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Feeble b-Supra Open Soft Sets and the Essential Topological Operators Inspired by Them

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Abstract. In the article at hand, we expand the classes of feeble α-supra open soft sets, feeble *semi*-supra open soft sets, and feeble *pre*-supra open soft sets by introducing the class of feeble b-supra open soft sets. This class is defined by b-supra open subsets of parametric supra topological spaces, so it associates the frameworks of supra topologies and soft supra topologies. We study the basic properties of feeble b-supra open soft sets and point out their behaviours under certain kinds of soft functions and the finite Cartesian product spaces. We also reveal the relationships between this class and certain existing classes, with the aid of illustrative examples. Following this line of research, we make use of the class of feeble b-supra open soft sets and b-closed sets to set up some topological operators in soft settings. We investigate the master features of these operators and derive formulas linking them.

1. Introduction

Soft set theory, set up by Molodtsov [1], is one of the recent approaches to deal with the uncertainty and imprecision we face in our daily issues. Molodtsov [1] pointed out that soft set theory provides a parameterized framework for effectively managing uncertainty, addressing limitations of earlier techniques such as fuzzy sets and rough sets, which require a membership function and an equivalence relation, respectively. Maji et al. [2], in 2002, demonstrated an application of this theory to select the optimal choices. Subsequently, many researchers have

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addressed numerous practical problems we face in various fields using the soft set approach as investigated in [3–6].

Maji et al. [7] put forward the basic principle of soft set theory by defining the primary operations and operators between soft sets. Afterwards, numerous contributions and research works proposed fresh operations on soft set theory and developed some previously known ideas, so that they will be suitable for building upon in the definitions of soft spaces and soft algebraic structures; see, for example, [8–10].

The soft topology concept was independently familiarized by Çağman et al. [12] and by Shabir and Naz [11]. Following that, soft topology rapidly emerged as an active research area, with a particular focus on showing the behaviors of classical topological notions within the soft framework. Min [13] looked at the main properties of soft regular spaces and proved that every soft T_3 is T_2. A new classification of soft spaces was proposed by El-Shafei et al. [14] with the aim of keeping more properties of classical separation axioms. Al-shami et al. [15] applied topological operators via soft setting to establish fresh types of soft separation axioms. Subsequently, Arar and Al-shami [16] developed a method for generating diverse families of soft separation axioms, corrected several interrelationships among existing ones, and answered open questions from the literature. Alqurashi [17], on the other hand, characterized R_i separation axioms using soft somewhat open sets. The framework of soft topology has also inspired a wide range of further investigations. These include studies on connectedness [18], separation axioms [19], compactness and Lindelöf properties [20–22], functions between soft topological spaces [23], continuity [24], Menger-type properties [25], maximal soft topologies [26], the Vietoris soft topology [27], fixed point theorems [28], expandable soft spaces [29], as well as various generalizations of soft open sets [30–33]. In a pioneering investigation, Al-shami and Kočinac [34] established a bridge between soft and classical topological operators and properties. Their findings reveal that many topological features can be transferred back and forth between enriched and extended soft topologies and their underlying parameter topologies.

Mashour et al. [35], in 1983, introduced the notion of supra topological spaces. Then, several authors studied topological ideas via supra topologies, such as separation axioms [36] and generalizations of open sets [37, 38]. Also, they applied supra topologies to model some practical problems, such as image processing [39] and decision making [40]. In 2014, El-Sheikh et al. [41] familiarized the idea of soft supra topological spaces and probed its main features. The behavior of topological concepts within soft supra topological spaces has been a subject of recent inquiry. A central question has been which classical properties survive in these generalized frameworks and which do not. Several authors have addressed this issue from different perspectives. For instance, Al-shami et al. [42] proposed a fresh two-category classification of soft supra topological spaces based on separation axioms. New notions of supra continuity and the decomposition of supra soft locally closed sets were introduced by Abd El-latif [43]. The properties of supra soft sd-operators and supra soft δ-closure operators were examined in [44, 45]. Meanwhile, [46–49] focused on defining and studying new families of supra soft sets, and [50] contributed new forms of covering properties.

1.1. **Motivation for the present work.** The current study is motivated by the following three factors:

- **(i):** To present a novel class of soft sets that expand the given supra soft topology, which allows us to reexamine the notions and properties of soft supra topological spaces.
- **(ii):** To broaden the existing classes of generalizations of supra soft open sets, such as feeble α-supra open soft sets [51], feeble *semi*-supra open soft sets [52], and feeble *pre*-supra open soft sets [53].
- **(iii):** To emphasize the significance of soft structures (i.e., soft topology, soft supra topology, soft infra soft topology) to establish several forms for each concept defined on supra topological spaces.

It is worth noting that the approach adopted herein was also explored and debated in several published manuscripts via soft topologies, such as those defined by using semi open soft sets [54], β-open soft sets [55], pre open soft sets [56], α-open soft sets [57], and parametric somewhat-open soft sets [58].

1.2. **Organization of the present study.** We lay out this work as follows. In Section 2, we mention the definitions and preliminary results that the reader needs to understand this study. Then, in Section 3, we define the core concept of this study " feeble b-supra open soft sets", which forms a novel generalization class of supra open soft sets; this definition is inspired by classical supra topological spaces that are built by the given soft supra topological spaces. Basic properties of this class are explored, and the relationships associated with famous previous classes are illustrated with some concrete counterexamples. In Section 4, we familiarize some topological operators induced from feeble b-supra open soft sets and feeble b-supra closed soft sets and infer some formulas that link them. Finally, we clarify the importance of the proposed approach and present the key contributions of this work in Section 5.

2. Preliminaries

This section provides the essential background (definitions and results) and notation used throughout the article.

Definition 2.1. *[1] If we have a set of objects or alternatives Ω that are described by a set of parameters or attributes $\mathcal{P}$, then we name a mapping Θ associated each object in Ω with its attributes in $\mathcal{P}$ a soft set. That is, a soft set is a set-valued mapping given by $\Theta : \mathcal{P} \to 2^{\Omega}$, where 2^{Ω} is the power set of Ω. For simplicity, we use $(\Theta, \mathcal{P})$ to refer to a soft set, and we mathematically represent it as follows*

$$(\Theta, \mathcal{P}) = \{(p, \Theta(p)) : p \in \mathcal{P} \text{ and } \Theta(p) \in 2^{\Omega}\}.$$

The next examples illustrates how we make use of a soft set to describe a practical situation.

Example 2.1. *Let the universal set of apartments be:*

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$$

Let the parameter set be:

$$\mathcal{P} = \{p_1, p_2, p_3, p_4\},$$

where:

- $p_1 =$ *modern*
- $p_2 =$ *quiet*
- $p_3 =$ *affordable*
- $p_4 =$ *furnished*

Define the soft set $(\Omega, \mathcal{P})$*:* $\Omega : \mathcal{P} \to 2^{\Omega}$ *by the tabular representation displayed in Table 1, where* ✓ *means to that the element has the characteristic and* $\times$ *means means it does not.*

Apartments	p_1 (*modern*)	p_2 (*quiet*)	p_3 (*affordable*)	p_4 (*furnished*)
ω_1	✓	$\times$	✓	$\times$
ω_2	✓	✓	$\times$	✓
ω_3	✓	✓	$\times$	$\times$
ω_4	$\times$	$\times$	✓	✓

Table 1. Soft set representation of apartments with respect to given parameters

Interpretation:

- $\Theta(p_1) = \{\omega_1, \omega_2, \omega_3\}$ *(modern apartments)*
- $\Theta(p_2) = \{\omega_2, \omega_3\}$ *(quiet apartments)*
- $\Theta(p_3) = \{\omega_1, \omega_4\}$ *(affordable apartments)*
- $\Theta(p_4) = \{\omega_2, \omega_4\}$ *(furnished apartments)*

This soft set can be used for decision-making; e.g., selecting an apartment satisfying $p_3 \wedge p_4$ *yields* ω_4 *or opting for an apartment meeting as many attributes as possible under consideration yields* ω_2*.*

