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Essential And 0-Essential Soft Intersection Ideals of Semigroups

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ABSTRACT. In this paper, we define the essential and 0-essential soft intersection ideals of semigroups and examine in terms of some soft set operations. Our study shows that, although every ideal of a semigroup with zero is an essential ideal, this is not the case as regards essential soft intersection ideal. Moreover, our fundamental theorem, which states that if an ideal of a semigroup is an essential ideal, then its soft characteristic function is an essential soft intersection ideal, and vice versa, enables us to bridge the gap between semigroup theory and soft set theory. Additionally, we showed that if an ideal of a semigroup is an essential ideal, then its support set is an essential ideal. Furthermore, we introduce the concept of nontrivial soft intersection ideal of a semigroup to define the concept of 0-essential soft intersection ideal of a semigroup. Another fundamental theorem that allows us to bridge the gap between semigroup theory and soft set theory states that if a nonzero ideal of a semigroup with zero is a 0-essential ideal, then its soft characteristic function is a 0-essential soft intersection ideal, and vice versa. Besides, we showed that if an ideal of a semigroup with zero is a 0-essential ideal, then its support set is a 0-essential ideal. Furthermore, we define the concepts of minimal (prime, semiprime) essential and 0-essential soft intersection ideals of a semigroup. Moreover, we demonstrated the connection between (0-) essential ideals and (0-) essential soft intersection ideals of a semigroup corresponding with minimality, primeness, and semiprimeness.

1. Introduction

The concept of semigroups first emerged in the early 20th century and has since been developed through a series of fragmented studies. Semigroups are utilized in various branches of mathematics, including theoretical computer science, automata, coding theory, formal languages, and probability theory. Moreover, semigroups serve as fundamental algebraic

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structures in graph theory and optimization theory.

Ideals play a central role in the advanced study of algebraic structures and their applications. The concept of an ideal was first introduced by Dedekind to facilitate a more systematic and simplified investigation of algebraic structures. Subsequently, Noether generalized this concept to associative rings. The idea of a one-sided ideal of any algebraic structure is an extension of the idea of an ideal, and both one-sided and two-sided ideals continue to be considered fundamental constructs in ring theory. The concept of a bi-ideal in semigroup theory was first introduced by Good and Hughes [1]. Steinfeld [2] developed the notion of quasi-ideals for semigroups and later extended this concept to rings. Quasi-ideals are a generalization of left ideals and right ideals, while bi-ideals are a generalization of quasi-ideals. The concept of an interior ideal of a semigroup was defined by Lajos [3] and later studied by Szasz [4,5]. The concept of the interior ideal of any algebraic structure is a generalization of the concept of ideal. Molodtsov [6] introduced "Soft Set Theory" to model uncertainty and apply it to problems involving uncertainty. Subsequently, Feng et al. [7] investigated the concept of soft semirings, thereby extending the algebraic foundations of soft set theory. Later, Çağman and Enginoğlu [8] redefined definition of a soft set of Molodtsov [6] and soft set operations to make them more useful. [9–55] represent some of the significant contributions to the development of soft set theory. In addition, Sezer et al. [56] introduced the notions of soft intersection semigroups, ideals, and (generalized) bi-ideals in semigroups. Furthermore, they examined the relationships among various soft intersection ideals and systematically compiled results concerning soft set operations and concepts available in the existing literature. Lately, Sezgin and İlgin [57] studied soft intersection almost subsemigroups of a semigroup as a generalization of soft intersection subsemigroups of semigroups. Various soft intersection almost ideals of semigroups have been studied in [58-68]. Medhi et al. [69] introduced the concept of fuzzy essential ideals in ring theory and examined their fundamental properties. Recently, Baupradist et al. [70] studied the essential ideals and essential fuzzy ideals of semigroups. Chinram et al. [71] further extended the concept of essential ideals of semigroups by defining the concepts of essential (m,n)-ideals and fuzzy essential (m,n)-ideals. In addition, Panpetch et al. [72] and Gaketem and Iampan [73] explored the essential bi-ideals and essential fuzzy bi-ideals in semigroups.

In this paper, we define the essential and 0-essential soft intersection ideals of semigroups and examine in terms of some soft set operations. Our study shows that, although every ideal of a semigroup with zero is an essential ideal, this is not the case as regards essential soft intersection ideal. Moreover, the finite union of essential ideals of a semigroup is an essential ideal. Our fundamental theorem, which states that if an ideal of a semigroup is an essential ideal, then its soft characteristic function is an essential soft intersection ideal, and vice versa, enables us to bridge the gap between semigroup theory and soft set theory. Additionally, we showed that if an ideal of a semigroup is an essential ideal, then its support set is an essential ideal. Furthermore, we introduce the concept of nontrivial soft intersection ideal of a semigroup to define the concept of 0-essential soft intersection ideal of a semigroup. Moreover, the finite union of 0-essential ideals of a semigroup is a 0-essential ideal. Another fundamental theorem that allows us to bridge the gap between semigroup theory and soft set theory states that if a non-zero ideal of a semigroup containing zero is a 0-essential ideal, then its soft characteristic function is a 0-essential soft intersection ideal, and vice versa. Besides, we showed that if an ideal of a semigroup with zero is a 0-essential ideal, then its support set is a 0-essential ideal. Furthermore, we define the concepts of minimal (prime, semiprime) essential and 0-essential soft intersection ideals of a semigroup. Moreover, we demonstrated the connection between (0-) essential ideals and (0-) essential soft intersection ideals of a semigroup corresponding with minimality, primeness, and semiprimeness. This paper is organized into five sections. Section 1 provides an introductory overview of the subject. Section 2 presents the fundamental definitions and propositions concerning soft sets and essential ideals. In Sections 3 and 4, respectively, the concept of essential soft intersection ideals and the concept of 0-essential soft intersection ideals of semigroups was introduced, their properties are examined and supported with illustrative examples. Lastly, Section 5 concludes the paper with a summary of the principal results and offers directions for future research.

2. Preliminaries

In this section, we remind the basic concepts regarding semigroups, soft sets, and the concept of essential ideals and 0-essential ideals of semigroups. Throughout this paper, S denotes a semigroup. A nonempty subset K of S is called a subsemigroup of S if $KK \subseteq K$, is called a left ideal of S if $SK \subseteq K$, is called a right ideal of S if $KS \subseteq K$, and is called an ideal of S when is both a left ideal of S and a right ideal of S.

Definition 2.1 [6,8]. Let E be the parameter set, U be the universal set, $P(U)$ be the power set of U, and $\mathrm{V} \subseteq E$. The soft set $ƒ_{\mathrm{V}}$ over U is a function such that $ƒ_{\mathrm{V}}: E \to P(U)$, where for all $\mathrm{n} \notin \mathrm{V}$, $ƒ_{\mathrm{V}}(\mathrm{n}) = \emptyset$. That is, $ƒ_{\mathrm{V}} = \{(\mathrm{n}, ƒ_{\mathrm{V}}(\mathrm{n})) : \mathrm{n} \in E, ƒ_{\mathrm{V}}(\mathrm{n}) \in P(U)\}$.

The set of all soft sets over U is designated by $S_E(U)$ throughout this paper.

Definition 2.2 [8]. Let $ƒ_{Ꝁ} \in S_E(U)$. If $ƒ_{Ꝁ}(ꞥ) = \emptyset$ for all $ꞥ \in E$, then $ƒ_{Ꝁ}$ is called a null soft set and indicated by $\emptyset_E$.

