -continuous multifunctions and lower weakly  $(\tau_1, \tau_2)$ -continuous multifunctions. Khampakdee et al. [14] introduced classes of continuous multifunctions defined between an ideal topological space and a bitopological space, namely upper  $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower  $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions. Viriyapong et al. [28] introduced and studied the notions of upper almost  $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower almost  $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions. Pue-on et al. [25] introduced and investigated the concepts of upper weakly  $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower weakly  $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions. Quite recently, Chiangpradit et al. [7] presented new classes of continuous multifunctions defined between bitopological spaces, called upper  $s$ - $(\tau_1, \tau_2)$ -continuous multifunctions and lower  $s$ - $(\tau_1, \tau_2)$ -continuous multifunctions. In this paper, we introduce the concepts of upper  $s$ - $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower  $s$ - $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions. We also investigate several characterizations of upper  $s$ - $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower  $s$ - $\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions.

## 2. PRELIMINARIES

Throughout the present paper, spaces  $(X, \tau_1, \tau_2)$  and  $(Y, \sigma_1, \sigma_2)$  (or simply  $X$  and  $Y$ ) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let  $A$  be a subset of a bitopological space  $(X, \tau_1, \tau_2)$ . The closure of  $A$  and the interior of  $A$  with respect to  $\tau_i$  are denoted by  $\tau_i\text{-Cl}(A)$  and  $\tau_i\text{-Int}(A)$ , respectively, for  $i = 1, 2$ . A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called  $\tau_1\tau_2$ -closed [6] if  $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$ . The complement of a  $\tau_1\tau_2$ -closed set is called  $\tau_1\tau_2$ -open. The intersection of all  $\tau_1\tau_2$ -closed sets of  $X$  containing  $A$  is called the  $\tau_1\tau_2$ -closure [6] of  $A$  and is denoted by  $\tau_1\tau_2\text{-Cl}(A)$ . The union of all  $\tau_1\tau_2$ -open sets of  $X$  contained in  $A$  is called the  $\tau_1\tau_2$ -interior [6] of  $A$  and is denoted by  $\tau_1\tau_2\text{-Int}(A)$ .

**Lemma 2.1.** [6] *Let  $A$  and  $B$  be subsets of a bitopological space  $(X, \tau_1, \tau_2)$ . For the  $\tau_1\tau_2$ -closure, the following properties hold:*

- (1)  $A \subseteq \tau_1\tau_2\text{-Cl}(A)$  and  $\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Cl}(A)) = \tau_1\tau_2\text{-Cl}(A)$ .
- (2) If  $A \subseteq B$ , then  $\tau_1\tau_2\text{-Cl}(A) \subseteq \tau_1\tau_2\text{-Cl}(B)$ .
- (3)  $\tau_1\tau_2\text{-Cl}(A)$  is  $\tau_1\tau_2$ -closed.
- (4)  $A$  is  $\tau_1\tau_2$ -closed if and only if  $A = \tau_1\tau_2\text{-Cl}(A)$ .
- (5)  $\tau_1\tau_2\text{-Cl}(X - A) = X - \tau_1\tau_2\text{-Int}(A)$ .

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called  $(\tau_1, \tau_2)$ - $r$ -open [31] (resp.  $(\tau_1, \tau_2)$ - $s$ -open [5],  $(\tau_1, \tau_2)$ - $p$ -open [5],  $(\tau_1, \tau_2)$ - $\beta$ -open [5],  $\alpha(\tau_1, \tau_2)$ -open [30]) if  $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$  (resp.

