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Innerness of Continuous Derivations on Algebras of Measurable Operators Associated with Finite Real W*-Algebras

Ulugbek Karimov[1,*], **Khasanbek Nazarov**[2]

[1] *National university of Uzbekistan, Uzbekistan*
[2] *"Tashkent Institute of Irrigation and Agricultural Mechanization Engineers" National Research University, Uzbekistan*

[*] *Corresponding author:* karimovulugbe07@gmail.com

Abstract. Derivations on algebras of measurable, locally measurable, and τ-measurable operators associated with real type I von Neumann algebras are investigated. By carefully adapting the methods from the complex case and taking into account the specific algebraic and topological features of real operator algebras, a complete characterization of all derivations on the algebras under consideration is obtained. These results generalize known theorems for complex von Neumann algebras to the real setting and contribute to a deeper understanding of derivations on algebras of unbounded operators associated with real operator algebras.

1. Introduction

Derivations on operator algebras acting on a Hilbert space have been extensively studied. In particular, it is well known that every derivation on a von Neumann algebra is inner, whereas a C^*-algebra may admit non-inner derivations. Recently, considerable attention has been devoted to the study of derivations on algebras of unbounded operators.

In papers [1–5], a complete description of all derivations on algebras of measurable, locally measurable, and τ-measurable operators associated with type I von Neumann algebras was obtained. Papers [6–9] investigate derivations on algebras of measurable and locally measurable operators associated with von Neumann algebras, focusing on their continuity and structural properties with respect to natural measure-type topologies.

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In the present paper, a real analogue of these results is considered. More precisely, we obtain descriptions of derivations on certain algebras of measurable, locally measurable, and τ-measurable operators associated with real von Neumann algebras of type I.

2. Preliminaries

Let M be a $*$-subalgebra of $B(H)$, the algebra of all bounded linear operators on a complex Hilbert space H. The set $M' = \{x \in B(H) : xy = yx,\ \forall y \in M\}$ is called the commutant of M. The set $Z(M) = M \cap M'$ is called the center of M. A *-subalgebra M satisfying the condition $M = M''$ is called a W^*-algebra, where M'' denotes the double commutant of M. Equivalently, M is weakly closed and contains the identity operator (i.e., $\mathbf{1} \in M$). W^*-algebras are also known as von Neumann algebras. A W^*-algebra M is called a factor if its center is trivial. The algebra M is called discrete, or of type I, if for every nonzero central projection z there exists a nonzero abelian projection q such that $q \le z$. The algebra M is said to be of type I_∞ if M is a properly infinite von Neumann algebra of type I.

A functional $\tau : M_+ \to [0, \infty]$ is called a trace on M_+ if

(1) $\tau(T + S) = \tau(T) + \tau(S)$ for all $T, S \in M_+$;
(2) $\tau(\lambda T) = \lambda\tau(T)$ for every $T \in M_+$ and $\lambda \ge 0$ (with the convention $0 \cdot \infty = 0$);
(3) $\tau(U^*TU) = \tau(T)$ for every $T \in M_+$ and every unitary operator $U \in M$.

A trace τ is called *finite* if $\tau(T) < \infty$ for every $T \in M_+$; *semifinite* if $\tau(T) = \sup\{\tau(S) : 0 \le S \le T,\ \tau(S) < \infty\}$, for every $T \in M_+$; *faithful* if $\tau(T) = 0$ for $T \in M_+$ implies $T = 0$; *normal* if from $T_\alpha \uparrow T$ with $T, T_\alpha \in M_+$ it follows that $\tau(T_\alpha) \uparrow \tau(T)$.

A linear subspace $D \subseteq H$ is said to be associated with the algebra M (denoted by $D\eta M$) if $U(D) \subseteq D$ for every unitary operator $U \in M'$. If D is a closed linear subspace of H and P_D denotes the orthogonal projection onto D, then $D\eta M \iff P_D \in P(M)$, where $P(M)$ denotes the complete lattice of projections of M.