Definition 2.3 [8]. Let $ƒ_{Ꝁ}, ƒ_{Ƀ} \in S_E(U)$. If $ƒ_{Ꝁ}(\rho) \subseteq ƒ_{Ƀ}(\rho)$ for all $\rho \in E$, then $ƒ_{Ꝁ}$ is a soft subset of $ƒ_{Ƀ}$ and indicated by $ƒ_{Ꝁ} \,\tilde{\subseteq}\, ƒ_{Ƀ}$. If $ƒ_{Ꝁ}(\rho) = ƒ_{Ƀ}(\rho)$, for all $\rho \in E$, then $ƒ_{Ꝁ}$ is called soft equal to $ƒ_{Ƀ}$ and denoted by $ƒ_{Ꝁ} = ƒ_{Ƀ}$.

Definition 2.4 [8]. Let $ƒ_{Ᵽ}, ƒ_{Ꝃ} \in S_E(U)$. The union (intersection) of $ƒ_{Ᵽ}$ and $ƒ_{Ꝃ}$ is the soft set $ƒ_{Ᵽ} \,\tilde{\cup}\, ƒ_{Ꝃ}$ $(ƒ_{Ᵽ} \,\tilde{\cap}\, ƒ_{Ꝃ})$ where $(ƒ_{Ᵽ} \,\tilde{\cup}\, ƒ_{Ꝃ})(ꝗ) = ƒ_{Ᵽ}(ꝗ) \cup ƒ_{Ꝃ}(ꝗ)$ $\left((ƒ_{Ᵽ} \,\tilde{\cap}\, ƒ_{Ꝃ})(ꝗ) = ƒ_{Ᵽ}(ꝗ) \cap ƒ_{Ꝃ}(ꝗ)\right)$, for all $ꝗ \in E$, respectively.

Definition 2.5 [7]. For a soft set f_A, the support of f_A is defined by $supp(f_A) = \{x \in A: f_A(x) \neq \emptyset\}$. It is obvious that a soft set with an empty support is a null soft set, otherwise the soft set is nonnull.
Note 2.6 [57]. Let $f_A, f_B \in S_E(U)$. If $f_A \tilde{\subseteq} f_B$, then $supp(f_A) \subseteq supp(f_B)$.
Definition 2.7 [56]. Let $\emptyset \neq Ꝛ \subseteq S$. The soft characteristic function of Ꝛ, denoted by Ꝛ, is defined as

$$S_{Ꝛ}(ƀ) = \begin{cases} U, & if\ ƀ \in Ꝛ \\ \emptyset, & if\ ƀ \in S \backslash Ꝛ \end{cases}$$

Lemma 2.8 [57]. Let $f_S \in S_E(U)$. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$.
Corollary 2.9 [57]. $supp(S_A) = A$.
Theorem 2.10 [56,57]. Let $\emptyset \neq Ѧ, \mathcal{I} \subseteq S$. Then, $Ѧ \subseteq \mathcal{I}$ if and only if $S_{Ѧ} \tilde{\subseteq} S_{\mathcal{I}}$.
Definition 2.11 [56]. $ɓ_S \in S_S(U)$ is called a soft intersection left (right) ideal of S over U if $ɓ_S(yᴖ) \supseteq ɓ_S(ᴖ)$ $\left(ɓ_S(yᴖ) \supseteq ɓ_S(y)\right)$ for all $y, ᴖ \in S$, and is called a soft intersection ideal of S over U if it is both a soft intersection left ideal of S over U and a soft intersection right ideal of of S over U.
For the sake of brevity, soft intersection (left/right) ideal of S over U is abbreviated by S-int (L/R)-ideal. It is easy to see that if $ɓ_S(\wp) = U$ for all $\wp \in S$, then $ɓ_S$ is an S-int ideal. We denote such a kind of S-int ideal by $\widetilde{\mathbb{S}}$. It is obvious that $\widetilde{\mathbb{S}} = S_S$, that is, $\widetilde{\mathbb{S}}(\wp) = U$ for all $\wp \in S$ [56].
Theorem 2.12 [56]. Let $\emptyset \neq X \subseteq S$. Then, X is an ideal of S if and only if S_X is an S-int ideal.
For additional information on operation of sets, we refer to [74], and more on soft sets, we refer to [75-109].
Now, we remind the concept of essential ideal and 0-essential ideal of a semigroup and give their illustrating examples.
Definition 2.13 [70]. An ideal $\mathcal{L}$ of S is called an essential ideal of S if $\mathcal{L} \cap \mathcal{J} \neq \emptyset$ for every ideal $\mathcal{J}$ of S.
It is obvious that if the semigroup has at least one nonessential ideal, then it has at least two. Moreover, S is a trivial essential ideal of S.
For the sake of brevity, essential ideal of S is abbreviated by Ɛ-ideal.
Corollary 2.14 [70]. Let S is a semigroup with zero. Then, every ideal of S is automatically an Ɛ-ideal, since every ideal contains the zero of S.
Example 2.15. Let $S = \{\mathfrak{p}, \mathfrak{t}, \mathfrak{w}\}$ be the semigroup with the following Cayley Table.

Table 1. Cayley Table of binary operation

$\circledast$	$\mathfrak{p}$	$\mathfrak{t}$	$\mathfrak{w}$
$\mathfrak{p}$	$\mathfrak{p}$	$\mathfrak{t}$	$\mathfrak{w}$
$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{w}$
$\mathfrak{w}$	$\mathfrak{w}$	$\mathfrak{w}$	$\mathfrak{t}$

It is obvious that S is a semigroup without zero, and all the ideals are $\{\mathfrak{t},\mathfrak{w}\}$ and $\{\mathfrak{p},\mathfrak{t},\mathfrak{w}\}$. They are also $\mathcal{E}$-ideals.

Definition 2.16 [70]. Let S be a semigroup with zero. A nonzero ideal $\mathcal{L}$ of S is called a 0-essential ideal of S if $\mathcal{L} \cap \mathcal{J} \neq \{0_S\}$ for every nonzero ideal $\mathcal{J}$ of S.

From now on, 0-essential ideal of S is abbreviated by 0-$\mathcal{E}$-ideal.

Example 2.17. Let $S = \{l, y, n, a\}$ be the semigroup with the following Cayley Table.

Table 2. Cayley Table of binary operation

$\cdot$	l	y	n	a
l	l	l	l	l
y	l	y	y	y
n	l	y	n	n
a	l	y	n	a

It is obvious that S has the zero element "l" and all the nonzero $\mathcal{J}$-ideals are $\mathcal{J}_1 = \{l, y\}$, $\mathcal{J}_2 = \{l, y, n\}$, and $\mathcal{J}_3 = S = \{l, y, n, a\}$. Here, all of the ideals are 0-$\mathcal{E}$-ideals.

In fact, $\mathcal{J}_1 \cap \mathcal{J}_2 = \{l, y\} \neq \{l\}$ and $\mathcal{J}_1 \cap \mathcal{J}_3 = \{l, y\} \neq \{l\}$. Thus, $\mathcal{J}_1$ is a 0-$\mathcal{E}$-ideal. Similarly, $\mathcal{J}_2 \cap \mathcal{J}_1 = \{l, y\} \neq \{l\}$ and $\mathcal{J}_2 \cap \mathcal{J}_3 = \{l, y, n\} \neq \{l\}$. Hence, $\mathcal{J}_2$ is a 0-$\mathcal{E}$-ideal. It is obvious that $\mathcal{J}_3$ is a 0-$\mathcal{E}$-ideal.

3. Essential Soft Intersection Ideals of Semigroups

In this section, we define essential soft intersection ideals of semigroups and study its basic properties.

Definition 3.1. A nonnull S-int ideal f_S of S over U such is called an essential soft intersection ideal of S over U if $f_S \tilde{\cap} h_S \neq \emptyset_S$ for every nonnull S-int ideal h_S of S over U.

For the sake of brevity, essential soft intersection ideal of S over U is abbreviated by $\mathcal{E}$S-int ideal.

Example 3.2. Let $S = \{a, i\}$ be the semigroup with the following Cayley Table.

