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$h\alpha$ −Almost Weakly Continuous Functions in Ideal Topological Spaces

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ABSTRACT. In this paper, we discuss $h\alpha$ −continuous functions modulo ideals. We examine the features and relationships between $h\alpha$ −irresolute continuous and $h\alpha$ −homeomorphism functions modulo ideals. We present almost $h\alpha$ −continuous functions modulo ideals and provide some illustrations of them. We ascertain the relationship between h −homeomorphism, $h\alpha$ −irresolute, almost $h\alpha$ −continuous, and continuous functions modulo ideals. Strongly $h\alpha$ −continuous functions modulo ideals are presented and explored.

1. Introduction and Preliminaries

The advent of generalized open sets has made it possible to define continuity more broadly. For example, a function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called generalized continuous if $f^{-1}(G)$ is a generalized open subset of (X,τ) for any generalized open subset G of (Y,σ).

The study of mappings that retain a lesser type of continuity but do not strictly preserve classical open sets has been made possible by generalized continuity. Chawalit [1] presented certain connections between continuous and generalized continuous functions and conducted additional research on generalized continuous mapping. Abbas [2] introduced the concept of h −open sets in 2022. Abdullah et al. [3] introduced the concept of $h\alpha$ −open sets using these ideas. Dontchev [4] first proposed the concept of a $contra$ −continuous function in 1996.

Nearly $contra$ −continuous and contra $almost-\beta$ −continuous functions were defined by Baker [5]. Dontchev et al. introduced a new family of functions called regular set linked functions [6]. Singh and Singal introduced the class of nearly continuous functions in [7].

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Recent years have also seen considerable research on these functions; see [8, 19-23]. Numerous researchers contributed to this a few decades ago, see [9]. The concept of a strongly h −continuous function was recently created by Sharma et al. [10]. In the study of generalizations of continuity in topological spaces, sets [14,16-18] are crucial. Many writers used these sets to introduce and research different kinds of functions' weak kinds of continuity. Levine [15] first proposed the idea of weakly topological spaces with continuous functions.

Throughout this paper, we denoted the ideal topological spaces (X,τ,I) and (Y,σ,J) simply by X and Y, respectively. In section two, we discussed $h\alpha$ −continuous functions modulo ideals. In the third section, we define $h\alpha$ −irresolute continuous and $h\alpha$ −homeomorphism functions modulo ideals; we explore their properties and relations between them.

In the fourth section, we introduce almost $h\alpha$ −continuous functions modulo ideals and give some examples describing them. We determine the relation between continuous, almost $h\alpha$ −continuous, h −homeomorphism and $h\alpha$ −irresolute continuous functions modulo ideals. In the last section, we present strongly $h\alpha$ −continuous functions modulo ideals.

In this paper, we focus on the relationships between lots of types of $h\alpha$ −continuous functions modulo ideals. Some fundamental definitions and findings that will be utilized in the following sections are covered in this section. Unless otherwise noted, X, Y, and Z represent topological spaces in this work without assuming separation axioms.

Definition 1.1: Consider the space X and $H \subseteq X$, then:

(i) H is an h −open set [2] if $\forall O \subset X, O \in \tau, H \subseteq int(H \cup O)$.

(ii) H is an α −open set [11] if $H \subseteq int(cl(int(H)))$.

(iii) The collection of h −open (α −open) sets of τ is denoted by $\tau^{h}(\tau^{\alpha})$.

(iv) H is an $h\alpha$ −open set [3] if $\forall O \subset X, O \in \tau^{h\alpha},\ H \subseteq int\_\{I\}(H \cup O)$.

(v) If $\forall O \subset X, \forall O \in \tau$,then $H \subseteq int(H \cup O)$

(vi) H is a $semi$ −open set (s −open) [4] if $\exists V \in \tau$: $V \subseteq H \subseteq cl(H)$.

(vii) If H is open and closed, then it is a clopen set.

The collection of $h\alpha$ −open is denoted by $\tau^{h\alpha}$.

Example 1.2: If $X = \{x,y,z\}$, $\tau = \{X,\phi,\{x\},\{x,z\}\}$, and $I = \{X,\phi,\{x,z\}\}$, then $\tau^{h\alpha} = \{X,\phi,\{x\},\{x,z\},\{y,z\}\}$.

2. $h\alpha$ −Continuous Functions Modulo Ideals

Definition 2.1: If X and Y are two ideal spaces modulo I,J , then the function $f: X \to Y$ is:

(i) $h\alpha$ −continuous modulo I,J if $f^{-1}(O)$ is an hα-open subset of X modulo $I, \forall O \subseteq Y$ an open subset modulo J.