A linear subspace $D \subseteq H$ is called *strongly dense* in H with respect to M if $D\eta M$ and there exists a sequence of projections $\{P_n\}_{n=1}^\infty \subset P(M)$ such that $P_n \uparrow \mathbf{1}$, $P_n(H) \subseteq D$ and $P_n^\perp$ is a finite projection for every $n = 1, 2, \ldots$. In this case one says that the strongly dense linear subspace D is determined by the sequence of projections $\{P_n\}_{n=1}^\infty$. Since $P_n \uparrow \mathbf{1}$, every strongly dense linear subspace is dense in H.

A linear operator T with domain $D(T)$ acting on the Hilbert space H is called associated with M (denoted by $T\eta M$) if $UT \subseteq TU$, for every unitary operator $U \in M'$, that is, $D(T)\eta M$ and $UT\xi = TU\xi$, $\forall\, \xi \in D(T)$.

It is easy to see that a bounded operator $T \in B(H)$ is associated with M if and only if $T \in M$. A closed linear operator T acting on H is called measurable with respect to M if $T\eta M$ and its domain $D(T)$ is strongly dense in H.

We denote by $S(M)$ the set of all operators measurable with respect to M. Clearly, $M \subseteq S(M)$. A closed linear operator T acting on H is called locally measurable with respect to M if $T\eta M$ and there exists a sequence of central projections $\{Z_n\}_{n=1}^{\infty} \subseteq P(Z(M))$ such that $Z_n \uparrow \mathbf{1}$, $TZ_n \in S(M)$, $n = 1, 2, \ldots$.

We denote by $LS(M)$ the set of all operators locally measurable with respect to M. It is clear that $S(M) \subseteq LS(M)$ and $S(M) = LS(M)$ whenever M is a factor.

3. Derivations on the Algebra $LS(R)$

Let A be an algebra over the complex field. A linear mapping $D : A \to A$ is called a derivation if it satisfies the Leibniz rule

$$D(xy) = D(x)y + xD(y), \qquad x, y \in A.$$

Every element $a \in A$ defines a derivation D_a on A by

$$D_a(x) = ax - xa, \qquad x \in A.$$

Such derivations are called *inner derivations*.

If the element a defining the derivation D_a belongs to a larger algebra B containing A, then D_a is called a *spatial derivation*.

In the special case when A is commutative, all inner derivations vanish and hence are trivial. One of the central problems in the theory of derivations is to determine when derivations are automatically inner or spatial, and whether non-inner derivations exist, especially nontrivial derivations on commutative algebras.

Recall that a real $*$-subalgebra $R \subset B(H)$ is called a real W^*-algebra if it is weakly closed, $\mathbf{1} \in R$, and $R \cap iR = \{0\}$. Real W^*-algebras are also known as real von Neumann algebras. Let R be a real von Neumann algebra and let $M = R + iR$. Consider the algebra $S(R)$ of all measurable operators associated with the real von Neumann algebra R. It was proved (see [10]) that

$$S(M) = S(R) + iS(R),$$

and similarly

$$LS(M) = LS(R) + iLS(R).$$

Below we give a complete description of derivations on the algebra $LS(R)$ of all locally measurable operators associated with a type I real von Neumann algebra R.

It is clear that if a derivation D on $LS(R)$ is inner, then it is Z-linear, i.e.,

$$D(\lambda x) = \lambda D(x), \qquad \lambda \in Z, \quad x \in LS(R),$$

where Z denotes the center of R. The following result shows that the converse statement is also true.

Theorem 3.1. *Let R be a type I real von Neumann algebra with center Z. Then every Z-linear derivation on the algebra $LS(R)$ is inner.*

Proof. Since

$$LS(R,\tau)+iLS(R,\tau)=LS(R+iR,\tau),$$

the derivation D can be extended by Z_c-linearity to a derivation $\overline{D}$ on $LS(R+iR)$ by setting

$$\overline{D}(x+iy)=D(x)+iD(y),\qquad x,y\in LS(R),$$

where

$$Z_c=Z+iZ$$

is the center of $LS(R+iR)$.