Table 3. Cayley Table of binary operation

$\star$	a	i
a	a	a
i	a	i

Let f_S and h_S be soft sets over $U = \{1,2\}$ as follows:

$$f_S = \{(a, U), (i, \{2\})\}$$
$$h_S = \{(a, \{1\}), (i, \{1\})\}$$

Here, while both f_S and h_S are S-int ideals, f_S is an $\mathcal{E}$S-int ideal, however h_S is not. In fact;

$$f_S(aa) = f_S(a) \supseteq f_S(a), f_S(ai) = f_S(a) \supseteq f_S(i), f_S(ia) = f_S(a) \supseteq f_S(a), f_S(ii) = f_S(i) \supseteq f_S(i)$$

Consequently, f_S is an S-int L-ideal. Similarly,

$$f_S(aa) = f_S(a) \supseteq f_S(a), f_S(ai) = f_S(a) \supseteq f_S(a), f_S(ia) = f_S(a) \supseteq f_S(i), f_S(ii) = f_S(i) \supseteq f_S(i)$$

Hence, f_S is an S-int R-ideal, and so f_S is an S-int ideal.

Similarly,

$$h_S(aa) = h_S(a) \supseteq h_S(a), h_S(ai) = h_S(a) \supseteq h_S(i),$$
$$h_S(ia) = h_S(a) \supseteq h_S(a), h_S(ii) = h_S(i) \supseteq h_S(i)$$

Consequently, h_S is an S-int L-ideal. Similarly,

$$h_S(aa) = h_S(a) \supseteq h_S(a), h_S(ai) = h_S(a) \supseteq h_S(a)$$
$$h_S(ia) = h_S(a) \supseteq h_S(i), h_S(ii) = h_S(i) \supseteq h_S(i)$$

Consequently, h_S is an S-int R-ideal. Therefore, h_S is an S-int ideal.

Now, we show that f_S is an ƐS-int ideal. Considering all the nonnull S-int ideals of S over U, that is,

$$b_S = \{(a, U), (i, U)\} = \widetilde{\mathbb{S}}, d_S = \{(a, U), (i, \emptyset)\}, g_S = \{(a, U), (i, \{1\})\}, f_S = \{(a, U), (i, \{2\})\},$$
$$j_S = \{(a, \{1\}), (i, \emptyset)\},\ k_S = \{(a, \{2\}), (i, \emptyset)\}, h_S = \{(a, \{1\}), (i, \{1\})\}, t_S = \{(a, \{2\}), (i, \{2\})\}$$

f_S is an ƐS-int ideal. Similarly, one can show that b_S, d_S, g_S are also ƐS-int ideals. However,

$$h_S \widetilde{\cap}\, t_S = \{(a, \emptyset), (i, \emptyset)\} = \emptyset_S$$

Thereby, h_S is not an ƐS-int ideal.

Corollary 3.3. $\widetilde{\mathbb{S}}$ is an ƐS-int ideal. Moreover, although every ideal of S, where S is a semigroup with zero, is an Ɛ-ideal, this is not the case as regards ƐS-int ideal as is seen in Example 3.2.

Proposition 3.4. Let f_S be an ƐS-int ideal. If g_S is an S-int ideal such that $f_S \widetilde{\subseteq} g_S$, then g_S is also an ƐS-int ideal.

Proof: Let f_S be an ƐS-int ideal and g_S be an S-int ideal such that $f_S \widetilde{\subseteq} g_S$. Let k_S be any S-int ideal. We need to show that $g_S \widetilde{\cap}\, k_S \neq \emptyset_S$. Since

$$\emptyset_S \neq f_S \widetilde{\cap}\, k_S \widetilde{\subseteq} g_S \widetilde{\cap}\, k_S$$

$g_S \widetilde{\cap}\, k_S \neq \emptyset_S$ is obtained. Hence, g_S is an ƐS-int ideal.

Proposition 3.5. Let f_S be an ƐS-int ideal and k_S be an S-int ideal. Then, $f_S \widetilde{\cup}\, k_S$ is an ƐS-int ideal.

Proof: Let f_S be an ƐS-int ideal and k_S be an S-int ideal. Thus, f_S is an S-int ideal. Therefore, $f_S \widetilde{\cup}\, k_S$ is an S-int ideal. In fact, let $b, e \in S$. Then,

$$(f_S \widetilde{\cup}\, k_S)(be) = f_S(be) \cup k_S(be) \supseteq f_S(b) \cup k_S(b) = (f_S \widetilde{\cup}\, k_S)(b)$$

and

$$(f_S \widetilde{\cup}\, k_S)(be) = f_S(be) \cup k_S(be) \supseteq f_S(e) \cup k_S(e) = (f_S \widetilde{\cup}\, k_S)(e)$$

Thus, $f_S \widetilde{\cup}\, k_S$ is an S-int ideal. Since $f_S \widetilde{\subseteq} f_S \widetilde{\cup}\, k_S$, $f_S \widetilde{\cup}\, k_S$ is an ƐS-int ideal by Proposition 3.4.

Corollary 3.6. The finite union of ƐS-int ideals is an ƐS-int ideal.

Now, we give the relationship between Ɛ-ideals and ƐS-int ideals of a semigroup. Firstly, we give the following Proposition:

Proposition 3.7. If f_S is an S-int ideal, then $supp(f_S)$ is an ideal.

Proof: Assume that f_S is an S-int ideal. Thus, $f_S(c\ell) \supseteq f_S(\ell)$ and $f_S(c\ell) \supseteq f_S(c)$ for all $c, \ell \in S$. Let $\mathscr{n} \in [S \cdot supp(f_S)]$. Then, there exists an element $\mathscr{w} \in S$ and $\mathscr{z} \in supp(f_S)$ such that $\mathscr{n} = \mathscr{w}\mathscr{z}$. Thus, $f_S(\mathscr{z}) \neq \emptyset$. Since f_S is an S-int ideal, hence an S-int left ideal,

$$f_S(\mathscr{n}) = f_S(\mathscr{w}\mathscr{z}) \supseteq f_S(\mathscr{z}) \neq \emptyset$$

Hence, $f_S(\mathscr{n}) \neq \emptyset$, implying that $\mathscr{n} \in supp(f_S)$, and so $S \cdot supp(f_S) \subseteq supp(f_S)$. Thus, $supp(f_S)$ is a left ideal.

Similarly, let $\mathscr{m} \in [supp(f_S) \cdot S]$. Then, there exists an element $\mathscr{h} \in supp(f_S)$ and $\mathscr{b} \in S$ such that $\mathscr{m} = \mathscr{h}\mathscr{b}$. Thus, $f_S(\mathscr{h}) \neq \emptyset$. Since f_S is an S-int ideal, hence an S-int right ideal,

$$f_S(\mathscr{m}) = f_S(\mathscr{h}\mathscr{b}) \supseteq f_S(\mathscr{h}) \neq \emptyset$$

Hence, $f_S(\mathscr{m}) \neq \emptyset$, implying that $\mathscr{m} \in supp(f_S)$, and so $supp(f_S) \cdot S \subseteq supp(f_S)$. Thus, $supp(f_S)$ is a right ideal, and hence $supp(f_S)$ is an ideal.

Theorem 3.8. Let $Ⱥ$ be an ideal of S. Then, $Ⱥ$ is an Ɛ-ideal of S if and only if $S_{Ⱥ}$ is an ƐS-int ideal.

Proof: Assume that $Ⱥ$ is an Ɛ-ideal of S. Let k_S be a nonnull S-int ideal. Then, $supp(k_S)$ is an ideal by Proposition 3.7. Then, $Ⱥ \cap supp(k_S) \neq \emptyset$ which implies that there exists an element $\mathscr{s} \in S$ such that $\mathscr{s} \in Ⱥ \cap supp(k_S)$. Therefore, $S_{Ⱥ}(\mathscr{s}) = U$ and $k_S(\mathscr{s}) \neq \emptyset$. Hence, $(S_{Ⱥ} \tilde{\cap}\, k_S)(\mathscr{s}) \neq \emptyset$, implying that $S_{Ⱥ} \tilde{\cap}\, k_S \neq \emptyset$. Thus, $S_{Ⱥ}$ is an ƐS-int ideal.