Definition 2.2. *[7, 59] The soft set set* $(\Theta, \mathcal{P})$ *given by* $\Theta(p) = \Omega$ *(resp.,* $\Theta(p) = \emptyset$*,* $\Theta(p) = \Omega$ *or* $\emptyset$*) for all* $p \in \mathcal{P}$ *is named an absolute (resp., a null, a pseudo constant) soft set, and it symbolized by* $\widetilde{\Omega}$ *(resp.,* Φ*,* $\widetilde{\Omega} * \Phi$*). If "*$\Theta(p) = \{\omega\}$ *for a fixed parameter* p *and* $\Theta(a) = \emptyset$ *for all* $a \in \mathcal{P} - \{p\}$*" hold, then we name* $(\Theta, \mathcal{P})$ *a soft point and assign by* ω_p*. We write* $\omega_p \in (\Theta, \mathcal{P})$ *if* $\omega \in \Theta(p)$*.*

Definition 2.3. *[60] We write* $(\Theta, \mathcal{P}) \widetilde{\subseteq} (O, \mathcal{P})$*, as an expression of that* $(\Theta, \mathcal{P})$ *is subset of* $(O, \mathcal{P})$*, if for each* $p \in \mathcal{P}$ *we have* $\Theta(p) \subseteq O(p)$ *holds.*

Definition 2.4. *[8] We write* $(\Theta^c, \mathcal{P})$*, as an expression of the complement of* $(\Theta, \mathcal{P})$*, if for each* $p \in \mathcal{P}$ *we have* $\Theta^c(p) = \Omega - \Theta(p)$*.*

Throughout this work, we refer to the complement by $(\Theta^c,\mathcal{P})$ or $(\Theta,\mathcal{P})^c$.

Definition 2.5. *The intersection, union, difference, and Cartesian product of soft sets $(\Theta,\mathcal{P})$ and $(O,\mathcal{P})$ are respectively defined by the following:*

- **(i):** $(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P}) = (W,\mathcal{P})$, *where* $W(p) = \Theta(p)\bigcap O(p)$ *for all* $p\in\mathcal{P}$.
- **(ii):** $(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P}) = (W,\mathcal{P})$, *where* $W(p) = \Theta(p)\bigcup O(p)$ *for all* $p\in\mathcal{P}$.
- **(iii):** $(\Theta,\mathcal{P})\setminus(O,\mathcal{P}) = (W,\mathcal{P})$, *where* $W(p) = \Theta(p)\setminus O(p)$ *for all* $p\in\mathcal{P}$.
- **(iv):** $(\Theta,\mathcal{P})\times(O,\mathcal{P}) = (W,\mathcal{P})$, *where* $W(p_1,p_2) = \Theta(p_1)\times O(p_2)$ *for all* $(p_1,p_2)\in\mathcal{P}\times\mathcal{P}$.

Soft functions have been defined in [61] as follows.

Definition 2.6. *We name $\mathcal{H}_\varphi : 2^{\Omega_\mathcal{P}} \to 2^{\Lambda_\mathcal{Q}}$, where $\mathcal{H} : \Omega \to \Lambda$ and $\varphi : \mathcal{P} \to \mathcal{Q}$ are crisp functions, a soft function providing that the image of each soft point ω_p in $2^{\Omega_\mathcal{P}}$ is one and only one soft point λ_q in $2^{\Lambda_\mathcal{Q}}$ such that*

$$\mathcal{H}_\varphi(\omega_p) = \mathcal{H}(\omega)_{\varphi(p)} \text{ for all } \omega_p \in 2^{\Omega_\mathcal{P}}.$$

In addition, $\mathcal{H}_\varphi^{-1}(\lambda_q) = \widetilde{\bigcup}_{\substack{\omega\in\mathcal{H}^{-1}(\lambda)\\ p\in\varphi^{-1}(q)}} \omega_p$ *for each* $\lambda_q \in 2^{\Lambda_\mathcal{Q}}$.

Accordingly, we have:

$$\mathcal{H}_\varphi(\Theta,\mathcal{P}) = \widetilde{\bigcup}_{\omega_p\in(\Theta,\mathcal{P})}\mathcal{H}_\varphi(\omega_p),$$

and

$$\mathcal{H}_\varphi^{-1}(O,\mathcal{Q}) = \widetilde{\bigcup}_{\lambda_q\in(O,\mathcal{Q})}\mathcal{H}_\varphi^{-1}(\lambda_q).$$

We say that a soft function $\mathcal{H}_\varphi$ is injective (surjective) providing that the functions $\mathcal{H}$ and φ are injective (surjective).

Theorem 2.1. *[23] If $\mathcal{H}_\varphi$: $2^{\Omega_\mathcal{P}} \to 2^{\Lambda_\mathcal{Q}}$ is a soft function and $(\Theta,\mathcal{P})\widetilde{\subseteq}\widetilde{\Omega}$ and $(O,\mathcal{Q})\widetilde{\subseteq}\widetilde{\Lambda}$, then:*

- **(i):** $(\Theta,\mathcal{P})\widetilde{\subseteq}\mathcal{H}_\varphi^{-1}(\mathcal{H}_\varphi(\Theta,\mathcal{P}))$, *and the equality holds when* $\mathcal{H}_\varphi$ *is injective.*
- **(ii):** $\mathcal{H}_\varphi(\mathcal{H}_\varphi^{-1}(O,\mathcal{Q}))\widetilde{\subseteq}(O,\mathcal{Q})$, *and the equality holds when* $\mathcal{H}_\varphi$ *is surjective.*

Definition 2.7. *[1] A class S of soft sets defined over the universe Ω with a set of parameters $\mathcal{P}$ is said to be a soft supra topology (briefly, soft ST) if it meets the subsequent axioms:*

- **(i):** $\Phi,\widetilde{\Omega}\in\mathrm{S}$.
- **(ii):** S *is closed under the arbitrary unions.*

A soft supra topological space (soft ST-space, in short) is symbolized by $(\Omega,\mathrm{S},\mathcal{P})$. The term of the supra open soft sets is given for each soft set that belongs to S, and the term of the supra closed soft sets is given for each soft set whose complement belongs to S.

Definition 2.8. *[1] For a soft subset $(\Theta,\mathcal{P})$ of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$, the supra soft closure and supra soft interior of $(\Theta,\mathcal{P})$, denoted respectively by $cl(\Theta,\mathcal{P})$ and $int(\Theta,\mathcal{P})$, are defined as follows:*

- **(i):** $cl(\Theta,\mathcal{P}) = \widetilde{\bigcap}\{(W,\mathcal{P}) : (\Theta,\mathcal{P})\widetilde{\subseteq}(W,\mathcal{P}) \text{ and } (W^c,\mathcal{P})\in\mathrm{S}\}$.
- **(ii):** $int(\Theta,\mathcal{P}) = \widetilde{\bigcup}\{(O,\mathcal{P})\in\mathrm{S} : (O,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P})\}$.

Theorem 2.2. *[1] Let $(\Omega, \mathrm{S}, \mathcal{P})$ be a soft ST-space. Then for every $p \in \mathcal{P}$*

$$\mathrm{S}_p = \{\Theta(p) : (\Theta, \mathcal{P}) \in \mathrm{S} \text{ is a supra topology on } \Omega\}.$$

A supra topology built by Theorem 2.2 is known as a parametric supra topology.