Since every derivation on $LS(R+iR)$ is inner (see [2, Theorem 2.1]), the derivation $\overline{D}$ is inner. Hence there exists an element

$$z=a+ib,\qquad a,b\in LS(R),$$

such that

$$\overline{D}(x+iy)=[z,x+iy]=z(x+iy)-(x+iy)z,$$

for all $x,y\in LS(R)$.

Therefore,

$$D(x)=\overline{D}(x)=[z,x]=[a+ib,x]=[a,x]+i[b,x].$$

Since $x\in LS(R)$ and $D(x)\in LS(R)$, it follows that

$$[b,x]=0.$$

Hence

$$D(x)=[a,x],$$

and therefore D is inner. ◻

Theorem 3.2. *If R is a type I_∞ real von Neumann algebra, then every derivation on the algebra $LS(R)$ is inner.*

The proof of Theorem 3.2 is analogous to that of Theorem 3.1.

Suppose that A is a commutative algebra and let $M_n(A)$ be the algebra of all $n\times n$ matrices over A. If $e_{ij},\qquad i,j=1,\ldots,n$, are the matrix units in $M_n(A)$, then every element $x\in M_n(A)$ can be written in the form

$$x=\sum_{i,j=1}^{n}\lambda_{ij}e_{ij},\qquad \lambda_{ij}\in A.$$

Let $\delta:A\to A$ be a derivation constructed in the paper [6] (see also [7]). Define

$$D_\delta\left(\sum_{i,j=1}^{n}\lambda_{ij}e_{ij}\right)=\sum_{i,j=1}^{n}\delta(\lambda_{ij})e_{ij}. \tag{1}$$

Then D_δ is a well-defined linear operator on $M_n(A)$. Moreover, D_δ is a derivation on $M_n(A)$, and its restriction to the center of $M_n(A)$ coincides with the derivation δ.

Now let R be a homogeneous real von Neumann algebra of type I_n, $n \in \mathbb{N}$, with center Z. Since $Z_c = Z + iZ$ is the center of M, it follows that $S(Z_c) = S(Z) + iS(Z)$ and $LS(Z_c) = LS(Z) + iLS(Z)$. Let $\delta : S(Z) \to S(Z)$ be a derivation constructed in the paper [6] and let D_δ be the derivation on the algebra $M_n(S(Z))$ defined by (1). Extend D_δ to

$$M_n(S(Z_c)) = M_n(S(Z)) + iM_n(S(Z))$$

by setting

$$\overline{D}_\delta\left(\sum_{i,j=1}^{n} \lambda_{ij}e_{ij}\right) = \sum_{i,j=1}^{n} \overline{\delta}(\lambda_{ij})e_{ij}, \tag{2}$$

where

$$\overline{\delta} : M \to M$$

is the extension of δ to $S(Z_c)$, defined by $\overline{\delta}(x + iy) = \delta(x) + i\delta(y)$.

It is clear that D_δ is a derivation on $M_n(S(Z))$, and $\overline{D}_\delta$ is a derivation on $M_n(S(Z_c))$.

The next lemma describes the structure of derivations on the algebra of measurable operators associated with homogeneous real von Neumann algebras of type I_n, $n \in \mathbb{N}$.

Lemma 3.1. *Let R be a homogeneous real von Neumann algebra of type I_n, $n \in \mathbb{N}$. Then every derivation D on the algebra $LS(R)$ admits a unique decomposition*

$$D = D_a + D_\delta,$$

where D_a is an inner derivation implemented by an element $a \in LS(R)$.

Proof. The proof is very similar to that of Lemma 3.3 in [2]; therefore, we only sketch the argument.