Conversely, assume that $S_{Ⱥ}$ is an ƐS-int ideal. Let $\mathcal{B}$ be an ideal of S. Then $S_{\mathcal{B}}$ is a nonnull S-int ideal by Theorem 2.12. Then, $S_{Ⱥ} \tilde{\cap}\, S_{\mathcal{B}} \neq \emptyset_S$. By the definition of soft characteristic function, it is obvious that $S_{Ⱥ}(\mathscr{m}) = \emptyset$ or $S_{Ⱥ}(\mathscr{m}) = U$ and $S_{\mathcal{B}}(\mathscr{m}) = \emptyset$ or $S_{\mathcal{B}}(\mathscr{m}) = U$ for $\mathscr{m} \in S$. Hence, $S_{Ⱥ} \tilde{\cap}\, S_{\mathcal{B}} \neq \emptyset_S$ implies that there exists an element $a \in S$ such that $S_{Ⱥ}(a) = U$ and $S_{\mathcal{B}}(a) = U$. Thereby, $a \in Ⱥ$ and $a \in \mathcal{B}$. So, $a \in Ⱥ \cap \mathcal{B}$, and $Ⱥ \cap \mathcal{B} \neq \emptyset$. Consequently, $Ⱥ$ is an Ɛ-ideal of S.

Here note that since $Ⱥ$ is an ideal, $Ⱥ \neq \emptyset$. Otherwise, if $Ⱥ = \emptyset$, then $S_{Ⱥ} = \emptyset_S$, and thus $S_{Ⱥ}$ cannot be an ƐS-int ideal.

Example 3.9. Let the semigroup $S = \{\sigma, ω, Ɛ\}$ be defined by the following table and let the soft sets over $U = \{ɱ, ɲ\}$ as follows:

Table 4. Cayley Table of binary operation

⊛	σ	ω	Ɛ
σ	Ɛ	σ	Ɛ
ω	σ	ω	Ɛ
Ɛ	Ɛ	Ɛ	Ɛ

All the nonnull S-int ideals of S over U are as below:

$$\mathscr{b}_S = \{(\sigma,\emptyset),(ω,\emptyset),(Ɛ,\{ɱ\})\},\ d_S = \{(\sigma,\emptyset),(ω,\emptyset),(Ɛ,\{ɲ\})\},\ \mathscr{g}_S = \{(\sigma,\emptyset),(ω,\emptyset),(Ɛ,U)\},$$
$$f_S = \{(\sigma,\{ɱ\}),(ω,\emptyset),(Ɛ,\{ɱ\})\},\ \mathscr{j}_S = \{(\sigma,\{ɱ\}),(ω,\emptyset),(Ɛ,U)\},\ \mathscr{k}_S = \{(\sigma,\{ɲ\}),(ω,\emptyset),(Ɛ,\{ɲ\})\},$$
$$h_S = \{(\sigma,\{ɲ\}),(ω,\emptyset),(Ɛ,U)\},\ \ell_S = \{(\sigma,U),(ω,\emptyset),(Ɛ,U)\},\ \mathscr{t}_S = \{(\sigma,\{ɱ\}),(ω,\{ɱ\}),(Ɛ,\{ɱ\})\},$$

$$u_S = \{(\sigma, \{\mathfrak{m}\}), (\omega, \{\mathfrak{m}\}), (\varepsilon, U)\},\ y_S = \{(\sigma, U), (\omega, \{\mathfrak{m}\}), (\varepsilon, U)\},\ q_S = \{(\sigma, \{\mathfrak{n}\}), (\omega, \{\mathfrak{n}\}), (\varepsilon, \{\mathfrak{n}\})\},$$
$$r_S = \{(\sigma, \{\mathfrak{n}\}), (\omega, \{\mathfrak{n}\}), (\varepsilon, U)\},\ p_S = \{(\sigma, U), (\omega, \{\mathfrak{n}\}), (\varepsilon, U)\},\ o_S = \{(\sigma, U), (\omega, U), (\varepsilon, U)\} = \widetilde{\mathbb{S}}$$

By Definition 3.1, $g_S, j_S, h_S, \ell_S, u_S, y_S, r_S, p_S, o_S$ are the ƐS-int ideals. Now, one can show that $\mathrm{¥} = \{\sigma, \varepsilon\}$ is an ƐS-int ideal, whose soft characteristic function

$$S_{¥} = \{(\sigma, U), (\omega, \emptyset), (\varepsilon, U)\}$$

is an ƐS-int ideal. Conversely, by choosing the ƐS-int ideal as

$$g_S = \{(\sigma, \emptyset), (\omega, \emptyset), (\varepsilon, U)\}$$

which is the soft characteristic function of Ƀ $= \{\varepsilon\}$, one can show that Ƀ is an Ɛ-ideal.

Theorem 3.10. Let t_S be an S-int ideal. If t_S is an ƐS-int ideal, then $supp(t_S)$ is an Ɛ-ideal.

Proof: We prove the theorem with two different proofs:

Proof (1): Assume that t_S is ƐS-int ideal. Then, t_S is an S-int ideal by Definition 3.1, and $supp(t_S)$ is an ideal by Theorem 3.7. By Theorem 3.8, $S_{supp(t_S)}$ is an S-int ideal. Since $t_S \mathrel{\widetilde{\subseteq}} S_{supp(t_S)}$ by Lemma 2.8, and t_S is an ƐS-int ideal, and $S_{supp(t_S)}$ is an S-int ideal, $S_{supp(t_S)}$ is an ƐS-int ideal by Proposition 3.4. Hence, $supp(t_S)$ is an an Ɛ-ideal of S by Theorem 3.8.

Proof (2): Assume that t_S is an ƐS-int ideal. Then, $supp(t_S)$ is an ideal. Let $\mathcal{D}$ be an ideal of S. Then, $S_{\mathcal{D}}$ is an S-int ideal by Theorem 2.12. Hence, $t_S \mathrel{\widetilde{\cap}} S_{\mathcal{D}} \neq \emptyset_S$. Thus, there exists an element $d \in S$ such that $t_S(d) \neq \emptyset$ and $S_{\mathcal{D}}(d) = U$. Otherwise, if $t_S(d) = \emptyset$ or $S_{\mathcal{D}}(d) = \emptyset$ for all $d \in S$, then $t_S \mathrel{\widetilde{\cap}} S_{\mathcal{D}} = \emptyset_S$. Thereby, $d \in supp(t_S) \cap \mathcal{D}$. Consequently, $supp(t_S) \cap \mathcal{D} \neq \emptyset$, thus $supp(t_S)$ is an Ɛ-ideal.

Note that the converse of Theorem 3.10 and Proposition 3.7 are not true as shown in the following example:

Example 3.11. Consider the semigroup in Example 3.2 and the S-int ideal

$$t_S = \{(a, \{2\}), (i, \{2\})\}$$

It is obvious that $supp(t_S) = \{a, i\}$ is an Ɛ-ideal; however, t_S is not an ƐS-int ideal.

Also, consider the semigroup $S = \{\sigma, \omega, \varepsilon\}$ be defined in Example 3.9 and let the soft set be over $U = \{\mathfrak{m}, \mathfrak{n}\}$ as follows:

$$\vartheta_S = \{(\sigma, \{\mathfrak{m}\}), (\omega, \emptyset), (\varepsilon, \{\mathfrak{n}\})\}$$

One can show that $supp(\vartheta_S) = \{\sigma, \varepsilon\}$ is an ideal; however, ϑ_S is not an S-int ideal since $\vartheta_S(\sigma\sigma) = \vartheta_S(\varepsilon) \not\supseteq \vartheta_S(\sigma)$.