Definition 2.9. *A soft subset $(\Theta, \mathcal{P})$ of soft ST-space $(\Omega, \mathrm{S}, \mathcal{P})$ is said to be:*

- **(i):** *a feeble α-supra open soft set [51] if $\Theta(p)$ is nonempty α-supra open subset of (Ω, S_p) for some $p \in \mathcal{P}$ or $\Theta(p) = \emptyset$ for all $p \in \mathcal{P}$.*
- **(ii):** *a feeble semi-supra open soft set [52] if $\Theta(p)$ is nonempty semi-supra open subset of (Ω, S_p) for some $p \in \mathcal{P}$ or $\Theta(p) = \emptyset$ for all $p \in \mathcal{P}$.*
- **(iii):** *a feeble pre-supra open soft set [53] if $\Theta(p)$ is nonempty pre-supra open subset of (Ω, S_p) for some $p \in \mathcal{P}$ or $\Theta(p) = \emptyset$ for all $p \in \mathcal{P}$.*

Remark 2.1. *The concept of a feeble pre-supra open soft set was studied in [53] under the name of WPS supra-open set.*

Definition 2.10. *[24] In a soft function $\mathcal{H}_\varphi : (\Omega, \mathrm{S}, \mathcal{P}) \to (\Lambda, \mathrm{T}, \mathcal{P})$, if $\mathcal{H}_\varphi^{-1}(\Theta, \mathcal{P})$ is a supra open soft set for each supra soft open set $(\Theta, \mathcal{P})$, then we term $\mathcal{H}_\varphi$ a supra soft continuous function.*

3. Feeble b-supra open soft sets and their main features

In this section, we put forward the idea of feeble b-supra open soft sets as a new generalization of supra open soft sets. We look at its basic features and illustrate, with the help of counterexamples, that the family of feeble b-supra open soft sets expands the classes of feeble α-supra open soft sets, feeble *semi*-supra open soft sets, and feeble *pre*-supra open soft sets.

We formulate our main concept in the next definition.

Definition 3.1. *Let $(\Omega, \mathrm{S}, \mathcal{P})$ be a soft ST-space and $(\Theta, \mathcal{P})$ be a soft set over Ω. If $(\Theta, \mathcal{P})$ is a null soft set or there is some $\Theta(p)$ are b-supra open subsets of (Ω, S_p), then $(\Theta, \mathcal{P})$ is said to be a feeble b-supra open soft set. That is, $(\Omega, \mathrm{S}, \mathcal{P})$ a feeble b-supra open soft set if*

$$\Theta(p) = \emptyset \text{ for all } p \in \mathcal{P}$$

or

$$\emptyset \neq \Theta(p) \subseteq cl(int(\Theta(p))) \cup int(cl(\Theta(p))) \text{ for some } p \in \mathcal{P}.$$

Remark 3.1. *The closure and interior operators appearing in the above definition are computed with respect to the parametric supra topological space (Ω, S_p), which was introduced via Theorem 2.2.*

Definition 3.2. *Let $(\Omega, \mathrm{S}, \mathcal{P})$ be a soft ST-space and $(\Theta, \mathcal{P})$ be a soft set over Ω. We call $(\Theta, \mathcal{P})$ a feeble b-supra closed soft set if $(\Theta^c, \mathcal{P})$ is a feeble b-supra open soft set.*

Proposition 3.1. *Let $(\Omega, \mathrm{S}, \mathcal{P})$ be a soft ST-space and $(\Theta, \mathcal{P})$ be a soft set over Ω. Then, $(\Theta, \mathcal{P})$ is a feeble b-supra closed soft set if and only if*

$$(\Theta, \mathcal{P}) = \widetilde{\Omega} \text{ or } cl(int(\Theta(p))) \cap int(cl(\Theta(p))) \subseteq \Theta(p) \neq \Omega \text{ for some } p \in \mathcal{P}.$$

Proof. Necessity: Let $(\Theta,\mathcal{P})$ be a feeble b-supra closed soft set. Then,

$$(\Theta^c,\mathcal{P}) = \Phi,$$

or

$$\emptyset \neq \Theta^c(p) \subseteq cl(int(\Theta^c(p))) \cup int(cl(\Theta^c(p))),$$

for some $p \in \mathcal{P}$. This automatically leads to that

$$\text{either } (\Theta,\mathcal{P}) = \widetilde{\Omega} \text{ or } cl(int(\Theta(p))) \cap int(cl(\Theta(p))) \subseteq \Theta(p) \neq \Omega \text{ for some } p \in \mathcal{P}.$$

This ends the proof of this side.
Sufficiency: Let $(\Theta,\mathcal{P})$ be a soft set such that

$$(\Theta,\mathcal{P}) = \widetilde{\Omega},$$

or

$$int(cl(\Theta(p))) \cap cl(int(\Theta(p))) \subseteq \Theta(p) \neq \Omega \text{ for some } p \in \mathcal{P}.$$

By taking the complement we obtain

$$(\Theta^c,\mathcal{P}) = \Phi,$$

or

$$\emptyset \neq \Theta^c(p) \subseteq cl(int(\Theta^c(p))) \cup int(cl(\Theta^c(p))) \text{ for some } p \in \mathcal{P}.$$

Accordingly, $(\Theta^c,\mathcal{P})$ is a feeble b-supra open soft set, and so $(\Theta,\mathcal{P})$ is a feeble b-supra closed soft set. □

We will provide the following example to explain the previous definitions and clarify some of the relationships revealed later in this section.

Example 3.1. *Let the next soft sets be defined over the universe $\Omega = \{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6\}$ with a set of parameters $\mathcal{P} = \{p_1,p_2\}$ as follows:*

$$(\Theta_1,\mathcal{P}) = \{(p_1,\{\omega_1\}),(p_2,\{\omega_1,\omega_2\})\};$$

$$(\Theta_2,\mathcal{P}) = \{(p_1,\{\omega_6\}),(p_2,\{\omega_2,\omega_3\})\};$$

$$(\Theta_3,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_6\}),(p_2,\{\omega_1,\omega_2,\omega_3\})\};$$

$$(\Theta_4,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_3,\omega_5\}),(p_2,\{\omega_1,\omega_2,\omega_4,\omega_5,\omega_6\})\};$$

$$(\Theta_5,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_3,\omega_5,\omega_6\}),(p_2,\Omega)\};$$

$$(\Theta_6,\mathcal{P}) = \{(p_1,\{\omega_3,\omega_4,\omega_5,\omega_6\}),(p_2,\Omega)\}; \text{ and}$$

$$(\Theta_7,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_3,\omega_4,\omega_5,\omega_6\}),(p_2,\Omega)\}.$$

Then, $\mathrm{S} = \{\Phi,\widetilde{\Omega},(\Theta_j,\mathcal{P}) : j = 1,2,...,7\}$ is a soft supra topology over Ω with $\mathcal{P}$. To determine whether a soft set is feeble b-supra open soft set, we first specify the parametric supra topologies inspired by the given soft supra topology S:

$\mathrm{S}_{p_1} = \{\emptyset,\Omega,\{\omega_1\},\{\omega_6\},\{\omega_1,\omega_6\},\{\omega_1,\omega_3,\omega_5\},\{\omega_1,\omega_3,\omega_5,\omega_6\},\{\omega_3,\omega_4,\omega_5,\omega_6\},\{\omega_1,\omega_3,\omega_4,\omega_5,\omega_6\}\}$, *and*

$$S_{p_2} = \{\emptyset, \Omega, \{\omega_1, \omega_2\}, \{\omega_2, \omega_3\}, \{\omega_1, \omega_2, \omega_3\}, \{\omega_1, \omega_2, \omega_4, \omega_5, \omega_6\}\}.$$

By taking the following soft sets:

$$(O_1, \mathcal{P}) = \{(p_1, \{\omega_2\}), (p_2, \{\omega_3, \omega_4\})\};$$

$$(O_2, \mathcal{P}) = \{(p_1, \{\omega_1, \omega_5\}), (p_2, \{\omega_1\})\}; \textit{ and}$$

$$(O_3, \mathcal{P}) = \{(p_1, \emptyset), (p_2, \Omega)\}.$$

We remark that:

- **(i):** *For $(O_1, \mathcal{P})$, we have by simple calculations that $cl^{p_1}(int^{p_1}(\{\omega_2\})) = int^{p_1}(cl^{p_1}(\{\omega_2\})) = \emptyset$ and $cl^{p_2}(int^{p_2}(\{\omega_3, \omega_4\})) = int^{p_2}(cl^{p_2}(\{\omega_3, \omega_4\})) = \emptyset$. Thus, $(O_1, \mathcal{P})$ is not a feeble b-supra open soft set since neither its first component $\{\omega_2\}$ is a b-supra open subset of (Ω, S_{p_1}) nor its second component $\{\omega_3, \omega_4\}$ is a b-supra open subset of (Ω, S_{p_2}).*
- **(ii):** *For $(O_2, \mathcal{P})$, we have by simple calculations that $cl^{p_1}(int^{p_1}(\{\omega_1, \omega_5\})) = \{\omega_1, \omega_2\}$ and $int^{p_1}(cl^{p_1}(\{\omega_1, \omega_5\})) = \{\omega_1, \omega_3, \omega_5\}$. Thus, $(O_2, \mathcal{P})$ is a feeble b-supra open soft set since its first component $\{\omega_1, \omega_5\}$ is a b-supra open subset of (Ω, S_{p_1}).*
- **(iii):** *For $(O_3, \mathcal{P})$, it is clear that its second component Ω is a b-supra open subset of (Ω, S_{p_2}), so $(O_3, \mathcal{P})$ is a feeble b-supra open soft set.*

Remark 3.2. *The following set of properties remains valid.*

- **(i):** *Every pseudo constant soft subset is a feeble b-supra open soft set.*
- **(ii):** *If $\emptyset$ (resp., Ω) is a component of a soft set $(\Theta, \mathcal{P})$, then this soft set is feeble b-supra closed soft (resp., feeble b-supra open soft).*

In what follows, we prove that the class of feeble b-supra open soft sets is wider than the classes of supra open soft sets, feeble α-supra open soft sets, feeble *semi*-supra open soft sets, and feeble *pre*-supra open soft sets.

Proposition 3.2. *Every supra open soft set (feeble α-supra open soft set, feeble semi-supra open soft set, feeble pre-supra open soft set) is a feeble b-supra open soft set.*

Proof. The proof is warranted by the well-known relationship between these classes via (classical) supra topological spaces, which states that every b-supra open set is supra open, α-supra open, *semi*-supra open, and *pre*-supra open. □

Corollary 3.1. *Every supra closed soft set (feeble α-supra closed soft set, feeble semi-supra closed soft set, feeble pre-supra closed soft set) is a feeble b-supra closed soft set.*

We clarify, in the subsequent instance, that the converse of Proposition 3.2 is not always true.

Example 3.2. *We assume a soft ST-space $(\Omega, S, \mathcal{P})$ displayed in Example 3.1. Let a soft set $(W, \mathcal{P})$ be given as follows:*

$$(W, \mathcal{P}) = \{(p_1, \emptyset), (p_2, \{\omega_1, \omega_2, \omega_3\})\}.$$

By simple calculations, we obtain $cl^{p_2}(int^{p_2}(\{\omega_1,\omega_2,\omega_3\})) = \{\omega_1,\omega_2\}$ *and* $int^{p_2}(cl^{p_2}(\{\omega_1,\omega_2,\omega_3\})) = \{\omega_1,\omega_3,\omega_5\}$. *Then,* $W(p_2) = \{\omega_1,\omega_2,\omega_3\} \subseteq cl^{p_2}(int^{p_2}(W(p_2))) \cup int^{p_2}(cl^{p_2}(W(p_2)))$, *so* $W(p_2)$ *is a b-supra open subset of* (Ω, S_{p_2}). *Hence,* $(W,\mathcal{P})$ *is a feeble b-supra open soft set. On the other hand, we note that*

(i): $W(p_2) = \{\omega_1,\omega_2,\omega_3\} \nsubseteq cl^{p_2}(int^{p_2}(\{\omega_1,\omega_2,\omega_3\})) = \{\omega_1,\omega_2\}$, *so* $(W,\mathcal{P})$ *is not a feeble semi-supra open soft set since its second component* $\{\omega_1,\omega_2,\omega_3\}$ *is not a semi-supra open subset of* (Ω, S_{p_2}) *and its first component is an empty set.*

(ii): $W(p_2) = \{\omega_1,\omega_2,\omega_3\} \nsubseteq int^{p_2}(cl^{p_2}(\{\omega_1,\omega_2,\omega_3\})) = \{\omega_1,\omega_3,\omega_5\}$, *so* $(W,\mathcal{P})$ *is not a feeble pre-supra open soft set since its second component* $\{\omega_1,\omega_2,\omega_3\}$ *is not a pre-supra open subset of* (Ω, S_{p_2}) *and its first component is an empty set.*

(iii): *By (i) and (ii) above,* $(W,\mathcal{P})$ *is not a feeble* α*-supra open soft set.*

We pay your attention to the fact that the concepts of b-supra open soft sets and feeble b-supra open soft sets are unrelated, as illustrated by the next examples.

Example 3.3. *We assume a soft ST-space* $(\Omega, S, \mathcal{P})$ *displayed in Example 3.1. We showed that* $(O_2,\mathcal{P})$ *is a feeble b-supra open soft set. In contrast,* $cl(int(O_2,\mathcal{P})) = int(cl(O_2,\mathcal{P})) = \Phi$*; therefore,* $(O_2,\mathcal{P})$ *is not a b-supra open soft set.*

Example 3.4. *Let the next soft sets be defined over the universe* $\Omega = \{\omega_1,\omega_2,\omega_3\}$ *with a set of parameters* $\mathcal{P} = \{p_1, p_2\}$ *as follows:*

$$(\Theta_1,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_2\}),(p_2,\{\omega_1,\omega_2\})\} \text{ and}$$

$$(\Theta_2,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_3\}),(p_2,\{\omega_2,\omega_3\})\}.$$

Then, $S = \{\Phi, \widetilde{\Omega}, (\Theta_j,\mathcal{P}) : j = 1,2\}$ *is a soft supra topology over* Ω *with* $\mathcal{P}$. *The parametric supra topologies inspired by the given soft supra topology* S *are:*

$$S_{p_1} = \{\emptyset, \Omega, \{\omega_1,\omega_2\}, \{\omega_1,\omega_3\} \text{ and}$$

$$S_{p_2} = \{\emptyset, \Omega, \{\omega_1,\omega_2\}, \{\omega_2,\omega_3\}\}.$$

Defining a soft set $(W,\mathcal{P})$ *as follows*

$$(W,\mathcal{P}) = \{(p_1,\{\omega_2\}),(p_2,\{\omega_3\})\}.$$

Now, $cl(int(W,\mathcal{P})) = \emptyset$ *and* $int(cl(W,\mathcal{P})) = \widetilde{\Omega}$, *so* $(W,\mathcal{P})$ *is a b-supra open soft set. In contrast,* $(W,\mathcal{P})$ *is not a feeble b-supra open soft set since* $cl^{p_1}(int^{p_1}(\{\omega_2\})) = \emptyset$ *and* $int^{p_2}(cl^{p_2}(\{\omega_3\})) = \emptyset$.

In Figure 1, we outline the relationships between the proposed class and the previous ones defined within soft ST-spaces.