$A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$ ,  $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ ,  $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$ ,  $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$ . The complement of a  $(\tau_1, \tau_2)r$ -open (resp.  $(\tau_1, \tau_2)s$ -open,  $(\tau_1, \tau_2)p$ -open,  $(\tau_1, \tau_2)\beta$ -open,  $\alpha(\tau_1, \tau_2)$ -open) set is called  $(\tau_1, \tau_2)r$ -closed (resp.  $(\tau_1, \tau_2)s$ -closed,  $(\tau_1, \tau_2)p$ -closed,  $(\tau_1, \tau_2)\beta$ -closed,  $\alpha(\tau_1, \tau_2)$ -closed). A bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $\tau_1\tau_2$ -connected [6] if  $X$  cannot be written as the union of two nonempty disjoint  $\tau_1\tau_2$ -open sets.

An ideal  $\mathcal{I}$  on a topological space  $(X, \tau)$  is a nonempty collection of subsets of  $X$  satisfying the following properties: (1)  $A \in \mathcal{I}$  and  $B \subseteq A$  imply  $B \in \mathcal{I}$ ; (2)  $A \in \mathcal{I}$  and  $B \in \mathcal{I}$  imply  $A \cup B \in \mathcal{I}$ . A topological space  $(X, \tau)$  with an ideal  $\mathcal{I}$  on  $X$  is called an ideal topological space and is denoted by  $(X, \tau, \mathcal{I})$ . For an ideal topological space  $(X, \tau, \mathcal{I})$  and a subset  $A$  of  $X$ ,  $A^*(\mathcal{I})$  is defined as follows:

$$A^*(\mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I} \text{ for every open neighbourhood } U \text{ of } x\}.$$

In case there is no chance for confusion,  $A^*(\mathcal{I})$  is simply written as  $A^*$ . In [18],  $A^*$  is called the local function of  $A$  with respect to  $\mathcal{I}$  and  $\tau$  and  $\text{Cl}^*(A) = A^* \cup A$  defines a Kuratowski closure operator for a topology  $\tau^*(\mathcal{I})$  finer than  $\tau$ . A subset  $A$  is said to be  $\star$ -closed [12] if  $A^* \subseteq A$ . The interior of a subset  $A$  in  $(X, \tau^*(\mathcal{I}))$  is denoted by  $\text{Int}^*(A)$ . A subset  $A$  of an ideal topological space  $(X, \tau, \mathcal{I})$  is said to be *semi* $\star$ - $\mathcal{I}$ -open [9] (resp. *semi*- $\mathcal{I}$ -open [11]) if  $A \subseteq \text{Cl}(\text{Int}^*(A))$  (resp.  $A \subseteq \text{Cl}^*(\text{Int}(A))$ ). The complement of a *semi* $\star$ - $\mathcal{I}$ -open (resp. *semi*- $\mathcal{I}$ -open) set is said to be *semi* $\star$ - $\mathcal{I}$ -closed [9] (resp. *semi*- $\mathcal{I}$ -closed [11]).

By a multifunction  $F : X \rightarrow Y$ , we mean a point-to-set correspondence from  $X$  into  $Y$ , and we always assume that  $F(x) \neq \emptyset$  for all  $x \in X$ . For a multifunction  $F : X \rightarrow Y$ , we shall denote the upper and lower inverse of a set  $B$  of  $Y$  by  $F^+(B)$  and  $F^-(B)$ , respectively, that is,  $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$  and  $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$ . In particular,  $F^-(y) = \{x \in X \mid y \in F(x)\}$  for each point  $y \in Y$ . For each  $A \subseteq X$ ,  $F(A) = \cup_{x \in A} F(x)$ .

### 3. UPPER AND LOWER $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -CONTINUOUS MULTIFUNCTIONS

In this section, we introduce the notions of upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions. Moreover, some characterizations of upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions and lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunctions are considered.

**Definition 3.1.** A multifunction  $F : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous at  $x \in X$  if for each  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  containing  $F(x)$  and having  $\sigma_1\sigma_2$ -connected complement, there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $F(U) \subseteq V$ . A multifunction  $F : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous if  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous at each point  $x$  of  $X$ .