Definition 3.12. Let S be a semigroup. Then,

i. An Ɛ-ideal $\mathcal{L}$ is called minimal if for every Ɛ-ideal $\mathcal{K}$ of S such that $\mathcal{K} \subseteq \mathcal{L}$ we have $\mathcal{K} = \mathcal{L}$.

ii. An ƐS-int ideal f_S is called minimal if for every ƐS-int ideal h_S such that $h_S \mathrel{\widetilde{\subseteq}} f_S$, we have $supp(f_S) = supp(h_S)$.

Example 3.13. The Ɛ-ideal $\{\mathfrak{p}, \mathfrak{t}, \mathfrak{w}\}$ in Example 2.13 is not minimal, since $\{\mathfrak{t}, \mathfrak{w}\} \subseteq \{\mathfrak{p}, \mathfrak{t}, \mathfrak{w}\}$ but $\{\mathfrak{t}, \mathfrak{w}\} \neq \{\mathfrak{p}, \mathfrak{t}, \mathfrak{w}\}$. Similarly, the ƐS-int ideal $g_S = \{(a, U), (i, \{1\})\}$ in Example 3.2 is not minimal, since $d_S \mathrel{\widetilde{\subseteq}} g_S$, but $supp(d_S) \neq supp(g_S)$ where $d_S = \{(a, U), (i, \emptyset)\}$ is an ƐS-int ideal.

We give the relationship between minimal Ɛ-ideals and minimal ƐS-int ideals of a semigroup.

Theorem 3.14. Let $\mathcal{A}$ be a nonempty subset of S. Then, $\mathcal{A}$ is a minimal Ɛ-ideal if and only if $S_{\mathcal{A}}$ is a minimal ƐS-int ideal.

Proof: Assume that $\mathcal{A}$ is a minimal Ɛ-ideal. Thus, $\mathcal{A}$ is an Ɛ-ideal of S, and so $S_{\mathcal{A}}$ is an ƐS-int ideal by Theorem 3.8. Let ℓ_S be an ƐS-int ideal such that $\ell_S \tilde{\subseteq} S_{\mathcal{A}}$. By Theorem 3.10, $supp(\ell_S)$ is an Ɛ-ideal and by Note 2.6, and Corollary 2.9, $supp(\ell_S) \subseteq supp(S_{\mathcal{A}}) = \mathcal{A}$. Since $\mathcal{A}$ is a minimal Ɛ-ideal, $supp(\ell_S) = supp(S_{\mathcal{A}}) = \mathcal{A}$. Thus, $S_{\mathcal{A}}$ is a minimal ƐS-int ideal by Definition 3.12 (ii).

Conversely, let $S_{\mathcal{A}}$ be a minimal ƐS-int ideal. Thus, $S_{\mathcal{A}}$ is an ƐS-int ideal of S, and so $\mathcal{A}$ is an Ɛ-ideal by Theorem 3.8. Let $\mathcal{B}$ be an Ɛ-ideal such that $\mathcal{B} \subseteq \mathcal{A}$. By Theorem 3.8, $S_{\mathcal{B}}$ is an ƐS-int ideal, and by Theorem 2.10, $S_{\mathcal{B}} \tilde{\subseteq} S_{\mathcal{A}}$. Since $S_{\mathcal{A}}$ is a minimal ƐS-int ideal,

$$\mathcal{B} = supp(S_{\mathcal{B}}) = supp(S_{\mathcal{A}}) = \mathcal{A}$$

by Corollary 2.9. Thus, $\mathcal{A}$ is a minimal Ɛ-ideal.

Definition 3.15. Let S be a semigroup. Then,

i. An Ɛ-ideal $\mathcal{L}$ is called prime if $c\jmath \in \mathcal{L}$ implies $c \in \mathcal{L}$ or $\jmath \in \mathcal{L}$, for all $c, \jmath \in \mathcal{L}$.

ii. An ƐS-int ideal f_S is called prime if $f_S(c\jmath) \subseteq f_S(c) \cup f_S(\jmath)$, for all $c, \jmath \in \mathcal{L}$.

Example 3.16. Let $S = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}\}$ be the semigroup with the following Cayley Table

Table 5. Cayley Table of binary operation

::	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$	$\mathfrak{d}$
$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{a}$
$\mathfrak{b}$	$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{a}$
$\mathfrak{c}$	$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{a}$
$\mathfrak{d}$	$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{b}$

All the ideals are $\{\mathfrak{a}\}, \{\mathfrak{a}, \mathfrak{b}\}, \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}, \{\mathfrak{a}, \mathfrak{b}, \mathfrak{d}\}$, and $\{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}\}$. Here, since the semigroup is with zero "$\mathfrak{a}$", all the of ideals are Ɛ-ideals. Here, $\{\mathfrak{a}, \mathfrak{b}\}$ is not a prime Ɛ-ideal. In fact; $\mathfrak{c}\mathfrak{d} = \mathfrak{a} \in \{\mathfrak{a}, \mathfrak{b}\}$, however $\mathfrak{c} \notin \{\mathfrak{a}, \mathfrak{b}\}$ and $\mathfrak{d} \notin \{\mathfrak{a}, \mathfrak{b}\}$. Similarly, in Example 3.9, $u_S = \{(\sigma, \{\mathfrak{m}\}), (\omega, \{\mathfrak{m}\}), (\mathcal{E}, U)\}$ is not a prime ƐS-int ideal, since $u_S(\sigma\sigma) = u_S(\mathcal{E}) \nsubseteq u_S(\sigma)$.

Now we give relationship between prime Ɛ-ideals and prime ƐS-int ideals of a semigroup.

Theorem 3.17. Let $\mathcal{A}$ be a nonempty subset of S. If $\mathcal{A}$ is a prime Ɛ-ideal, then $S_{\mathcal{A}}$ is a prime ƐS-int ideal.

Proof: Let $\mathcal{A}$ be a prime Ɛ-ideal. Thus, $\mathcal{A}$ is an Ɛ-ideal of S, and so $S_{\mathcal{A}}$ is an ƐS-int ideal by Theorem 3.8. Let $c, \jmath \in \mathcal{A}$. Then,

Case 1: $c\jmath \in \mathcal{A}$. Thereby, $c \in \mathcal{A}$ or $\jmath \in \mathcal{A}$. Hence, $S_{\mathcal{A}}(c\jmath) = U \subseteq S_{\mathcal{A}}(c) \cup S_{\mathcal{A}}(\jmath) = U$.

Case 2: $c\jmath \notin \mathcal{A}$. This implies that, $S_{\mathcal{A}}(c\jmath) = \emptyset \subseteq S_{\mathcal{A}}(c) \cup S_{\mathcal{A}}(\jmath)$.

Thereby, $S_{\mathcal{A}}$ is a prime ƐS-int ideal by Definition 3.15 (ii).

Definition 3.18. Let S be a semigroup. Then,

i. An Ɛ-ideal $\mathcal{L}$ is called semiprime if $c^2 \in \mathcal{L}$ implies $c \in \mathcal{L}$, for all $c \in \mathcal{L}$.

ii. An ƐS-int ideal f_S is called semiprime if $f_S(c^2) \subseteq f_S(c)$, for all $c \in \mathcal{L}$.

It is obvious that if an Ɛ-ideal is prime, then it is semiprime, and if an ƐS-int ideal is prime, then it is semiprime.