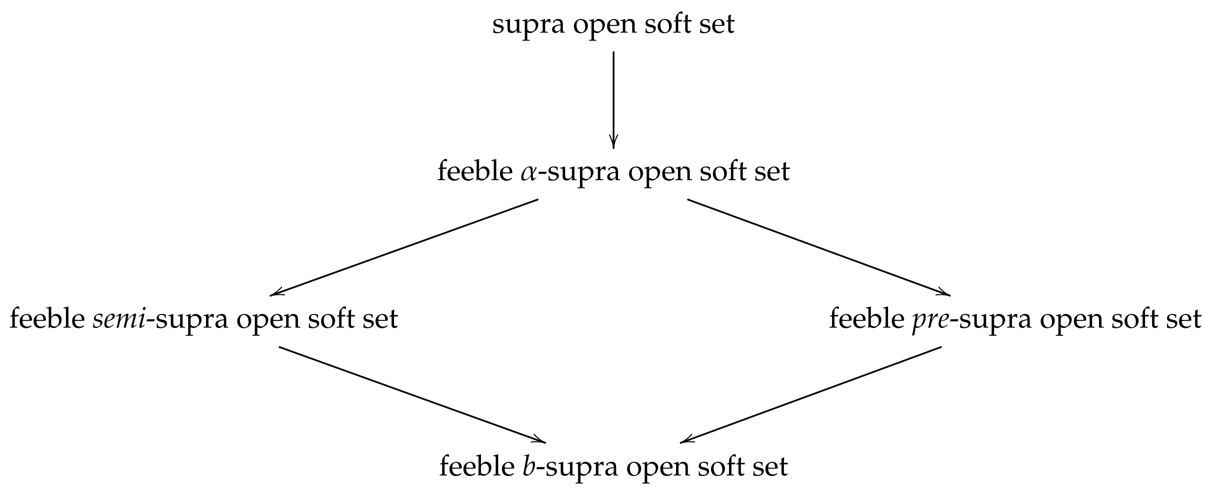

Figure 1. The relationships between the class of feeble b-supra open soft sets and some celebrated foregoing classes

Theorem 3.1. *Let $\mathcal{H}_{\varphi}$: $(\Omega, \mathrm{S}, \mathcal{P}) \to (\Lambda, \mathrm{R}, \mathcal{Q})$ be a soft function. If $\varphi : \mathcal{P} \to \mathcal{Q}$ is an injective function and for each p we have $\mathcal{H}$: $(\Omega, \mathrm{S}_p) \to (\Lambda, \mathrm{R}_{\varphi(p)=q})$ is a supra bicontinuous function. Then, for each feeble b-supra open soft set $(\Theta, \mathcal{P})$, $\mathcal{H}_{\varphi}(\Theta, \mathcal{P})$ is a feeble b-supra open soft set.*

Proof. Let $(\Theta, \mathcal{P})$ be a feeble b-supra soft subset of a soft ST-space $(\Omega, \mathrm{S}, \mathcal{P})$ such that $(\Theta, \mathcal{P}) \neq \widetilde{\Phi}$. Then, there is $p \in \mathcal{P}$ such that $\Theta(p)$ is a nonempty b-open subset of (Ω, S_p). Assume $\varphi(p) = q$. Now, we have

$$\mathcal{H}(\Theta(p)) \subseteq \mathcal{H}(cl^q(int^q(\Theta(p))) \cup int^q(cl^q(\Theta(p)))) = \mathcal{H}(cl^q(int^q(\Theta(p)))) \cup \mathcal{H}(int^q(cl^q(\Theta(p)))). \tag{3.1}$$

By hypothesis of supra bicontinuity of $\mathcal{H}$: $(\Omega, \mathrm{S}_p) \to (\Lambda, \mathrm{R}_{\varphi(p)=q})$ it follows for any set $V \subseteq \Omega$: $\mathcal{H}(int(V)) \subseteq int(\mathcal{H}(V))$ (by supra open) and $\mathcal{H}(cl(V)) \subseteq cl(\mathcal{H}(V))$ (by supra continuity). This implies that

$$\mathcal{H}(cl^q(int^q(\Theta(p)))) \cup \mathcal{H}(int^q(cl^q(\Theta(p)))) \subseteq cl^q(int^q(\mathcal{H}(\Theta(p)))) \cup int^q(cl^q(\mathcal{H}(\Theta(p)))). \tag{3.2}$$

By (3.1) and (3.2), it follows that $\mathcal{H}(\Theta(p)) \subseteq cl^q(int^q(\mathcal{H}(\Theta(p)))) \cup int^q(cl^q(\mathcal{H}(\Theta(p))))$. It yields by injectiveness of φ that $\mathcal{H}(\Theta(p))$ is a nonempty b-supra open subset of (Λ, R_q). Hence, $\mathcal{H}_{\varphi}(\Theta, \mathcal{P})$ is a feeble b-supra open soft subset of $(\Lambda, \mathrm{R}, \mathcal{Q})$. □

Corollary 3.2. *The property of being a feeble b-supra open soft set is a soft ST property.*

Theorem 3.2. *The finite product of feeble b-supra open soft sets is a feeble b-supra open soft set.*

Proof. Let $(\Theta, \mathcal{P})$ and $(O, \mathcal{Q})$ be feeble b-supra open soft sets of soft ST-spaces $(\Omega, \mathrm{S}, \mathcal{P})$ and $(\Lambda, \mathrm{R}, \mathcal{Q})$, respectively. Now, we can find $p \in \mathcal{P}$ and $q \in \mathcal{Q}$ s. t. $\Theta(p) \neq \emptyset$ and $O(q) \neq \emptyset$ are b-supra-open subsets of parametric supra topological spaces (Ω, S_p) and (Λ, R_q), respectively. Putting

$(W, \mathcal{P} \times Q) = (\Theta, \mathcal{P}) \times (O, Q)$, then $W(p,q) = \Theta(p) \times O(q)$, where $(p,q) \in \mathcal{P} \times Q$. As we know that the product of two nonempty b-supra open sets is nonempty b-supra open, so $W(p,q)$ is a nonempty b-supra open set of $(\Omega \times \Lambda, \tau_{p \times q})$, where τ is product topology on $\Omega \times \Lambda$ induced from the soft supra topologies S and R. Hence, $(\Theta, \mathcal{P}) \times (O, Q)$ is a feeble b-supra open soft subset of $(\Omega \times \Lambda, \tau, \mathcal{P} \times Q)$. □

As we showed for the previous classes of generalizations of supra open (supra closed) soft sets, the current class of feeble b-supra open soft sets (feeble b-supra closed soft sets) is not closed under the soft intersection and union operators. The next instance clarifies this remark.

Example 3.5. *Let the next soft sets be defined over the universe $\Omega = \{\omega_1, \omega_2, \omega_3\}$ with a set of parameters $\mathcal{P} = \{p_1, p_2\}$ as follows:*

$$(\Theta_1, \mathcal{P}) = \{(p_1, \Omega), (p_2, \{\omega_1\})\};$$

$$(\Theta_2, \mathcal{P}) = \{(p_1, \{\omega_1\}), (p_2, \Omega)\};$$

$$(\Theta_3, \mathcal{P}) = \{(p_1, \{\omega_2\}), (p_2, \{\omega_3\})\};$$

$$(\Theta_4, \mathcal{P}) = \{(p_1, \{\omega_1, \omega_2\}), (p_2, \Omega)\};$$

$$(\Theta_5, \mathcal{P}) = \{(p_1, \Omega), (p_2, \{\omega_1, \omega_3\})\};$$

$$(\Theta_6, \mathcal{P}) = \{(p_1, \{\omega_2, \omega_3\}), (p_2, \{\omega_1, \omega_2\})\};$$

$$(\Theta_7, \mathcal{P}) = \{(p_1, \Omega), (p_2, \{\omega_1, \omega_2\})\};\ \text{and}$$

$$(\Theta_8, \mathcal{P}) = \{(p_1, \{\omega_2, \omega_3\}), (p_2, \Omega)\}.$$

Then, $\mathsf{S} = \{\Phi, \widetilde{\Omega}, (\Theta_j, \mathcal{P}) : j = 1, 2, ..., 8\}$ is a soft supra topology over Ω with $\mathcal{P}$. The parametric supra topologies inspired by the given soft supra topology S are::

$$\mathsf{S}_{p_1} = \{\emptyset, \Omega, \{\omega_1\}, \{\omega_2\}, \{\omega_1, \omega_2\}, \{\omega_2, \omega_3\}\},\ \text{and}$$

$$\mathsf{S}_{p_2} = \{\emptyset, \Omega, \{\omega_1\}, \{\omega_3\}, \{\omega_1, \omega_3\}, \{\omega_1, \omega_2\}\}.$$