**Theorem 3.1.** For a multifunction  $F : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma_1, \sigma_2)$ , the following properties are equivalent:

- (1)  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous;
- (2)  $F^+(V)$  is  $\star$ -open in  $X$  for every  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  having  $\sigma_1\sigma_2$ -connected complement;
- (3)  $F^-(K)$  is  $\star$ -closed in  $X$  for every  $\sigma_1\sigma_2$ -connected  $\sigma_1\sigma_2$ -closed set  $K$  of  $Y$ ;

- (4)  $Cl^*(F^-(B)) \subseteq F^-(\sigma_1\sigma_2-Cl(B))$  for every subset  $B$  of  $Y$  having the  $\sigma_1\sigma_2$ -connected  $\sigma_1\sigma_2$ -closure;  
 (5)  $F^+(\sigma_1\sigma_2-Int(B)) \subseteq Int^*(F^+(B))$  for every subset  $B$  of  $Y$  such that  $Y - \sigma_1\sigma_2-Int(B)$  is  $\sigma_1\sigma_2$ -connected.

*Proof.* (1)  $\Rightarrow$  (2): Let  $V$  be any  $\sigma_1\sigma_2$ -open set of  $Y$  having  $\sigma_1\sigma_2$ -connected complement and

$$x \in F^+(V).$$

Then, there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $F(U) \subseteq V$ . Therefore, we have  $x \in U \subseteq F^+(V)$  and hence  $x \in Int^*(F^+(V))$ . Thus,  $F^+(V) \subseteq Int^*(F^+(V))$  and so  $F^+(V)$  is  $\star$ -open in  $X$ .

(2)  $\Rightarrow$  (3): The proof follows immediately from the fact that  $F^+(Y - B) = X - F^-(B)$  for every subset  $B$  of  $Y$ .

(3)  $\Rightarrow$  (4): Let  $B$  be any subset of  $Y$  having the  $\sigma_1\sigma_2$ -connected  $\sigma_1\sigma_2$ -closure. Thus by (3),

$$Cl^*(F^-(B)) \subseteq Cl^*(F^-(\sigma_1\sigma_2 Cl(B))) = F^-(\sigma_1\sigma_2 Cl(B)).$$

(4)  $\Rightarrow$  (5): Let  $B$  be any subset of  $Y$  such that  $Y - \sigma_1\sigma_2-Int(B)$  is  $\sigma_1\sigma_2$ -connected. By (4), we have

$$\begin{aligned} X - Int^*(F^+(B)) &= Cl^*(X - F^+(B)) \\ &= Cl^*(F^-(Y - B)) \\ &\subseteq F^-(\sigma_1\sigma_2 Cl(Y - B)) \\ &= F^-(Y - \sigma_1\sigma_2 Int(B)) \\ &= X - F^+(\sigma_1\sigma_2 Int(B)) \end{aligned}$$

and hence  $F^+(\sigma_1\sigma_2-Int(B)) \subseteq Int^*(F^+(B))$ .

(5)  $\Rightarrow$  (1): Let  $x \in X$  and  $V$  be any  $\sigma_1\sigma_2$ -open set of  $Y$  containing  $F(x)$  and having  $\sigma_1\sigma_2$ -connected complement. By (5),  $F^+(V) = F^+(\sigma_1\sigma_2-Int(V)) \subseteq Int^*(F^+(V))$ . Then, there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $U \subseteq F^+(V)$ . Thus,  $F(U) \subseteq V$ . This shows that  $F$  is upper  $s-\tau^*(\sigma_1, \sigma_2)$ -continuous.  $\square$

**Definition 3.2.** A multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be lower  $s-\tau^*(\sigma_1, \sigma_2)$ -continuous at  $x \in X$  if for each  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  such that  $F(x) \cap V \neq \emptyset$  and having  $\sigma_1\sigma_2$ -connected complement, there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $F(z) \cap V \neq \emptyset$  for each  $z \in U$ . A multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be lower  $s-\tau^*(\sigma_1, \sigma_2)$ -continuous if  $F$  is lower  $s-\tau^*(\sigma_1, \sigma_2)$ -continuous at each point  $x$  of  $X$ .