Let D be an arbitrary derivation on the algebra $LS(R) \cong M_n(S(Z))$. Consider the restriction $\delta = D|_{S(Z)}$ of D to the center $S(Z)$ of this algebra, and let D_δ be the derivation on $M_n(S(Z))$ constructed by (1). Set $D_1 = D - D_\delta$. Then for every $\lambda \in S(Z)$ we have $D_1(\lambda) = 0$, that is, D_1 vanishes identically on $S(Z)$. Hence D_1 is Z-linear, and by Theorem 3.1 it follows that D_1 is an inner derivation. Therefore $D_1 = D_a$, for some $a \in M_n(S(Z))$, and consequently $D = D_a + D_\delta$.

Suppose now that

$$D = D_{a_1} + D_{\delta_1} = D_{a_2} + D_{\delta_2}.$$

Then

$$D_{a_1} - D_{a_2} = D_{\delta_2} - D_{\delta_1}.$$

Since $D_{a_1} - D_{a_2}$ vanishes on the center of $M_n(S(Z))$, it follows that $D_{\delta_2} - D_{\delta_1}$ also vanishes on the center. Hence $\delta_1 = \delta_2$, and therefore $D_{a_1} = D_{a_2}$. Thus the decomposition is unique. □

Remark 3.1. *Under the assumptions of Lemma 3.1 , the derivation D can be extended to $LS(M)$ by*

$$\overline{D}(x + iy) = D(x) + iD(y), \qquad x, y \in LS(R).$$

By Lemma 3.1 (see also [2]), $\overline{D} = \overline{D}_a + \overline{D}_\delta$, where $\overline{D}_a = [a, \cdot],\ a \in LS(M)$, and $\overline{D}_\delta$ is the derivation defined by (2).

From the proof of Lemma 3.1 it follows that

$$\overline{D}_\delta\big|_{LS(Z)} = D_\delta,$$

and if $a = a_1 + ia_2$, $a_i \in LS(R)$, then

$$\overline{D}_a\big|_{LS(R)} = D_{a_1}.$$

Consequently,

$$D = D_{a_1} + D_\delta.$$

Now let R be an arbitrary finite real von Neumann algebra of type I with center Z. Then there exists a family of central projections

$$\{z_n\}_{n\in F}, \qquad F \subseteq \mathbb{N},$$

such that

$$\sup_{n\in F} z_n = \mathbf{1},$$

and

$$R \cong \bigoplus_{n\in F} z_n R,$$

where each $z_n R$ is a real von Neumann algebra of type I_n.

Analogously to Proposition 1.1 (see [2]), one can prove that

$$LS(R) \cong \prod_{n\in F} LS(z_n R).$$

Let D be a derivation on $LS(R)$, and let δ be its restriction to the center $S(Z)$. Since δ maps each

$$z_n S(Z) \cong Z(LS(z_n R))$$

into itself, it induces a derivation δ_n on $z_n S(Z)$ for every $n \in F$.

Let D_{δ_n} be the derivation on the matrix algebra

$$M_n\Big(z_n Z(LS(R))\Big) \cong LS(z_n R)$$

defined by (1). Define

$$D_\delta\Big(\{x_n\}_{n\in F}\Big) = \{D_{\delta_n}(x_n)\}, \qquad \{x_n\}_{n\in F} \in LS(R). \tag{3}$$

It is easy to verify that the mapping D_δ is a derivation on $LS(R)$.

Lemma 3.1 now implies the following result.

Lemma 3.2. Let R be a finite real von Neumann algebra of type I. Then every derivation D on the algebra $LS(R)$ admits a unique decomposition

$$D = D_a + D_\delta,$$

where D_a is an inner derivation implemented by an element $a \in LS(R)$, and D_δ is the derivation defined by (3).

We now consider derivations on the algebra $LS(R)$ of locally measurable operators associated with an arbitrary type I real von Neumann algebra R.