By taking the following soft sets:

$$(W_1, \mathcal{P}) = \{(p_1, \{\omega_1\}), (p_2, \{\omega_2\})\};$$

$$(W_2, \mathcal{P}) = \{(p_1, \{\omega_3\}), (p_2, \{\omega_3\})\};$$

$$(W_3, \mathcal{P}) = \{(p_1, \{\omega_1, \omega_3\}), (p_2, \Omega)\};\ \text{and}$$

$$(W_4, \mathcal{P}) = \{(p_1, \Omega), (p_2, \{\omega_2, \omega_3\})\}.$$

We remark that:

- **(i):** *$(W_1, \mathcal{P})$ and $(W_2, \mathcal{P})$ are feeble b-supra open soft sets. Now, $(W_1, \mathcal{P}) \widetilde{\cup} (W_2, \mathcal{P}) = \{(p_1, \{\omega_1, \omega_3\}), (p_2, \{\omega_2, \omega_3\})\}$ and by simple calculations it follows that $cl^{p_1}(int^{p_1}(\{\omega_1, \omega_3\})) = int^{p_1}(cl^{p_1}(\{\omega_1, \omega_3\})) = \{\omega_1\}$ and $cl^{p_2}(int^{p_2}(\{\omega_2, \omega_3\})) = int^{p_2}(cl^{p_2}(\{\omega_2, \omega_3\})) = \{\omega_3\}$. Accordingly, $\{\omega_1, \omega_3\}$ is not a b-supra open subset of $(\Omega, \mathsf{S}_{p_1})$ and $\{\omega_2, \omega_3\}$ is not a b-supra open subset of $(\Omega, \mathsf{S}_{p_2})$. Thus, $(W_1, \mathcal{P}) \widetilde{\cup} (W_2, \mathcal{P})$ is not a feeble b-supra open soft set.*

(ii): $(W_3,\mathcal{P})$ *and* $(W_4,\mathcal{P})$ *are feeble b-supra open soft sets. Now,* $(W_3,\mathcal{P})\widetilde{\cap}(W_4,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_3\}),(p_2,\{\omega_2,\omega_3\})\}$ *According to the computations made in (i) above, we obtain* $(W_3,\mathcal{P})\widetilde{\cap}(W_4,\mathcal{P})$ *is not a feeble b-supra open soft set.*

4. Feeble b-supra interior and feeble b-supra closure operators

In this segment, we employ the classes of feeble b-supra open soft sets and feeble b-supra closed soft sets to introduce the notions of interior, closure, frontier, and accumulation operators. We discuss their core properties and conclude the relationships among them. Furthermore, through counterexamples, elucidate that the feeble b-supra interior and feeble b-supra closure of a soft set fail to be a feeble b-supra open set and a feeble b-supra closed set, respectively.

Definition 4.1. *The feeble b-supra interior points of a soft subset* $(\Theta,\mathcal{P})$ *of a soft ST-space* $(\Omega,\mathrm{S},\mathcal{P})$*, denoted by* $int^b(\Theta,\mathcal{P})$*, is defined as the union of all feeble b-supra open soft sets that are contained in* $(\Theta,\mathcal{P})$*.*

Remark 4.1. *One of the divergence we obtain herein is that the equality between a soft set* $(\Theta,\mathcal{P})$ *and its feeble b-supra interior points* $int^b(\Theta,\mathcal{P})$ *does not imply that* $(\Theta,\mathcal{P})$ *is a feeble b-supra open soft set. To illustrate this point, consider a soft set* $(\Theta,\mathcal{P}) = \{(p_1,\{\omega_1,\omega_3\}),(p_2,\{\omega_2,\omega_3\})\}$ *and a soft ST-space* $(\Omega,\mathrm{S},\mathcal{P})$ *displayed in Example 3.5. One can check that* $(\Theta,\mathcal{P}) = int^b(\Theta,\mathcal{P})$*; however, we showed that this soft set is not a feeble b-supra open soft set.*

The proofs of the next propositions are direct consequences of the definitions, so we leave their proofs.

Proposition 4.1. *Let* $(\Theta,\mathcal{P})$ *be a soft subset of a soft ST-space* $(\Omega,\mathrm{S},\mathcal{P})$ *and* $\omega_p \in \widetilde{\Omega}$*. Then* $\omega_p \in int^b(\Theta,\mathcal{P})$ *iff there is a feeble b-supra open soft set* $(O,\mathcal{P})$ *contains* ω_p *such that* $(O,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P})$*.*

Proposition 4.2. *The following set of properties remains applicable for any two soft subsets* $(\Theta,\mathcal{P})$ *and* $(O,\mathcal{P})$ *of a soft ST-space* $(\Omega,\mathrm{S},\mathcal{P})$*.*

(i): $int^b(\Theta,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P})$.

(ii): *if* $(\Theta,\mathcal{P})\widetilde{\subseteq}(O,\mathcal{P})$*, then* $int^b(\Theta,\mathcal{P})\widetilde{\subseteq}int^b(O,\mathcal{P})$.

Corollary 4.1. *The following set of properties remains applicable for any two soft subsets* $(\Theta,\mathcal{P})$ *and* $(O,\mathcal{P})$ *of a soft ST-space* $(\Omega,\mathrm{S},\mathcal{P})$*.*

(i): $int^b[(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P})]\widetilde{\subseteq}int^b(\Theta,\mathcal{P})\widetilde{\bigcap}int^b(O,\mathcal{P})$.

(ii): $int^b(\Theta,\mathcal{P})\widetilde{\bigcup}int^b(O,\mathcal{P})\widetilde{\subseteq}int^b[(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P})]$.

Proof. It is a direct consequence of the following relations

(i): $(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P})$ and $(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P})\widetilde{\subseteq}(O,\mathcal{P})$.

(ii): $(\Theta,\mathcal{P})\widetilde{\subseteq}[(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P})]$ and $(O,\mathcal{P})\widetilde{\subseteq}[(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P})]$

□

Definition 4.2. *The feeble b-supra closure points of a soft subset $(\Theta,\mathcal{P})$ of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$, denoted by $cl^b(\Theta,\mathcal{P})$, is defined as the intersection of all feeble b-supra closed soft sets containing $(\Theta,\mathcal{P})$.*

Remark 4.2. *One of the divergence we obtain herein is that the equality between a soft set $(\Theta,\mathcal{P})$ and its feeble b-supra closure points $cl^b(\Theta,\mathcal{P})$ does not imply that $(\Theta,\mathcal{P})$ is a feeble b-supra closed soft set. To illustrate this point, consider a soft set $(\Theta,\mathcal{P}) = \{(p_1,\{\omega_2\}),(p_2,\{\omega_1\})\}$ and a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$ displayed in Example 3.5. One can check that $(\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P})$; however, this soft set is not a feeble b-supra closed soft set.*

The next proposition follows immediately from the definition of feeble *b*-supra closure points; therefore, we omit its proof.

Proposition 4.3. *The following set of properties remains applicable for any two soft subsets $(\Theta,\mathcal{P})$ and $(O,\mathcal{P})$ of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$.*

- **(i):** $(\Theta,\mathcal{P})\widetilde{\subseteq}cl^b(\Theta,\mathcal{P})$.
- **(ii):** *if* $(\Theta,\mathcal{P})\widetilde{\subseteq}(O,\mathcal{P})$, *then* $cl^b(\Theta,\mathcal{P})\widetilde{\subseteq}cl^b(O,\mathcal{P})$.

Corollary 4.2. *The next properties hold for all subsets of $(\Theta,\mathcal{P})$, $(O,\mathcal{P})$ of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$.*

- **(i):** $cl^b[(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P})]\widetilde{\subseteq}cl^b(\Theta,\mathcal{P})\widetilde{\bigcap}cl^b(O,\mathcal{P})$.
- **(ii):** $cl^b(\Theta,\mathcal{P})\widetilde{\bigcup}cl^b(O,\mathcal{P})\widetilde{\subseteq}cl^b[(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P})]$.