(ii) $h\alpha$ −totally continuous modulo I,J if $f^{-1}(O)$ is a clopen subset of X modulo $I, \forall O \subseteq Y$ an $h\alpha$ −open subset modulo J.

(iii) $h\alpha$ −irresolute modulo I, J if $f^{-1}(O)$ is hα-open subset of X modulo $I, \forall O \subseteq Y$an $h\alpha$ −open subset modulo J.
(iv) Almost $h\alpha$ −continuous modulo I, J if $f^{-1}(O)$ is an $h\alpha$ −open subset of X modulo I for each regular open subset O of Y.
(v) If f is a bijective, $h\alpha$ −open and $h\alpha$ −continuous, then f is $h\alpha$ −homeomorphism.
Example 2.2: Consider $X = Y = \{1,3,5\}$, $\tau = \{\phi, X, \{3\}, \{3,5\}\}, \sigma = \{\phi, Y, \{3,5\}, \{3\}, \{1,5\}, \{5\}\}$,
$I = \{X, \phi, \{3,5\}\}, J = \{Y, \phi, \{3,5\}\}$, and $\tau^{h\alpha} = \{\phi, X, \{3\}, \{5\}, \{3,5\}, \{1,3\}\}$,
then $f(x) = x$ the identity function modulo I, J is an almost $h\alpha$ −continuous function modulo I, J. Clearly, f is $h\alpha$ −continuous but not an α −continuous function modulo I, J since if $O = \{3,5\}$, then $f^{-1}(O) = \{3,5\} \notin \tau^{\alpha}$.
Lemma 2.3: Every h −continuous function modulo ideals is $h\alpha$ −continuous.
Proof. If $f: X \to Y$ is an h −continuous function modulo I, J and $O \subseteq Y$ is open, then $f^{-1}(O)$ is an h −open subset of X modulo I.
But, $f^{-1}(O)$ is an $h\alpha$ −open subset of X modulo I, hence f is an $h\alpha$ −continuous modulo I, J.
Theorem2.4: Every continuous function modulo ideals is an $h\alpha$ −continuous function.
Proof. Consider $f: X \to Y$ an h −continuous function modulo I, J and $O \subseteq Y$ is open, then $f^{-1}(O)$ is an open subset of X modulo I.
But, $f^{-1}(O)$ is an hα-open subset of X modulo I, hence f is an $h\alpha$ −continuous modulo I, J .

3. $h\alpha$ −Irresolute Continuous and hα-Homeomorphism Functions Modulo Ideals

Definition 3.1: If X and Y are two ideal spaces modulo I, J , then the function $f: X \to Y$ is $h\alpha$ −irresolute modulo I, J if $f^{-1}(O)$ is $h\alpha$ −open subset of X modulo $I, \forall O \subseteq Y$ an $h\alpha$ −open subset modulo J.
Example 3.2: If $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{b\}, \{b, c\}\}, I = \{X, \phi, \{b, c\}\}$, then
$\tau^{h\alpha} = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Now, if $\sigma = \{Y, \phi, \{b\}\}$, $J = \sigma$, so $\sigma^{h\alpha} = \{Y, \varphi, \{b\}, \{a, c\}\}$
Obviously, the identity function is $h\alpha$ −irresolute modulo I, J.
Theorem 3.3: Every h −irresolute function modulo ideals is $h\alpha$ −irresolute [12].
Proof. Consider $f: X \to Y$ an h −irresolute function modulo I, J.
If $O \subseteq Y$ is h-open modulo J, then $f^{-1}(O)$ is an $h\alpha$ −open subset of X modulo I.
Hence, $f^{-1}(O)$ is $h\alpha$ −irresolute modulo I.

The converse of the previous theorem is not true.

Example 3.4: Let $X = Y = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$ and $I = \{X, \phi, \{a, b, c\}\}$, then $\tau^{h\alpha} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{b, c\}, \{b, c, d\}$.
Also, $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\{a, b, c\}\}$, $J = \{Y, \{a, b, c\}\}$, $\sigma^{h\alpha} = \{Y, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$.
If $f: X \to Y$ is given by $f(x) = x$, then f is an $h\alpha$ −irresolute function modulo I, J but not α −irresolute modulo I, J.

Theorem 3.5: Every continuous function modulo ideals is an $h\alpha$ −irresolute function.