**Theorem 3.2.** For a multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$ , the following properties are equivalent:

- (1)  $F$  is lower  $s-\tau^*(\sigma_1, \sigma_2)$ -continuous;
- (2)  $F^-(V)$  is  $\star$ -open in  $X$  for every  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  having  $\sigma_1\sigma_2$ -connected complement;
- (3)  $F^+(K)$  is  $\star$ -closed in  $X$  for every  $\sigma_1\sigma_2$ -connected  $\sigma_1\sigma_2$ -closed set  $K$  of  $Y$ ;
- (4)  $Cl^*(F^+(B)) \subseteq F^+(\sigma_1\sigma_2-Cl(B))$  for every subset  $B$  of  $Y$  having the  $\sigma_1\sigma_2$ -connected  $\sigma_1\sigma_2$ -closure;

- (5)  $F^-(\sigma_1\sigma_2\text{-Int}(B)) \subseteq \text{Int}^*(F^-(B))$  for every subset  $B$  of  $Y$  such that  $Y - \sigma_1\sigma_2\text{-Int}(B)$  is  $\sigma_1\sigma_2$ -connected.

*Proof.* The proof is similar to that of Theorem 3.1.  $\square$

**Corollary 3.1.** A multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous if  $F^-(V)$  is  $\star$ -closed in  $X$  for every  $\sigma_1\sigma_2$ -open  $\sigma_1\sigma_2$ -connected set  $V$  of  $Y$ .

*Proof.* Let  $V$  be any  $\sigma_1\sigma_2$ -open set of  $Y$  having  $\sigma_1\sigma_2$ -connected complement. Then,  $Y - V$  is  $\sigma_1\sigma_2$ -connected and  $\sigma_1\sigma_2$ -closed. By the hypothesis,  $F^-(Y - V)$  is  $\star$ -closed in  $X$ . Thus,  $F^+(V)$  is  $\star$ -open in  $X$  and by Theorem 3.1,  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous.  $\square$

**Corollary 3.2.** A multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous if  $F^+(V)$  is  $\star$ -closed in  $X$  for every  $\sigma_1\sigma_2$ -open  $\sigma_1\sigma_2$ -connected set  $V$  of  $Y$ .

*Proof.* The proof is similar to that of Corollary 3.1.  $\square$

**Definition 3.3.** A function  $f : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous at  $x \in X$  if for each  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  containing  $f(x)$  and having  $\sigma_1\sigma_2$ -connected complement, there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$ . A function  $f : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous if  $f$  is  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous at each point  $x$  of  $X$ .

**Corollary 3.3.** For a function  $f : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$ , the following properties are equivalent:

- (1)  $f$  is  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous;
- (2)  $f^{-1}(V)$  is  $\star$ -open in  $X$  for every  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  having  $\sigma_1\sigma_2$ -connected complement;
- (3)  $f^{-1}(K)$  is  $\star$ -closed in  $X$  for every  $\sigma_1\sigma_2$ -connected  $\sigma_1\sigma_2$ -closed set  $K$  of  $Y$ ;
- (4)  $\text{Cl}^*(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$  for every subset  $B$  of  $Y$  having the  $\sigma_1\sigma_2$ -connected  $\sigma_1\sigma_2$ -closure;
- (5)  $f^{-1}(\sigma_1\sigma_2\text{-Int}(B)) \subseteq \text{Int}^*(f^{-1}(B))$  for every subset  $B$  of  $Y$  such that  $Y - \sigma_1\sigma_2\text{-Int}(B)$  is  $\sigma_1\sigma_2$ -connected.