Example 3.19. In Example 3.9, all Ɛ-ideals are $\{\text{Ɛ}\}, \{\sigma, \text{Ɛ}\}$, and $\{\sigma, \omega, \text{Ɛ}\}$. Here, since $\sigma\sigma = \text{Ɛ} \in \{\text{Ɛ}\}$ and $\sigma \notin \{\text{Ɛ}\}$, $\{\text{Ɛ}\}$ is not a semiprime Ɛ-ideal. Besides, in Example 3.9, the ƐS-int ideals $\ell_S, \mathscr{y}_S, \mathscr{p}_S, \mathscr{s}_S$ are also semiprime ƐS-int ideal; however $\mathscr{g}_S = \{(\sigma, \emptyset), (\omega, \emptyset), (\text{Ɛ}, U)\}$ is not a semiprime ƐS-int ideal, since $\mathscr{g}_S(\sigma\sigma) = \mathscr{g}_S(\text{Ɛ}) \nsubseteq \mathscr{g}_S(\sigma)$.

Now, we give relationship between semiprime Ɛ-ideals and semiprime ƐS-int ideals of a semigroup.

Theorem 3.20. Let $\mathcal{A}$ be a nonempty subset of S. Then, $\mathcal{A}$ is a semiprime Ɛ-ideal if and only if $S_{\mathcal{A}}$ is a semiprime ƐS-int ideal.

Proof: Let $\mathcal{A}$ be a semiprime Ɛ-ideal. Thus, $\mathcal{A}$ is an Ɛ-ideal of S, and so $S_{\mathcal{A}}$ is an ƐS-int ideal by Theorem 3.8. Let $c \in \mathcal{A}$. Then,

Case 1: $c^2 \in \mathcal{A}$. Thereby, $c \in \mathcal{A}$. Hence, $S_{\mathcal{A}}(c^2) = U \subseteq S_{\mathcal{A}}(c) = U$.

Case 2: $c^2 \notin \mathcal{A}$. This implies that, $S_{\mathcal{A}}(c^2) = \emptyset \subseteq S_{\mathcal{A}}(c)$.

Thereby, $S_{\mathcal{A}}$ is a semiprime ƐS-int ideal by Definition 3.18 (ii).

Conversely, let $S_{\mathcal{A}}$ be a semiprime ƐS-int ideal. Thus, $S_{\mathcal{A}}$ is an ƐS-int ideal of S, and so $\mathcal{A}$ is an Ɛ-ideal by Theorem 3.8. Let $c^2 \in \mathcal{A}$. Thus, $S_{\mathcal{A}}(c^2) = U$. By assumption, $S_{\mathcal{A}}(c^2) \subseteq S_{\mathcal{A}}(c)$. This implies that $S_{\mathcal{A}}(c) = U$. Hence, $c \in \mathcal{A}$. Thus, $\mathcal{A}$ is a semiprime Ɛ-ideal.

4. 0-Essential Soft Intersection Ideals of Semigroups

In this section, we define 0-essential soft intersection ideals of semigroups and study its basic properties.

Definition 4.1. Let S be a semigroup with zero. An S-int ideal f_S of S over U is called a nontrivial S-int ideal if there exists a nonzero element $x \in S$ such that $f_S(x) \neq \emptyset$.

It is obvious from Definition 4.1 that a nontrivial S-int ideal is a nonnull soft set.

Example 4.2. Consider the semigroup in Example 3.9, whose zero element is "Ɛ". It is obvious that

$$\mathscr{b}_S = \{(\sigma, \emptyset), (\omega, \emptyset), (\text{Ɛ}, \{\text{ɱ}\})\},\ d_S = \{(\sigma, \emptyset), (\omega, \emptyset), (\text{Ɛ}, \{\text{ɲ}\})\},\ \mathscr{g}_S = \{(\sigma, \emptyset), (\omega, \emptyset), (\text{Ɛ}, U)\}$$

are NOT nontrivial S-int ideals, since $\mathscr{b}_S(x) = d_S(x) = \mathscr{g}_S(x) = \emptyset$ for all $x \in S - \{\text{Ɛ}\}$. The other S-int ideals in Example 3.9, that is

$$f_S = \{(\sigma, \{\text{ɱ}\}), (\omega, \emptyset), (\text{Ɛ}, \{\text{ɱ}\})\},\ \mathscr{j}_S = \{(\sigma, \{\text{ɱ}\}), (\omega, \emptyset), (\text{Ɛ}, U)\},\ \mathscr{k}_S = \{(\sigma, \{\text{ɲ}\}), (\omega, \emptyset), (\text{Ɛ}, \{\text{ɲ}\})\},$$

$$h_S = \{(\sigma, \{\text{ɲ}\}), (\omega, \emptyset), (\text{Ɛ}, U)\},\ \ell_S = \{(\sigma, U), (\omega, \emptyset), (\text{Ɛ}, U)\},\ \mathscr{t}_S = \{(\sigma, \{\text{ɱ}\}), (\omega, \{\text{ɱ}\}), (\text{Ɛ}, \{\text{ɱ}\})\},$$

$$\mathscr{u}_S = \{(\sigma, \{\text{ɱ}\}), (\omega, \{\text{ɱ}\}), (\text{Ɛ}, U)\},\ \mathscr{y}_S = \{(\sigma, U), (\omega, \{\text{ɱ}\}), (\text{Ɛ}, U)\},\ \mathscr{q}_S = \{(\sigma, \{\text{ɲ}\}), (\omega, \{\text{ɲ}\}), (\text{Ɛ}, \{\text{ɲ}\})\},$$

$$\mathscr{r}_S = \{(\sigma, \{\text{ɲ}\}), (\omega, \{\text{ɲ}\}), (\text{Ɛ}, U)\},\ \mathscr{p}_S = \{(\sigma, U), (\omega, \{\text{ɲ}\}), (\text{Ɛ}, U)\},\ \mathscr{s}_S = \{(\sigma, U), (\omega, U), (\text{Ɛ}, U)\} = \widetilde{\mathbb{S}}$$

are all nontrivial S-int ideals. For example,

$$f_S = \{(\sigma,\{\text{ɱ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},\{\text{ɱ}\})\}$$

is a nontrivial S-int ideal, since $f_S(\sigma) \neq \emptyset$, where $\sigma \neq$ Ɛ.

Now, we define the definition of 0-essential soft intersection ideal of a semigroup as follows:

Definition 4.3. Let S be a semigroup with zero. An S-int ideal f_S of S over U is called a 0-essential soft intersection ideal of S over U if $supp(f_S \tilde{\cap} h_S) \neq \{0_S\}$ for every nontrivial S-int ideal h_S of S over U.

For the sake of brevity, 0-essential soft intersection ideal of S over U is abbreviated by 0-ƐS-int ideal. It is obvious from Definition 4.3 that a 0-ƐS-int ideal is a nonnull soft set.

Example 4.4. Consider the semigroup and S-int ideals in Example 3.9. Considering all the nontrivial S-int ideals, we can show that

$$\ell_S = \{(\sigma,U),(\text{ɷ},\emptyset),(\text{Ɛ},U)\}$$

is a 0-ƐS-int ideal. In fact,

$$\ell_S \tilde{\cap} f_S = \{(\sigma,\{\text{ɱ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},\{\text{ɱ}\})\},\ supp(\ell_S \tilde{\cap} f_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} j_S = \{(\sigma,\{\text{ɱ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} j_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} k_S = \{(\sigma,\{\text{ɲ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},\{\text{ɲ}\})\},\ supp(\ell_S \tilde{\cap} k_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} h_S = \{(\sigma,\{\text{ɲ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} h_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} \ell_S = \{(\sigma,U),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} \ell_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} t_S = \{(\sigma,\{\text{ɱ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},\{\text{ɱ}\})\},\ supp(\ell_S \tilde{\cap} t_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} u_S = \{(\sigma,\{\text{ɱ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} u_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} y_S = \{(\sigma,U),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} y_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} q_S = \{(\sigma,\{\text{ɲ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},\{\text{ɲ}\})\},\ supp(\ell_S \tilde{\cap} q_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} r_S = \{(\sigma,\{\text{ɲ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} r_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} p_S = \{(\sigma,U),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} p_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$
$$\ell_S \tilde{\cap} o_S = \{(\sigma,U),(\text{ɷ},\emptyset),(\text{Ɛ},U)\},\ supp(\ell_S \tilde{\cap} o_S) = \{\sigma,\text{Ɛ}\} \neq \{\text{Ɛ}\}$$