Proof. Let $f: X \to Y$ be a continuous function modulo I, J. If $O \subseteq Y$ is $h\alpha$ −open modulo J, then $f^{-1}(O)$ is an open subset of X modulo I.

But, f is continuous, hence $f^{-1}(O)$ is an $h\alpha$ −open subset of X modulo I.

Thus, f is an $h\alpha$ −irresolute function modulo I, J.

Theorem 3.6: The composition of two $h\alpha$ −irresolute functions modulo ideals is an $h\alpha$ −irresolute function.

Corollary 3.7: The composition of an $h\alpha$ −irresolute function modulo ideals and an $h\alpha$ −continuous function modulo ideals is $h\alpha$ −irresolute.

Proof. If $f: X \to Y$ is an $h\alpha$ −irresolute function modulo I, J, and $g: Y \to Z$ is an $h\alpha$ −irresolute function modulo J, K, then if $V \subseteq Z$ is an open subset modulo K, then V is an $h\alpha$ −open subset of Z modulo K.

But g is an $h\alpha$ −continuous function modulo J, hence $f^{-1}(V)$ is an $h\alpha$ −open subset of Y modulo J. Now, f is an $h\alpha$ −irresolute function modulo I, so $f^{-1}(g^{-1}(V))$ is an $h\alpha$ −open subset of X modulo I. Therefore, $f \circ g$ is an $h\alpha$ −irresolute function modulo I, J.

Theorem 3.8: The composition of a continuous open function modulo ideals and an almost $h\alpha$ −continuous function modulo ideals is an almost $h\alpha$ −continuous function.

Proof. If $f: X \to Y$ is a continuous open function modulo I, J, and $g: Y \to Z$ is an almost $h\alpha$ −continuous function modulo J, K, then if $O \subseteq Z$ is a regular open subset modulo K, then $f^{-1}(O)$ is an $h\alpha$ −open subset of Y modulo J.

Since f is an open continuous function modulo I, then $f^{-1}(g^{-1}(V)) \subseteq X$ is an $h\alpha$ −open modulo I.

Thus, $g \circ f$ is an almost $h\alpha$ −continuous function modulo I, J.

Definition 3.9: If $f: X \to Y$ is a bijective, $h\alpha$ −open and $h\alpha$ −continuous, then f is $h\alpha$ −homeomorphism.

Theorem 3.10: Every homeomorphism is $h\alpha$ −homeomorphism.

Proof. Let $f: X \to Y$ be a continuous function modulo I, J, then f is an $h\alpha$ −continuous, $h\alpha$ −open function modulo I, J. Since f is a homeomorphism, then f is an $h\alpha$ −homeomorphism modulo I, J.

Theorem 3.11: Every $h\alpha$ −continuous function modulo ideals is an almost $h\alpha$ −continuous function modulo ideals.

Theorem 3.12: Every $h\alpha$ −continuous function modulo ideals is a regular set connected function modulo ideals.

Corollary 3.13: Every almost continuous function modulo ideals is an almost $h\alpha$ −continuous function modulo ideals.

4. Almost hα-Continuous Functions Modulo Ideals

Definition 4.1: A function $f: X \to Y$ is an $h\alpha$ −almost weakly continuous modulo I, J if $\forall x \in X$ and $\forall V \subseteq Y$ an $h\alpha$ −open subset containing $f(x)$, $x \in int^{h\alpha}(cl^{h\alpha}(f^{-1}(cl^{h\alpha}(V))))$.

Theorem 4.2: Consider $f: X \to Y$ a continuous function modulo I, J , and O an $h\alpha$ −open subset of Y, then the following are equivalent:

(i) f is an $h\alpha$ −almost weakly continuous function modulo I, J .

(ii) $f^{-1}(O) \subseteq int^{h\alpha}(cl^{h\alpha}(f^{-1}(cl^{h\alpha}(O))))$.

(iii) $cl^{h\alpha}(int^{h\alpha}(f^{-1}(O))) \subseteq f^{-1}(cl^{h\alpha}(O))$

Proof. (i)⇒(ii) Let O be an $h\alpha$ −open subset of Y, and $x \in f^{-1}(O)$, then $f(x) \in O$.

If $x \in int^{h\alpha}(cl^{h\alpha}(f^{-1}(cl^{h\alpha}(O))))$, hence $f^{-1}(O) \subseteq int^{h\alpha}(cl^{h\alpha}(f^{-1}(cl^{h\alpha}(O))))$.