Proof. It automatically comes from the following:

(i) $(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P})$ and $(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P})\widetilde{\subseteq}(O,\mathcal{P})$.

(ii) $(\Theta,\mathcal{P})\widetilde{\subseteq}[(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P})]$ and $(O,\mathcal{P})\widetilde{\subseteq}[(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P})]$. □

In the following result, we establish a criterion to determine whether a soft point belongs to the feeble *b*-supra closure points of a soft set.

Proposition 4.4. *Let $(\Theta,\mathcal{P})$ be a soft subset of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$ and $\omega_p \in \widetilde{\Omega}$. Then $\omega_p \in cl^b(\Theta,\mathcal{P})$ if and only if $(O,\mathcal{P})\widetilde{\bigcap}(\Theta,\mathcal{P}) \neq \Phi$ for each feeble b-supra open soft set $(O,\mathcal{P})$ contains ω_p.*

Proof. $[\Rightarrow]$ Let $\omega_p \in cl^b(\Theta,\mathcal{P})$. Suppose that there is feeble *b*-supra open soft set $(O,\mathcal{P})$ containing ω_p with

$$(O,\mathcal{P})\widetilde{\bigcap}(\Theta,\mathcal{P}) = \Phi.$$

Then

$$(\Theta,\mathcal{P})\widetilde{\subseteq}(O^c,\mathcal{P}).$$

Since $(O,\mathcal{P})$ is a feeble *b*-supra closed soft set,

$$cl^b(\Theta,\mathcal{P})\widetilde{\subseteq}(O^c,\mathcal{P}).$$

Accordingly,

$$\omega_p \notin cl^b(\Theta,\mathcal{P}), \text{which is a contradiction with assumption.}$$

Thus,

$$(O,\mathcal{P})\widetilde{\bigcap}(\Theta,\mathcal{P}) \neq \Phi \text{ is satisfied.}$$

[$\Leftarrow$] Let for every feeble b-supra open soft set $(O,\mathcal{P})$ contains ω_p we have

$$(O,\mathcal{P})\widetilde{\bigcap}(\Theta,\mathcal{P}) \neq \Phi.$$

Assume that

$$\omega_p \notin cl^b(\Theta,\mathcal{P}).$$

Then there is a feeble b-supra closed soft set $(W,\mathcal{P})$ containing $(\Theta,\mathcal{P})$ with $\omega_p \notin (W,\mathcal{P})$. So

$$\omega_p \in (W^c,\mathcal{P})$$

and

$$(W^c,\mathcal{P})\widetilde{\bigcap}(\Theta,\mathcal{P}) = \Phi.$$

But this contradicts our assumption. This ends the proof. □

Corollary 4.3. *For any feeble b-supra open soft set $(\Theta,\mathcal{P})$ and a soft set $(O,\mathcal{P})$ in $(\Omega,\mathrm{S},\mathcal{P})$ we have $(\Theta,\mathcal{P})\widetilde{\bigcap}cl^b(O,\mathcal{P}) = \Phi$ whenever $(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P}) = \Phi$.*

Proposition 4.5. *In a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$, the following equalities are satisfied for any soft set $(\Theta,\mathcal{P})$.*

- **(i):** $[int^b(\Theta,\mathcal{P})]^c = cl^b(\Theta^c,\mathcal{P})$.
- **(ii):** $[cl^b(\Theta,\mathcal{P})]^c = int^b(\Theta^c,\mathcal{P})$.

Proof. (i) If

$$\omega_p \notin [int^b(\Theta,\mathcal{P})]^c,$$

then there is a feeble b-supra open soft set $(O,\mathcal{P})$ with

$$\omega_p \in (O,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P}).$$

Therefore,

$$(\Theta^c,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P}) = \Phi,$$

and hence,

$$\omega_p \notin cl^b(\Theta^c,\mathcal{P}).$$

Conversely, if $\omega_p \notin cl^b(\Theta^c,\mathcal{P})$ one may verify $\omega_p \notin [int^b(\Theta,\mathcal{P})]^c$ by adapting the previous steps.
(ii) The proof follows an argument similar to (i). □

Definition 4.3. *In a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$, we name a soft point ω_p a feeble b-supra frontier point of a soft set $(\Theta,\mathcal{P})$ if ω_p belongs to the complement of $int^b(\Theta,\mathcal{P})\widetilde{\bigcup}int^b(\Theta^c,\mathcal{P})$.*

We denote a soft set consisting of all feeble b-supra frontier points of $(\Theta,\mathcal{P})$ by the symbol $fr^b(\Theta,\mathcal{P})$.

Theorem 4.1.

$$fr^b(\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P})\widetilde{\bigcap}cl^b(\Theta^c,\mathcal{P})$$

for every subset $(\Theta,\mathcal{P})$ of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$.

Proof.

$$\begin{aligned} fr^b(\Theta,\mathcal{P}) &= [int^b(\Theta,\mathcal{P})\widetilde{\bigcup}int^b(\Theta^c,\mathcal{P})]^c \\ &= [int^b(\Theta,\mathcal{P})]^c\widetilde{\bigcap}[int^b(\Theta^c,\mathcal{P})]^c \ (De\ \ Morgan's\ \ law) \\ &= cl^b(\Theta^c,\mathcal{P})\widetilde{\bigcap}cl^b(\Theta,\mathcal{P}) \ (Theorem\ \ 4.5(ii)) \end{aligned}$$

□

Corollary 4.4. *In a soft ST-space $(\Omega, S, \mathcal{P})$, the next formulas holds for any soft set $(\Theta,\mathcal{P})$:*

(i): $fr^b(\Theta,\mathcal{P}) = fr^b(\Theta^c,\mathcal{P})$.
(ii): $fr^b(\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P}) \setminus int^b(\Theta,\mathcal{P})$.
(iii): $cl^b(\Theta,\mathcal{P}) = int^b(\Theta,\mathcal{P})\widetilde{\bigcup} fr^b(\Theta,\mathcal{P})$.
(iv): $int^b(\Theta,\mathcal{P}) = (\Theta,\mathcal{P}) \setminus fr^b(\Theta,\mathcal{P})$.

Proof. (i) Obvious.

(ii) $fr^b(\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P})\widetilde{\bigcap}cl^b(\Theta^c,\mathcal{P}) = cl^b(\Theta,\mathcal{P}) \setminus [cl^b(\Theta^c,\mathcal{P})]^c$. By (ii) of Theorem 4.5, the desired relation follows.

(iii) $int^b(\Theta,\mathcal{P})\widetilde{\bigcup} fr^b(\Theta,\mathcal{P}) = int^b(\Theta,\mathcal{P})\widetilde{\bigcup} [cl^b(\Theta,\mathcal{P}) \setminus int^b(\Theta,\mathcal{P})] = cl^b(\Theta,\mathcal{P})$.

(iv)

$$\begin{aligned} (\Theta,\mathcal{P}) \setminus fr^b(\Theta,\mathcal{P}) &= (\Theta,\mathcal{P}) \setminus [cl^b(\Theta,\mathcal{P}) \setminus int^b(\Theta,\mathcal{P})] \\ &= (\Theta,\mathcal{P})\widetilde{\bigcap}[cl^b(\Theta,\mathcal{P})\widetilde{\bigcap}(int^b(\Theta,\mathcal{P}))^c]^c \\ &= (\Theta,\mathcal{P})\widetilde{\bigcap}[(cl^b(\Theta,\mathcal{P}))^c\widetilde{\bigcup} int^b(\Theta,\mathcal{P})] \\ &= [(\Theta,\mathcal{P})\widetilde{\bigcap}(cl^b(\Theta,\mathcal{P}))^c]\widetilde{\bigcup}[(\Theta,\mathcal{P})\widetilde{\bigcap} int^b(\Theta,\mathcal{P})] \\ &= int^b(\Theta,\mathcal{P}). \end{aligned}$$

□

Theorem 4.2. *In a soft ST-space $(\Omega, S, \mathcal{P})$, the next results are true for any soft set $(\Theta,\mathcal{P})$:*

(i): $fr^b(int^b(\Theta,\mathcal{P}))\widetilde{\subseteq}fr^b(\Theta,\mathcal{P})$.
(ii): $fr^b(cl^b(\Theta,\mathcal{P}))\widetilde{\subseteq}fr^b(\Theta,\mathcal{P})$.