**Definition 3.4.** [6] A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is said to be:

- (1)  $\tau_1\tau_2$ -paracompact if every cover of  $A$  by  $\tau_1\tau_2$ -open sets of  $X$  is refined by a cover of  $A$  which consists of  $\tau_1\tau_2$ -open sets of  $X$  and is  $\tau_1\tau_2$ -locally finite in  $X$ ;
- (2)  $\tau_1\tau_2$ -regular if for each  $x \in A$  and each  $\tau_1\tau_2$ -open set  $U$  of  $X$  containing  $x$ , there exists a  $\tau_1\tau_2$ -open set  $V$  of  $X$  such that  $x \in V \subseteq \tau_1\tau_2\text{-Cl}(V) \subseteq U$ .

**Lemma 3.1.** [6] If  $A$  is a  $\tau_1\tau_2$ -regular  $\tau_1\tau_2$ -paracompact set of a bitopological space  $(X, \tau_1, \tau_2)$  and  $U$  is a  $\tau_1\tau_2$ -open neighbourhood of  $A$ , then there exists a  $\tau_1\tau_2$ -open set  $V$  of  $X$  such that  $A \subseteq V \subseteq \tau_1\tau_2\text{-Cl}(V) \subseteq U$ .

For a multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$ , by  $\text{Cl}F_i : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  [29] we denote a multifunction defined as follows:  $\text{Cl}F_i(x) = \sigma_1\sigma_2\text{-Cl}(F(x))$  for each  $x \in X$ .

**Lemma 3.2.** [29] If  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is a multifunction such that  $F(x)$  is  $\sigma_1\sigma_2$ -regular and  $\sigma_1\sigma_2$ -paracompact for each  $x \in X$ , then  $\text{Cl}F_i^+(V) = F^+(V)$  for each  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$ .

**Theorem 3.3.** Let  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  be a multifunction such that  $F(x)$  is  $\sigma_1\sigma_2$ -paracompact and  $\sigma_1\sigma_2$ -regular for each  $x \in X$ . Then,  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous if and only if  $\text{Cl}F_i$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous.

*Proof.* We put  $G = \text{Cl}F_i$ . Suppose that  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous. Let  $x \in X$  and  $V$  be any  $\sigma_1\sigma_2$ -open set of  $Y$  containing  $G(x)$  and having  $\sigma_1\sigma_2$ -connected complement. By Lemma 3.2, we have  $x \in G^+(V) = F^+(V)$  and hence there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $F(U) \subseteq V$ . Since  $F(z)$  is  $\sigma_1\sigma_2$ -paracompact and  $\sigma_1\sigma_2$ -regular for each  $z \in U$ , by Lemma 3.1 there exists a  $\sigma_1\sigma_2$ -open set  $W$  of  $Y$  such that  $F(z) \subseteq W \subseteq \sigma_1\sigma_2\text{-Cl}(W) \subseteq V$ ; hence  $G(z) \subseteq \sigma_1\sigma_2\text{-Cl}(W) \subseteq V$  for each  $z \in U$ . Thus,  $G(U) \subseteq V$ . This shows that  $G$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous.

Conversely, suppose that  $G$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous. Let  $x \in X$  and  $V$  be any  $\sigma_1\sigma_2$ -open set of  $Y$  containing  $F(x)$  and having  $\sigma_1\sigma_2$ -connected complement. By Lemma 3.2, we have  $x \in F^+(V) = G^+(V)$  and hence  $G(x) \subseteq V$ . There exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $G(U) \subseteq V$ . Thus,  $U \subseteq G^+(V) = F^+(V)$  and so  $F(U) \subseteq V$ . This shows that  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous.  $\square$

**Lemma 3.3.** [29] For a multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$ ,  $\text{Cl}F_i^-(V) = F^-(V)$  for each  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$ .

**Theorem 3.4.** A multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous if and only if  $\text{Cl}F_i : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous.