Assume that R is a type I real von Neumann algebra. Then there exists a central projection $z_0 \in R$ such that

(1) z_0R is a finite real von Neumann algebra;
(2) $z_0^{\perp}R$ is a real von Neumann algebra of type I_∞.

Let D be a derivation on $LS(R)$ and let δ be its restriction to the center $Z(S)$. By Theorem 3.2,

$$z_0^{\perp}D$$

is inner, and therefore

$$z_0^{\perp}\delta = 0,$$

that is,

$$\delta = z_0\delta.$$

Let D_δ be the derivation on $z_0LS(R)$ defined by (3), and consider its extension to

$$LS(R) = z_0LS(R) \oplus z_0^{\perp}LS(R),$$

defined by

$$D_\delta(x_1 + x_2) := D_\delta(x_1), \qquad x_1 \in z_0LS(R), \quad x_2 \in z_0^{\perp}LS(R). \tag{4}$$

The following theorem gives the general form of derivations on the algebra $LS(R)$.

Combining Lemma 3.2 with Theorem 3.2, we obtain the following result.

Theorem 3.3. *Let R be a type I real von Neumann algebra. Then every derivation D on the algebra $LS(R)$ admits a unique decomposition*

$$D = D_a + D_\delta,$$

where D_a is an inner derivation implemented by an element $a \in LS(R)$, and D_δ is a derivation of the form (4) *generated by a derivation δ on the center of $LS(R)$.*

4. Derivations on the Algebra $S(R)$

In this section, we describe derivations on the algebra $S(R)$ of measurable operators associated with a type I real von Neumann algebra R. Let A be an arbitrary subalgebra of $LS(R)$ containing R.

Suppose that $D : A \to LS(R)$ is a derivation. We shall show that D can be extended to a derivation on the whole algebra $LS(R)$. Since R is of type I, for every element $x \in LS(R)$ there exists a sequence $\{z_n\}_{n=1}^{\infty}$ of mutually orthogonal central projections such that

$$\bigvee_{n\in\mathbb{N}} z_n = \mathbf{1}, \qquad z_nx \in R, \quad n \in \mathbb{N}.$$

Define

$$\widetilde{D}(x) = \sum_{n\geq 1} z_nD(z_nx). \tag{5}$$

Since every derivation $D : A \to LS(R)$ vanishes identically on the central projections of R, formula (5) defines a well-defined derivation

$$\widetilde{D} : LS(R) \to LS(R)$$

which coincides with D on A.

More precisely, the Z-linearity of D on A implies the Z-linearity of $\widetilde{D}$, and therefore, by Theorem 3.1, the derivation $\widetilde{D}$ is inner on $LS(R)$. Hence D is spatial on A, i.e., there exists an element $a \in LS(R)$ such that

$$D(x) = ax - xa, \qquad x \in A.$$

Thus we obtain the following result.

Theorem 4.1. *Let R be a type I real von Neumann algebra with center Z, and let A be an arbitrary subalgebra of $LS(R)$ containing R. Then every Z-linear derivation*

$$D : A \to LS(R)$$

is spatial and is implemented by an element of $LS(R)$.

Corollary 4.1. *Let R be a type I real von Neumann algebra with center Z, and let D be a Z-linear derivation on $S(R)$ or $S(R,\tau)$. Then D is spatial and is implemented by an element of $LS(R)$.*

Lemma 4.1. *If R is a real von Neumann algebra of type I_∞, then every derivation*

$$D : R \to S(R)$$

has the form

$$D(x) = ax - xa, \qquad x \in R,$$

for some element $a \in S(R)$.