Proof. Follows by (iii) of Corollary 4.4. □

Theorem 4.3. *Let $(\Theta,\mathcal{P})$ be a soft subset of a soft ST-space $(\Omega, S, \mathcal{P})$. Then*

(i): $(\Theta,\mathcal{P}) = int^b(\Theta,\mathcal{P})$ *iff* $fr^b(\Theta,\mathcal{P})\widetilde{\bigcap}(\Theta,\mathcal{P}) = \Phi$.
(ii): $(\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P})$ *iff* $fr^b(\Theta,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P})$.

Proof. (i) Suppose that

$$(\Theta,\mathcal{P}) = int^b(\Theta,\mathcal{P}).$$

Then by (iv) of Corollary 4.4,

$$(\Theta,\mathcal{P}) = int^b(\Theta,\mathcal{P}) = (\Theta,\mathcal{P}) \setminus fr^b(\Theta,\mathcal{P})$$

and hence,

$$fr^b(\Theta,\mathcal{P})\widetilde{\bigcap}(\Theta,\mathcal{P}) = \Phi.$$

Conversely, let $\omega_p \in (\Theta,\mathcal{P})$. Since $\omega_p \notin fr^b(\Theta,\mathcal{P})$ and $\omega_p \in cl^b(\Theta,\mathcal{P})$, by (iii) of Corollary 4.4, $\omega_p \in int^b(\Theta,\mathcal{P})$. Therefore,

$$int^b(\Theta,\mathcal{P}) = (\Theta,\mathcal{P}),$$

which establishes the claim.

(ii) Assume that

$$(\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P}).$$

Then

$$fr^b(\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P})\widetilde{\bigcap}cl^b(\Theta^c,\mathcal{P})\widetilde{\subseteq}cl^b(\Theta,\mathcal{P}) = (\Theta,\mathcal{P}),$$

which establishes the claim.

Conversely, if $fr^b(\Theta,\mathcal{P})\widetilde{\subseteq}(\Theta,\mathcal{P})$, then by (iii) of Corollary 4.4,

$$cl^b(\Theta,\mathcal{P})\widetilde{\subseteq}int^b(\Theta,\mathcal{P})\widetilde{\bigcup}(\Theta,\mathcal{P}) = (\Theta,\mathcal{P})$$

and hence

$$cl^b(\Theta,\mathcal{P}) = (\Theta,\mathcal{P}),$$

as required. □

Corollary 4.5. *Let $(\Theta,\mathcal{P})$ be a subset of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$. Then*

$$int^b(\Theta,\mathcal{P}) = (\Theta,\mathcal{P}) = cl^b(\Theta,\mathcal{P})$$

iff

$$fr^b(\Theta,\mathcal{P}) = \Phi.$$

Definition 4.4. *In a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$ we name a soft point ω_p a feeble b-supra accumulation point of a soft subset $(\Theta,\mathcal{P})$ if for each feeble b-supra open soft set $(O,\mathcal{P})$ containing ω_p we have*

$$[(O,\mathcal{P})\backslash\omega_p]\bigcap(\Theta,\mathcal{P}) \neq \Phi.$$

We denote a soft set consisting of all feeble b-supra accumulation points of $(\Theta,\mathcal{P})$ by the symbol $ac^b(\Theta,\mathcal{P})$.

Theorem 4.4. *Let $(\Theta,\mathcal{P})$ and $(O,\mathcal{P})$ be subsets of a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$. If $(\Theta,\mathcal{P})\widetilde{\subseteq}(O,\mathcal{P})$, then $ac^b(\Theta,\mathcal{P})\widetilde{\subseteq}ac^b(O,\mathcal{P})$.*

Proof. Warranted by Definition 4.4. □

Corollary 4.6. *In a soft ST-space $(\Omega,\mathrm{S},\mathcal{P})$, the next results holds for any soft sets $(\Theta,\mathcal{P})$ and $(O,\mathcal{P})$.*

- **(i):** $ac^b[(\Theta,\mathcal{P})\widetilde{\bigcap}(O,\mathcal{P})]\widetilde{\subseteq}ac^b(\Theta,\mathcal{P})\widetilde{\bigcap}\, ac^b(O,\mathcal{P})$.
- **(ii):** $ac^b(\Theta,\mathcal{P})\widetilde{\bigcup}\, ac^b(O,\mathcal{P})\widetilde{\subseteq}ac^b[(\Theta,\mathcal{P})\widetilde{\bigcup}(O,\mathcal{P})]$.

Theorem 4.5. *Let $(\Theta, \mathcal{P})$ be a subset of a soft ST-space $(\Omega, \mathrm{S}, \mathcal{P})$, then*

$$cl^b(\Theta, \mathcal{P}) = (\Theta, \mathcal{P}) \widetilde{\bigcup} ac^b(\Theta, \mathcal{P}).$$

Proof. The side

$$(\Theta, \mathcal{P}) \widetilde{\bigcup} ac^b(\Theta, \mathcal{P}) \widetilde{\subseteq} cl^b(\Theta, \mathcal{P})$$

is clear. To prove the reverse direction, let

$$\omega_p \notin [(\Theta, \mathcal{P}) \widetilde{\bigcup} ac^b(\Theta, \mathcal{P})].$$

Then $\omega_p \notin (\Theta, \mathcal{P})$ and $\omega_p \notin ac^b(\Theta, \mathcal{P})$. Therefore, there is feeble b-supra open soft $(O, \mathcal{P})$ containing ω_p with

$$(O, \mathcal{P}) \widetilde{\cap} (\Theta, \mathcal{P}) = \Phi.$$

Thus, $\omega_p \notin cl^b(\Theta, \mathcal{P})$. Hence, we find that

$$cl^b(\Theta, \mathcal{P}) = (\Theta, \mathcal{P}) \widetilde{\bigcup} ac^b(\Theta, \mathcal{P}).$$

□

Corollary 4.7. *In a soft ST-space $(\Omega, \mathrm{S}, \mathcal{P})$, if $(\Theta, \mathcal{P})$ is a feeble b-supra closed soft set, then $ac^b(\Theta, \mathcal{P}) \widetilde{\subseteq} (\Theta, \mathcal{P})$.*

5. Conclusion

The framework of a supra topological space arises when the finite intersection condition is disregarded from the axioms of a general topology. These spaces have several advantages as explained in the literature, for instance, the simplicity of exemplifying spaces that satisfy certain topological properties [36], whereas constructing examples for these properties is a difficult task in general topology, along with their suitability for modeling diverse real-world applications [39,40]. To benefit from these merits and the advantages of soft spaces, we have devoted this work to establishing a novel generalization class of supra open soft sets, namely, feeble b-open soft sets. We have initiated this class by linking a given soft supra topological space with its parametric supra topologies, which offer a wide area of study via soft settings.

We first explored the basic features of this class and determined under which conditions this class is preserved under soft functions. Also, we demonstrated the Cartesian product of finite numbers of feeble b-supra open soft sets is a feeble b-supra open soft set and elucidated, with the assistance of counterexamples, the relationships between this class and previous ones, i.e., feeble *semi*-supra open soft sets and feeble *pre*-supra open soft sets. Then, have employed the proposed class to put forward new topological operators via soft supra topological spaces, namely, feeble b-supra interior operator, feeble b-supra closure operator, feeble b-supra frontier operator, and feeble b-supra accumulation operator. We have established several relationships among these operators via formulas analogous to their classical counterparts and examined their fundamental properties.

Additionally, we have identified which classical properties fail to hold within the framework of soft supra topologies.

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