*Proof.* By using Lemma 3.3 this can be shown similarly to that of Theorem 3.3.  $\square$

The  $\star$ -frontier [13] of a subset  $A$  of an ideal topological space  $(X, \tau, \mathcal{J})$ , denoted by  $\text{fr}^*(A)$ , is defined by  $\text{fr}^*(A) = \text{Cl}^*(A) \cap \text{Cl}^*(X - A) = \text{Cl}^*(A) - \text{Int}^*(A)$ .

**Theorem 3.5.** The set of all points  $x$  of  $X$  at which a multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is not upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous is identical with the union of the  $\star$ -frontier of the upper inverse images of  $\sigma_1\sigma_2$ -open sets containing  $F(x)$  and having  $\sigma_1\sigma_2$ -connected complement.

*Proof.* Let  $x$  be a point of  $X$  at which  $F$  is not upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous. Then, there exists a  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  containing  $F(x)$  and having  $\sigma_1\sigma_2$ -connected complement such that

$$U \cap (X - F^+(V)) \neq \emptyset$$

for every  $\star$ -open set  $U$  of  $X$  containing  $x$ . Therefore, we have  $x \in \text{Cl}^*(X - F^+(V))$  and hence  $x \in \text{fr}^*(F^+(V))$ .

Conversely, suppose that  $V$  is a  $\sigma_1\sigma_2$ -open set of  $Y$  containing  $F(x)$  and having  $\sigma_1\sigma_2$ -connected complement such that  $x \in \text{fr}^*(F^+(V))$ . If  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous at  $x \in X$ , then there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $U \subseteq F^+(V)$ ; hence  $x \in \text{Int}^*(F^+(V))$ . This is a contradiction and so  $F$  is not upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous at  $x$ .  $\square$

**Theorem 3.6.** *The set of all points  $x$  of  $X$  at which a multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is not lower  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous is identical with the union of the  $\star$ -frontier of the lower inverse images of  $\sigma_1\sigma_2$ -open sets meeting  $F(x)$  and having  $\sigma_1\sigma_2$ -connected complement.*

*Proof.* The proof is similar to that of Theorem 3.5.  $\square$

For a multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$ , the graph  $G(F) = \{(x, F(x)) \mid x \in X\}$  is said to be strongly  $\tau^*$ -closed if for each  $(x, y) \in (X \times Y) - G(F)$ , there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  and a  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  containing  $y$  such that  $[U \times \sigma_1\sigma_2\text{-Cl}(V)] \cap G(F) = \emptyset$ .

**Lemma 3.4.** *A multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  has a strongly  $\tau^*$ -closed graph if and only if for each  $(x, y) \in (X \times Y) - G(F)$ , there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  and a  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  containing  $y$  such that  $F(U) \cap \sigma_1\sigma_2\text{-Cl}(V) = \emptyset$ .*

*Proof.* This proof is obvious.  $\square$

Recall that a bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $(\tau_1, \tau_2)$ -regular [8] if for each  $\tau_1\tau_2$ -closed set  $F$  and each point  $x \in X - F$ , there exist disjoint  $\tau_1\tau_2$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subseteq V$ .

**Definition 3.5.** [7] *A bitopological space  $(X, \tau_1, \tau_2)$  is said to be locally  $\tau_1\tau_2$ -connected if for each  $x \in X$  and each  $\tau_1\tau_2$ -open set  $G$  of  $X$  containing  $x$ , there exists a  $\tau_1\tau_2$ -open  $\tau_1\tau_2$ -connected set  $V$  such that  $x \in V \subseteq G$ .*

**Theorem 3.7.** *Let  $(Y, \sigma_1, \sigma_2)$  be a  $(\sigma_1, \sigma_2)$ -regular locally  $\sigma_1\sigma_2$ -connected space. If*

$$F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$$

*is an upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous multifunction such that  $F(x)$  is  $\sigma_1\sigma_2$ -closed for each  $x \in X$ , then  $G(F)$  is strongly  $\tau^*$ -closed.*