Proof. The derivation D extends to a derivation $\overline{D}$ on $M = R + iR$ by

$$\overline{D}(x + iy) = D(x) + iD(y), \qquad x, y \in R.$$

By Lemma 3.4 (see [2]), the derivation $\overline{D}$ on M is inner. Hence there exists an element

$$z = a + ib, \qquad a, b \in R,$$

such that

$$\overline{D}(x + iy) = [z, x + iy] = z(x + iy) - (x + iy)z, \qquad x, y \in R.$$

Therefore

$$D(x) = \overline{D}(x) = [a, x] + i[b, x].$$

Since

$$x \in R, \qquad D(x) \in R,$$

it follows that

$$[b, x] = 0.$$

Hence

$$D(x) = [a, x],$$

and therefore D is inner. □

Analogously, one proves the following lemma.

Lemma 4.2. *Let R be a type I real von Neumann algebra with center Z. Then every Z-linear derivation on the algebra $S(R)$ is inner.*

In particular, if R is of type I_∞, then every derivation on $S(R)$ is inner.

Finally, combining Lemmas 3.2 and 4.2, we obtain the main result of this section.

Theorem 4.2. *Let R be a type I real von Neumann algebra. Then every derivation D on the algebra $S(R)$ admits a unique representation*

$$D = D_a + D_\delta,$$

where D_a is an inner derivation implemented by an element

$$a \in S(R),$$

and D_δ is a derivation of the form (3) *generated by a derivation δ on the center of $S(R)$.*

Let τ be a faithful normal semifinite trace on a von Neumann algebra M. A linear subspace $D \subset H$ is called τ-dense if $D\eta M$ and for every $\varepsilon > 0$ there exists a projection $P \in P(M)$ such that $P(H) \subset D,\ \tau(I - P) \leq \varepsilon$.

By Proposition 3.15 (see [5]), every τ-dense subspace of H is strongly dense. A closed linear operator T acting on H is called τ-measurable with respect to M if $T\eta M$ and the domain $D(T)$ is τ-dense in H. We denote by $S(M, \tau)$ the algebra of all τ-measurable operators. Clearly,

$$M \subset S(M, \tau) \subset S(M).$$

Moreover, $T \in S(M, \tau)$ if and only if $T \in S(M)$ and $D(T)$ is τ-dense in H.

We now consider the general form of derivations on the algebra $S(R, \tau)$ of τ-measurable operators associated with a type I real von Neumann algebra R and a faithful normal semifinite trace τ.

Similarly to the proof of Theorem 2.1 (see [2]), one obtains the following theorem.

Theorem 4.3. *Let R be a type I real von Neumann algebra with center Z and a faithful normal semifinite trace τ. Then every Z-linear derivation*

$$D : S(R, \tau) \to S(R, \tau)$$

is inner.

In particular, if R is of type I_∞, then every derivation on $S(R, \tau)$ is inner.

In a manner analogous to (3) and formula (12) from [2], one can define a derivation D_δ on $S(R, \tau)$. Then, similarly to Lemma 3.2, the following statement holds.

Lemma 4.3. *Let R be a finite real von Neumann algebra of type I equipped with a faithful normal semifinite trace τ. Then every derivation D on the algebra $S(R,\tau)$ admits a unique representation*

$$D = D_a + D_\delta,$$

where D_a is an inner derivation implemented by an element

$$a \in S(R,\tau),$$

and D_δ is a derivation defined as in (5).

Finally, Theorem 4.3 and Lemma 4.3 imply the following result.

Theorem 4.4. *Let R be a type I real von Neumann algebra equipped with a faithful normal semifinite trace τ. Then every derivation D on the algebra $S(R,\tau)$ admits a unique representation*

$$D = D_a + D_\delta,$$

where D_a is an inner derivation implemented by an element $a \in S(R,\tau)$, and D_δ is the derivation described in Lemma 4.3.

If we consider the measure topology t_τ on the algebra $S(R,\tau)$ (see Section 1), then every nonzero derivation of the form D_δ is discontinuous with respect to t_τ. Therefore, Theorem 4.4 immediately yields the following corollary.

Corollary 4.2. *Let R be a type I real von Neumann algebra with a faithful normal semifinite trace τ. A derivation on the algebra $S(R,\tau)$ is inner if and only if it is continuous in the measure topology.*

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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