One can similarly show that p_S, o_S, y_S are also 0-ƐS-int ideals. On the other hand,

$$f_S = \{(\sigma,\{\text{ɱ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},\{\text{ɱ}\})\}$$

is not a 0-ƐS-int ideals. In fact,

$$f_S \tilde{\cap} h_S = \{(\sigma,\emptyset),(\text{ɷ},\emptyset),(\text{Ɛ},\{\text{ɱ}\})\},\ supp(f_S \tilde{\cap} h_S) = \{\text{Ɛ}\}$$

where $h_S = \{(\sigma,\{\text{ɲ}\}),(\text{ɷ},\emptyset),(\text{Ɛ},U)\}$ is a nontrivial S-int ideal. One can similarly show that $b_S, d_S, g_S, j_S, k_S, h_S, t_S, u_S, q_S, r_S$ are not 0-ƐS-int ideals.

Corollary 4.5. $\tilde{\mathbb{S}}$ is a 0-ƐS-int ideal.

Proposition 4.6. Let f_S be a 0-ƐS-int ideal. If g_S is an S-int ideal such that $f_S \tilde{\subseteq} g_S$, then g_S is also a 0-ƐS-int ideal.

Proof: Let f_S be a 0-ƐS-int ideal and g_S be an S-int ideal such that $f_S \tilde{\subseteq} g_S$. Let k_S be any nontrivial S-int ideal. We need to show that $supp(g_S \tilde{\cap} k_S) \neq \{0_S\}$. Since f_S is a 0-ƐS-int ideal, $supp(f_S \tilde{\cap} k_S) \neq \{0_S\}$, and also since $f_S \tilde{\cap} k_S \tilde{\subseteq} g_S \tilde{\cap} k_S$,

$$\{0_S\} \neq supp(f_S \tilde{\cap} k_S) \subseteq supp(g_S \tilde{\cap} k_S)$$

by Note 2.6. Thus, $supp(g_S \tilde{\cap} k_S) \neq \{0_S\}$, implying that g_S is a 0-ƐS-int ideal.

Proposition 4.7. Let f_S be a 0-ƐS-int ideal and k_S be an S-int ideal. Then, $f_S \tilde{\cup} k_S$ is a 0-ƐS-int ideal.

Proof: Let f_S be a 0-ƐS-int ideal and k_S be an S-int ideal. Thus, f_S is an S-int ideal. Therefore, $f_S \tilde{\cup} k_S$ is an S-int ideal by Proposition 3.5. Since $f_S \tilde{\subseteq} f_S \tilde{\cup} k_S$, $f_S \tilde{\cup} k_S$ is a 0-ƐS-int ideal by Proposition 4.6.

Corollary 4.8. The finite union of 0-ƐS-int ideals is a 0-ƐS-int ideal.

Now, we give the relationship between 0-Ɛ-ideals and 0-ƐS-int ideals of a semigroup.

Theorem 4.9. Let S be a semigroup with zero and $\mathbb{A}$ be a nonzero ideal. Then, $\mathbb{A}$ is a 0-Ɛ-ideal of S if and only if $S_{\mathbb{A}}$ is a 0-ƐS-int ideal.

Proof: Assume that $\mathbb{A}$ is a 0-Ɛ-ideal of S. Let k_S be a nontrivial S-int ideal. Then, $\emptyset \neq supp(k_S)$ is an ideal by Proposition 3.7. Then, $\mathbb{A} \cap supp(k_S) \neq \{0_S\}$, which implies that there exists a nonzero element $s \in S$ such that $s \in \mathbb{A} \cap supp(k_S)$. Thereby, $S_{\mathbb{A}}(s) = U$ and $k_S(s) \neq \emptyset$. Hence, $(S_{\mathbb{A}} \tilde{\cap} k_S)(s) \neq \emptyset$, where $0_S \neq s$. This implies that $supp(S_{\mathbb{A}} \tilde{\cap} k_S) \neq \{0_S\}$. Thus, $S_{\mathbb{A}}$ is a 0-ƐS-int ideal. Conversely, assume that $S_{\mathbb{A}}$ is a 0-ƐS-int ideal. Let $\mathcal{B}$ be a nonzero ideal of S. Then, $S_{\mathcal{B}}$ is a nontrivial S-int ideal of S. Then, $supp(S_{\mathbb{A}} \tilde{\cap} S_{\mathcal{B}}) \neq \{0_S\}$, which implies that there exists a nonzero element $k \in S$ such that $k \in supp(S_{\mathbb{A}} \tilde{\cap} S_{\mathcal{B}})$. Thus, $supp(S_{\mathbb{A}} \tilde{\cap} S_{\mathcal{B}})(k) \neq \emptyset$ for $0_S \neq k$. By the definition of soft characteristic function, $S_{\mathbb{A}}(k) = U$ and $S_{\mathcal{B}}(k) = U$. Otherwise, $(S_{\mathbb{A}} \tilde{\cap} S_{\mathcal{B}})(k) \neq \emptyset$. Thereby, $k \in \mathbb{A}$ and $k \in \mathcal{B}$, where for $0_S \neq k$. So, $0_S \neq k \in \mathbb{A} \cap \mathcal{B}$, and $\mathbb{A} \cap \mathcal{B} \neq \{0_S\}$. Consequently, $\mathbb{A}$ is a 0-Ɛ-ideal of S.

Example 4.10. Consider the semigroup in Example 3.9. One can show that $\mathcal{W} = \{\sigma, Ɛ\}$ is a 0-ƐS-int ideal, whose soft characteristic function

$$S_{\mathcal{W}} = \{(\sigma, U), (ω, \emptyset), (Ɛ, U)\}$$

is a 0-ƐS-int ideal. Conversely, by choosing the 0-ƐS-int ideal as

$$p_S = \{(\sigma, U), (ω, \{ɲ\}), (Ɛ, U)\}$$

which is the soft characteristic function of $\mathcal{K} = \{\sigma, ω, Ɛ\}$, one can show that $\mathcal{K}$ is a 0-Ɛ-ideal.

Theorem 4.11. Let S be a semigroup with zero and t_S be an S-int ideal. If t_S is a 0-ƐS-int ideal, then $supp(t_S)$ is a 0-Ɛ-ideal.

Proof: We prove the theorem with two different proofs:

Proof (1): Assume that t_S is a 0-ƐS-int ideal. Then, t_S is an S-int ideal by Definition 4.3, and $supp(t_S)$ is an ideal by Theorem 3.7. By Theorem 2.12, $S_{supp(t_S)}$ is an S-int ideal. Since $t_S \tilde{\subseteq} S_{supp(t_S)}$ by Lemma 2.8, and t_S is a 0-ƐS-int ideal, and $S_{supp(t_S)}$ is an S-int ideal, $S_{supp(t_S)}$ is a 0-ƐS-int ideal by Proposition 4.7. Hence, $supp(t_S)$ is a 0-Ɛ-ideal of S by Theorem 4.9.