*Proof.* Let  $(x, y) \in (X \times Y) - G(F)$ . Then,  $y \in Y - F(x)$ . Since  $(Y, \sigma_1, \sigma_2)$  is  $(\sigma_1, \sigma_2)$ -regular, there exist disjoint  $\sigma_1\sigma_2$ -open sets  $V$  and  $V'$  of  $Y$  such that  $F(x) \subseteq V$  and  $y \in V'$ . Moreover, since  $(Y, \sigma_1, \sigma_2)$  is locally  $\sigma_1\sigma_2$ -connected, there exists a  $\sigma_1\sigma_2$ -open  $\sigma_1\sigma_2$ -connected set  $W$  of  $Y$  such that  $y \in W \subseteq \sigma_1\sigma_2\text{-Cl}(W) \subseteq V'$ . Since  $F$  is upper  $s\text{-}\tau^*(\sigma_1, \sigma_2)$ -continuous and  $Y - \sigma_1\sigma_2\text{-Cl}(W)$  is a  $\sigma_1\sigma_2$ -open set having  $\sigma_1\sigma_2$ -connected complement, there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $F(U) \subseteq Y - \sigma_1\sigma_2\text{-Cl}(W)$ . Thus,  $F(U) \cap \sigma_1\sigma_2\text{-Cl}(W) = \emptyset$  and by Lemma 3.4,  $G(F)$  is strongly  $\tau^*$ -closed.  $\square$

**Definition 3.6.** [14] *A multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is called lower  $\tau^*(\sigma_1, \sigma_2)$ -continuous if for each  $x \in X$  and each  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$  such that  $F(x) \cap V \neq \emptyset$ , there exists a  $\star$ -open set  $U$  of  $X$  containing  $x$  such that  $F(z) \cap V \neq \emptyset$  for each  $z \in U$ .*

**Lemma 3.5.** [14] *For a multifunction  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$ , the following properties are equivalent:*

- (1)  $F$  is lower  $\tau^*(\sigma_1, \sigma_2)$ -continuous;

- (2)  $F^-(V)$  is  $\star$ -open in  $X$  for every  $\sigma_1\sigma_2$ -open set  $V$  of  $Y$ ;
- (3)  $F^+(K)$  is  $\star$ -closed in  $X$  for every  $\sigma_1\sigma_2$ -closed set  $K$  of  $Y$ ;
- (4)  $Cl^\star(F^+(B)) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(B))$  for every subset  $B$  of  $Y$ ;
- (5)  $F(Cl^\star(A)) \subseteq \sigma_1\sigma_2\text{-Cl}(F(A))$  for every subset  $A$  of  $X$ ;
- (6)  $F^-(\sigma_1\sigma_2\text{-Int}(B)) \subseteq Int^\star(F^-(B))$  for every subset  $B$  of  $Y$ .

**Theorem 3.8.** If  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is lower  $s\text{-}\tau^\star(\sigma_1, \sigma_2)$ -continuous and  $F(A)$  is  $\sigma_1\sigma_2$ -connected for every subset  $A$  of  $X$ , then  $F$  is lower  $\tau^\star(\sigma_1, \sigma_2)$ -continuous.

*Proof.* Let  $A$  be any subset of  $X$ . Since  $\sigma_1\sigma_2\text{-Cl}(F(A))$  is  $\sigma_1\sigma_2$ -closed and  $\sigma_1\sigma_2$ -connected, by Theorem 3.2 we have  $F^+(\sigma_1\sigma_2\text{-Cl}(F(A))) = Cl^\star(F^+(\sigma_1\sigma_2\text{-Cl}(F(A))))$  and

$$A \subseteq F^+(F(A)) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(F(A))).$$

Thus,  $F(Cl^\star(A)) \subseteq \sigma_1\sigma_2\text{-Cl}(F(A))$ . It follows from Lemma 3.5 that  $F$  lower  $\tau^\star(\sigma_1, \sigma_2)$ -continuous.  $\square$