(ii)⇒(iii)
$$\begin{aligned} X - f^{-1}(cl^{h\alpha}(O)) &= f^{-1}(Y - cl^{h\alpha}(O)) \\ &\subseteq int^{h\alpha}(cl^{h\alpha}(f^{-1}(cl^{h\alpha}(Y - cl^{h\alpha}(O))))) \\ &\subseteq int^{h\alpha}(cl^{h\alpha}(f^{-1}(Y - O))) \\ &= int^{h\alpha}(cl^{h\alpha}(X - f^{-1}(O))) \\ &= X - cl^{h\alpha}(int^{h\alpha}(f^{-1}(O))). \end{aligned}$$

(iii)⇒(i) Obvious.

Theorem 4.3: Consider $f: X \to Y$ is a continuous function modulo I, J, and O an $h\alpha$ −open subset of Y, then f is an hα-almost weakly continuous function modulo I, J iff $cl^{h\alpha}(int^{h\alpha}(f^{-1}(int^{h\alpha}(O)))) \subseteq f^{-1}(O)$.

Proof. If O an $h\alpha$ −open subset of Y, then $Y - O$ is an $h\alpha$ −closed subset of Y.

Now,
$$\begin{aligned} X - f^{-1}(O) &= f^{-1}(Y - O) \\ &\subseteq int^{h\alpha}(cl^{h\alpha}(f^{-1}(cl^{h\alpha}(Y - O)))) \\ &= int^{h\alpha}(cl^{h\alpha}(f^{-1}(Y - int^{h\alpha}(O)))) \\ &= int^{h\alpha}(cl^{h\alpha}(X - f^{-1}(int^{h\alpha}(O)))) \\ &= X - cl^{h\alpha}(int^{h\alpha}(f^{-1}(int^{h\alpha}(O)))). \end{aligned}$$

Thus, $cl^{h\alpha}(int^{h\alpha}(f^{-1}(int^{h\alpha}(O)))) \subseteq f^{-1}(O)$.

Theorem 4.4: If Y is a Hausdorff space, $f: X \to Y$ is an $h\alpha$ −almost continuous modulo I, J and $g: X \to Y$ is an $h\alpha$ −almost weakly continuous modulo I, J, then the $\{x \in X: f(x) = g(x)\}$ is an $h\alpha$ −closed subset of X.

Proof. If $D = \{x \in X: f(x) = g(x)\}$, then $\forall x \notin D$: $f(x) \neq g(x), \exists U, V$ two disjoint $h\alpha$ −open subsets of Y : $f(x) \in U$ and $g(x) \in V$.

Now, $int^{h\alpha}(cl^{h\alpha}(U)) \cap cl^{h\alpha}(V) = \phi$. But f is an $h\alpha$ −almost continuous modulo I, J, then $\exists W$ an $h\alpha$ −open subset of X : $f(W) \subseteq int^{h\alpha}(cl^{h\alpha}(U))$.

Furthermore, since g is an $h\alpha$ −almost weakly continuous modulo I, J, $\exists W'$ an $h\alpha$ −open subset of X : $x \in W'$and $g(W') \subseteq cl^{h\alpha}(V)$. Hence, $f(W)$ and $g(W')$ are disjoint.

Thus, $W \cap W'$ is an $h\alpha$ −open subsets of X. Consequently, D is an $h\alpha$ −closed subset of X.

Definition 4.5: A space X is $h\alpha$ −Urysohn modulo I if $\forall a \neq b \in X, \exists\, U_1, U_2$ two $h\alpha$ −open subsets of $X:\ a \in U_1, b \in U_2$ and $cl^{h\alpha}(U_1) \cap cl^{h\alpha}(U_2) = \phi$.

Theorem 4.6: Consider $f: X \to Y$ is an $h\alpha$ −almost weakly continuous one-to-one function modulo I, J and Y is an $h\alpha$ −Urysohn space, then X is Hausdorff.

Proof. Since Y is $h\alpha$ −Urysohn modulo J, then $\forall a \neq b \in X, \exists\, V_1, V_2$ two $h\alpha$ −open subsets of Y : $f(a) \in V_1, f(b) \in V_2$ and $cl^{h\alpha}(V_1) \cap cl^{h\alpha}(V_2) = \phi$.

Now, if the function f is one-to-one, then $f(a) \neq f(b)$. But f is an $h\alpha$ −almost weakly continuous function modulo I, J, then $\exists U_1, U_2$ two $h\alpha$ −open subsets of $X : a \in U_1, b \in U_2$ and $f(U_1) \subseteq cl^{h\alpha}(V_1), f(U_2) \subseteq cl^{h\alpha}(V_2)$. Thus, U_1 and U_2 are disjoint.