Proof (2): Assume that t_S is a 0-ƐS-int ideal of S. Then, $supp(t_S)$ is an ideal. Let $\mathcal{N}$ be a nonzero ideal of S. Then, $S_{\mathcal{N}}$ is a nontrivial S-int ideal. Hence, $supp(t_S \tilde{\cap} S_{\mathcal{N}}) \neq \{0_S\}$. Thus, there exists a nonzero element $n \in S$ such that $t_S(n) \neq \emptyset$ and $S_{\mathcal{N}}(n) \neq \emptyset$. Otherwise, if $t_S(n) = \emptyset$ or $S_{\mathcal{N}}(n) = \emptyset$ for all $n \in S$, then $n \notin supp(t_S \tilde{\cap} S_{\mathcal{N}})$, where $n \neq 0_S$. Thereby, $t_S(n) \neq \emptyset$ and $S_{\mathcal{N}}(n) = U$. This implies that $0_S \neq n \in supp(t_S)$ and $0_S \neq n \in \mathcal{N}$. Thus, $0_S \neq n \in supp(t_S) \cap \mathcal{N}$. Consequently, $supp(t_S) \cap \mathcal{N} \neq \{0_S\}$, and so $supp(t_S)$ is a 0-Ɛ-ideal.

Note that the converse of Theorem 4.10 is not true as shown in the following example:

Example 4.12. Consider the semigroup in Example 3.9 and the S-int ideal

$$r_S = \{(\sigma, \{\text{ɲ}\}), (\text{ɰ}, \{\text{ɲ}\}), (\text{Ɛ}, U)\}$$

It is obvious that $supp(r_S) = \{\sigma, \text{ɰ}, \text{Ɛ}\}$ is a 0-Ɛ-ideal; however, r_S is not a 0-ƐS-int ideal since $supp(r_S \tilde{\cap} u_S) = \{\text{Ɛ}\}$, where $u_S = \{(\sigma, \{\text{ɱ}\}), (\text{ɰ}, \{\text{ɱ}\}), (\text{Ɛ}, U)\}$ is a nontrivial S-int ideal.

Definition 4.13. Let S be a semigroup with zero. Then,

i. A 0-Ɛ-ideal $\mathcal{L}$ is called minimal if for every 0-Ɛ-ideal $\mathcal{K}$ of S such that $\mathcal{K} \subseteq \mathcal{L}$, we have $\mathcal{K} = \mathcal{L}$.

ii. A 0-ƐS-int ideal f_S is called minimal if for every 0-ƐS-int ideal h_S such that $h_S \tilde{\subseteq} f_S$, we have $supp(f_S) = supp(h_S)$.

Example 4.14. The 0-Ɛ-ideal $\{l, y, n\}$ in Example 3.4 is not minimal, since $\{l, y\} \subseteq \{l, y, n\}$, however $\{l, y\} \neq \{l, y, n\}$, where $\{l, y\}$ is a 0-Ɛ-ideal. Similarly, the 0-ƐS-int ideal $p_S = \{(\sigma, U), (\text{ɰ}, \{\text{ɲ}\}), (\text{Ɛ}, U)\}$ in Example 4.4 is not minimal, since $\ell_S \tilde{\subseteq} p_S$, but $supp(\ell_S) \neq supp(p_S)$, where $\ell_S = \{(\sigma, U), (\text{ɰ}, \emptyset), (\text{Ɛ}, U)\}$ is a 0-ƐS-int ideal.

We give the relationship between minimal 0-Ɛ-ideals and minimal 0-ƐS-int ideals of a semigroup.

Theorem 4.15. Let S be a semigroup with zero and $\mathcal{A}$ be a nonempty subset of S. Then, $\mathcal{A}$ is a minimal 0-Ɛ-ideal if and only if $S_{\mathcal{A}}$ is a minimal 0-ƐS-int ideal.

Proof: The proof is similar to the proof of Theorem 3.14.

Definition 4.16. Let S be a semigroup with zero. Then,

i. A 0-Ɛ-ideal $\mathcal{L}$ is called prime if $cj \in \mathcal{L}$ implies $c \in \mathcal{L}$ or $j \in \mathcal{L}$, for all $c, j \in \mathcal{L}$.

ii. A 0-ƐS-int ideal f_S is called prime if $f_S(cj) \subseteq f_S(c) \cup f_S(j)$, for all $c, j \in \mathcal{L}$.

Example 4.17. Consider the semigroup in Example 3.16. All the 0-Ɛ-idels are $\{\mathfrak{a}, \mathfrak{b}\}, \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}, \{\mathfrak{a}, \mathfrak{b}, \mathfrak{d}\}$, and $\{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}\}$. Here, $\{\mathfrak{a}, \mathfrak{b}\}$ is not a prime 0-Ɛ-ideal. In fact; $\mathfrak{c}\mathfrak{d} = \mathfrak{a} \in \{\mathfrak{a}, \mathfrak{b}\}$, however, $\mathfrak{c} \notin \{\mathfrak{a}, \mathfrak{b}\}$ and $\mathfrak{d} \notin \{\mathfrak{a}, \mathfrak{b}\}$. Moreover, all the 0-ƐS-int ideals in Example 4.4 are prime.

We give the relationship between prime 0-Ɛ-ideals and prime 0-ƐS-int ideals of a semigroup.

Theorem 4.18. Let S be a semigroup with zero and $\mathcal{A}$ be a nonempty subset of S. If $\mathcal{A}$ is a prime 0-Ɛ-ideal, then $S_{\mathcal{A}}$ is a prime 0-ƐS-int ideal.

Proof: The proof is similar to the proof of Theorem 3.17.

Definition 4.19. Let S be a semigroup with zero. Then,

i. A 0-Ɛ-ideal $\mathcal{L}$ is called semiprime if $c^2 \in \mathcal{L}$ implies $c \in \mathcal{L}$, for all $c \in \mathcal{L}$.

ii. A 0-ƐS-int ideal f_S is called semiprime if $f_S(c^2) \subseteq f_S(c)$, for all $c \in \mathcal{L}$.

Example 4.20. Consider the 0-ƐS-int ideal $\{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$ in Example 3.16. Since $\mathfrak{d}\mathfrak{d} = \mathfrak{b} \in \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$, however, $\mathfrak{d}\mathfrak{d} = \mathfrak{b} \notin \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$, $\{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$ is not a semiprime 0-Ɛ-ideal. Moreover, all the 0-ƐS-int ideals in Example 4.4 are semiprime.

We give the relationship between semiprime 0-Ɛ-ideals and semiprime 0-ƐS-int ideals of a semigroup.

Theorem 4.21. Let S be a semigroup with zero and $\mathcal{A}$ be a nonempty subset of S. Then, $\mathcal{A}$ is a semiprime 0-Ɛ-ideal if and only if $S_{\mathcal{A}}$ is a semiprime 0-ƐS-int ideal.

Proof: The proof is similar to the proof of Theorem 3.20.

5. Conclusions

In this paper, we define the essential and 0-essential soft intersection ideals of semigroups and examine in terms of some soft set operations. Our study shows that, although every ideal of a semigroup with zero is an essential ideal, this is not the case as regards essential soft intersection ideal. Moreover, the finite union of essential ideals of a semigroup is an essential ideal. Our fundamental theorem, which states that if an ideal of a semigroup is an essential ideal, then its soft characteristic function is an essential soft intersection ideal, and vice versa, enables us to bridge the gap between semigroup theory and soft set theory. Additionally, we showed that if an ideal of a semigroup is an essential ideal, then its support set is an essential ideal. Furthermore, we introduce the concept of nontrivial soft intersection ideal of a semigroup to define the concept of 0-essential soft intersection ideal of a semigroup. Moreover, the finite union of 0-essential ideals of a semigroup is a 0-essential ideal. Another fundamental theorem that allows us to bridge the gap between semigroup theory and soft set theory states that if a non-zero ideal of a semigroup containing zero is a 0-essential ideal, then its soft characteristic function is a 0-essential soft intersection ideal, and vice versa. Besides, we showed that if an ideal of a semigroup with zero is a 0-essential ideal, then its support set is a 0-essential ideal. Furthermore, we define the concepts of minimal (prime, semiprime) essential and 0-essential soft intersection ideals of a semigroup. Moreover, we demonstrated the connection between (0-) essential ideals and (0-) essential soft intersection ideals of a semigroup corresponding with minimality, primeness, and semiprimeness.

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