Recall that a subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $\tau_1\tau_2$ -clopen [6] if  $A$  is both  $\tau_1\tau_2$ -open and  $\tau_1\tau_2$ -closed. A bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $\tau_1\tau_2$ -connected [6] if  $X$  cannot be written as the union of two nonempty disjoint  $\tau_1\tau_2$ -open sets. An ideal topological space  $(X, \tau, \mathcal{J})$  is called  $\star\text{-}\mathcal{J}$ -connected [4] if  $X$  cannot be written as the union of two nonempty disjoint  $\star$ -open sets.

**Definition 3.7.** [7] A bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $s\text{-}\tau_1\tau_2$ -connected if  $X$  cannot be written as the union of two nonempty disjoint  $\tau_1\tau_2$ -open sets having  $\tau_1\tau_2$ -connected complement.

**Theorem 3.9.** If  $F : (X, \tau, \mathcal{J}) \rightarrow (Y, \sigma_1, \sigma_2)$  is an upper or lower  $s\text{-}\tau^\star(\sigma_1, \sigma_2)$ -continuous surjective multifunction such that  $F(x)$  is  $\sigma_1\sigma_2$ -connected for each  $x \in X$  and  $(X, \tau, \mathcal{J})$  is  $\star\text{-}\mathcal{J}$ -connected, then  $(Y, \sigma_1, \sigma_2)$  is  $s\text{-}\sigma_1\sigma_2$ -connected.

*Proof.* Suppose that  $(Y, \sigma_1, \sigma_2)$  is not  $s\text{-}\sigma_1\sigma_2$ -connected. There exist nonempty  $\sigma_1\sigma_2$ -open sets  $U$  and  $V$  of  $Y$  having  $\sigma_1\sigma_2$ -connected complement such that  $U \cap V = \emptyset$  and  $U \cup V = Y$ . Since  $F(x)$  is  $\sigma_1\sigma_2$ -connected for each  $x \in X$ , either  $F(x) \subseteq U$  or  $F(x) \subseteq V$ . If  $x \in F^+(U \cup V)$ , then  $F(x) \subseteq U \cup V$  and hence  $x \in F^+(U) \cup F^+(V)$ . Moreover, since  $F$  is surjective, there exist  $x$  and  $y$  in  $X$  such that  $F(x) \subseteq U$  and  $F(y) \subseteq V$ ; hence  $x \in F^+(U)$  and  $y \in F^+(V)$ . Therefore, we obtain the following:

- (1)  $F^+(U) \cup F^+(V) = F^+(U \cup V) = X$ ;
- (2)  $F^+(U) \cap F^+(V) = F^+(U \cap V) = \emptyset$ ;
- (3)  $F^+(U) \neq \emptyset$  and  $F^+(V) \neq \emptyset$ .

Next, we shall show that  $F^+(U)$  and  $F^+(V)$  are  $\star$ -open in  $X$ . (i) Let  $F$  be upper  $s\text{-}\tau^\star(\sigma_1, \sigma_2)$ -continuous. By Theorem 3.1,  $F^+(U)$  and  $F^+(V)$  are  $\star$ -open in  $X$ . (ii) Let  $F$  be lower  $s\text{-}\tau^\star(\sigma_1, \sigma_2)$ -continuous. Since  $V$  is a  $\sigma_1\sigma_2$ -clopen set with  $\sigma_1\sigma_2$ -connected complement, by Theorem 3.2  $F^+(V)$  is  $\star$ -closed in  $X$ . Therefore,  $F^+(U)$  is  $\star$ -open in  $X$ . Similarly, we obtain  $F^+(V)$  is  $\star$ -open in  $X$ . Thus,  $(X, \tau_1, \tau_2)$  is not  $\star\text{-}\mathcal{J}$ -connected.  $\square$

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