Consequently, X is a Hausdorff space.

5. Strongly $h\alpha$ −Continuous Functions Modulo Ideals

Definition 5.1: A function $f: X \to Y$ is strongly $h\alpha$ −continuous modulo I, J if $f^{-1}(O)$ is an open subset of X modulo I for each $h\alpha$ −open subset $O \subseteq Y$.

Example 5.2: If $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{c\}\}$,
$\tau^{h\alpha} = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}, I = \{X, \phi, \{a, b\}, \{a, c\}, \{b, c\}\}, \sigma = \{Y, \phi, \{a\}\}$,
$\sigma^{h\alpha} = \{Y, \phi, \{a\}, \{b, c\}\}, J = \{Y, \phi\}$, and $f: X \to Y$ is given by $f(a) = c,\ f(c) = a$ and $f(b) = b$, then f is a strongly $h\alpha$ −continuous function modulo I, J.

Theorem 5.3: If $f: X \to Y$ is a strongly $h\alpha$ −continuous open function modulo I, J, then f is an $h\alpha$ −irresolute function modulo I, J.

Proof. Let $f: X \to Y$ is a strongly $h\alpha$ −continuous open function modulo I, J, then for every $h\alpha$ −open subset $O \subseteq Y$, $f^{-1}(O) \subseteq X$ is open modulo I. Hence $f^{-1}(O)$ is $h\alpha$ −open.

Thus, f is an $h\alpha$ −irresolute function modulo I, J.

The converse of the previous theorem need not be true.

Example 5.4: Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau^{h\alpha} = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$,
$I = \{X, \phi, \{a, b\}\}$, $\sigma = \{Y, \phi, \{a\}\}, J = \{Y, \phi\}$, $\sigma^{h\alpha} = \{Y, \phi, \{a\}, \{b, c\}\}$, then $f(x) = x$ is an $h\alpha$ −irresolute function modulo I, J but it is not a strongly $h\alpha$ −continuous open function modulo I, J since $f^{-1}(\{b, c\}) = \{b, c\} \subseteq X$, which is not an $h\alpha$ −open set modulo I.

Definition 5.5: A function $f: X \to Y$ is $h\alpha$ −totally continuous modulo I, J if $\forall O \subseteq Y$ an $h\alpha$ −open set modulo J, $f^{-1}(O)$ is a clopen subset of X modulo I.

Theorem 5.6: If $f: X \to Y$ is an $h\alpha$ −totally continuous function modulo I, J, then f is strongly $h\alpha$ −continuous modulo I, J.

Proof. Let f:X→Y be an $h\alpha$ −totally continuous function modulo I, J, then $\forall O \subseteq Y$ an $h\alpha$ −open set modulo J, $f^{-1}(O)$ is a clopen subset of X modulo I. Now, $f^{-1}(O)$ is an open subset of X modulo I. Thus, f is a strongly $h\alpha$ −continuous function modulo I, J.

The converse of the previous theorem is needs not to be true.

Example 5.7: If $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}$, $\tau^{h\alpha} = \{X, \phi, \{a\}, \{a, c\}\}$, $I = \{X, \phi, \{a, c\}\}, \sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}, \sigma^{h\alpha} = \{Y, \phi, \{a\}, \{a, c\}\}$, and $J = \{Y, \phi, \{a, c\}\}$, then $f(x) = x$ is a strongly $h\alpha-$continuous function modulo I, J but not an $h\alpha-$totally continuous function modulo I, J.

Theorem 5.8: If $f: X \to Y$ is a strongly $h\alpha-$continuous function modulo I, J, then f is $h\alpha-$irresolute.

Proof. Let $f: X \to Y$ be a strongly $h\alpha-$continuous function modulo I, J, and $O \subseteq Y$ an $h\alpha-$open set modulo J, then $f^{-1}(O)$ is a clopen subset of X modulo I.

Now, $f^{-1}(O)$ is an open subset of X modulo I. But, f is a strongly $h\alpha-$continuous function modulo I, J. Thus, f is $h\alpha-$irresolute.

Theorem 5.9: If $f: X \to Y$ is a strongly $h\alpha-$continuous function modulo I, J, then f is $h\alpha-$continuous.

Theorem 5.10: If $f: X \to Y$ is a strongly $h\alpha-$continuous function modulo I, J, and $g: Y \to Z$ is an almost $h\alpha-$continuous function modulo J, K, then $g \circ f$ is strongly $h\alpha-$continuous function modulo I, K.

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