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Essential Interval Valued Fuzzy Ideals in Ternary Semigroups

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Abstract. In this paper, we give the concepts of essential interval valued fuzzy ideals in ternary semigroups. We proved properties and relationships between essential interval valued fuzzy ideals and essential ideals in ternary semigroups. Moreover, we investigate properties of 0-essential interval valued fuzzy ideals and relationships between 0-essential interval valued fuzzy ideals and 0-essential ideals in ternary semigroups.

1. Introduction

A ternary semigroup is defined as a nonempty set together with an associative ternary operation. The concept of ternary operations dates back to 1932, when Lehmer [6] introduced a ternary analogue of an abelian group. Although every semigroup naturally induces a ternary semigroup, the converse does not generally hold, as not all ternary semigroups arise from binary operations.

The theory of fuzzy sets was initiated by Zadeh in 1965 [17]. This theory was later applied to algebraic structures, and in the same year, Sioson, [14] examined fuzzy left ideals, fuzzy right ideals, and fuzzy bi-ideals in ternary semigroups. Since then, the notion of essential ideals has attracted increasing attention. Medhi et al. [10] first studied essential fuzzy ideals in rings in 1971. Subsequently, Medhi and Saikia [11] introduced T-fuzzy essential ideals and investigated their fundamental properties.

The extension of essential ideals to more general algebraic systems has continued over the years. Wani and Pawar [16] generalized the concept to ternary semirings in 2017. In 2020, Baupradist et al. [1] analyzed essential ideals and essential fuzzy ideals in semigroups, including the notions of 0-essential ideals and 0-essential fuzzy ideals. Chinram and Gaketem [2] further developed these ideas by introducing essential (m,n)-ideals and fuzzy essential (m,n)-ideals in semigroups. Later,

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Gaketem et al. [12] investigated essential bi-ideals and fuzzy essential bi-ideals, while Gaketem and Iampan [3,4] extended the framework to UP-algebras.

In addition, Khamrot and Gaketem [7,8] studied essential ideals within the settings of interval-valued fuzzy sets and bipolar fuzzy sets. More recently, Rittichuai et al. [13] examined essential ideals and essential fuzzy ideals in ternary semigroups. The concept was further generalized in 2024 by Kaewmanee and Gaketem [5], who introduced essential hyperideals and essential fuzzy hyperideals in hypersemigroups. In 2025, Khamrot et al. [9] proposed generalized essential interior ideals in semigroups.

Motivated by these developments, the present paper extends the study of essential interval valued fuzzy ideals to the setting of ternary semigroups. We establish several basic properties of essential interval-valued fuzzy ideals, investigate their relationships with classical essential ideals, and clarify the connections between essential interval-valued fuzzy ideals and essential ideals in ternary semigroups.

2. Preliminaries

In this section, we review the concept's basic definitions and the theorem used to prove all results in the next section.

Definition 2.1. *[14] A nonempty set $\mathcal{S}$ is called a* ternary semigroup *if there exists a ternary operation $\mathcal{S}\times\mathcal{S}\times\mathcal{S}\rightarrow\mathcal{S}$, written as $(\mathfrak{h},\mathfrak{r},\mathfrak{k})\longmapsto\mathfrak{h}\mathfrak{r}\mathfrak{k}$ such that $(\mathfrak{h}\mathfrak{r}\mathfrak{k})\mathfrak{x}\mathfrak{y}=\mathfrak{h}(\mathfrak{r}\mathfrak{k}\mathfrak{x})\mathfrak{y}=\mathfrak{h}(\mathfrak{r}\mathfrak{k}\mathfrak{x})\mathfrak{y}=\mathfrak{h}\mathfrak{r}(\mathfrak{k}\mathfrak{x}\mathfrak{y})$, for all $\mathfrak{h},\mathfrak{r},\mathfrak{k},\mathfrak{x},\mathfrak{y}\in\mathcal{S}$.*

Definition 2.2. *[14] A nonempty subset $\mathcal{B}$ of a ternary semigroup $\mathcal{S}$ is named as*

(1) *a ternary subsemigroup (TSSG) of $\mathcal{S}$ if $\mathcal{B}^3\subseteq\mathcal{B}$,*
(2) *a left ideal (LID) of $\mathcal{S}$ if $\mathcal{S}\mathcal{S}\mathcal{B}\subseteq\mathcal{B}$,*
(3) *a right ideal (RID) of $\mathcal{S}$ if $\mathcal{B}\mathcal{S}\mathcal{S}\subseteq\mathcal{B}$,*
(4) *a middle ideal (MID) of $\mathcal{S}$ if $\mathcal{S}\mathcal{B}\mathcal{S}\subseteq\mathcal{B}$. By an* ideal *$\mathcal{B}$ of a ternary semigroup $\mathcal{S}$ we mean an LID, an RID, and an MID of $\mathcal{S}$,*

For any $t,r\in[0,1]$, we have

$$t\vee r=\max\{t,r\}\quad\text{and}\quad t\wedge r=\min\{t,r\}.$$

A fuzzy set (FS) ω of a non-empty set $\mathfrak{T}$ is function from $\mathfrak{T}$ into unit closed interval $[0,1]$ of real numbers, i.e., $\omega:\mathfrak{T}\rightarrow[0,1]$.

For any two FSs of ω and τ of a non-empty set of $\mathfrak{T}$, we defined

(1) $\omega\leq\tau$ if $\omega(t)\leq\tau(t)$ for all $t\in\mathfrak{T}$
(2) $(\omega\vee\tau)(t)=\omega(t)\vee\tau(t)$ for all $t\in\mathfrak{T}$,
(3) $(\omega\wedge\tau)(t)=\omega(t)\wedge\tau(t)$ for all $t\in\mathfrak{T}$.

The *support* of ω instead of $\text{supp}(\omega)=\{t\in\mathfrak{T}\mid\omega(t)\neq 0\}$.

For two FSs ω and τ in an SG $\mathcal{S}$, define the product $\omega \circ \tau$ as follows : for all $t \in \mathcal{S}$,

$$(\omega \circ \tau)(t) = \begin{cases} \bigvee_{(w,d)\in F_t} \{\{\omega(w) \wedge \tau(d)\}\} & \text{if } F_t \neq \emptyset, \\ 0 & \text{if } F_t = \emptyset, \end{cases}$$

where $F_t := \{(w,d) \in \mathcal{S} \times \mathcal{S} \mid t = wd\}$.

Let ω be an FS of a ternary semigroup $\mathcal{S}$. Then

(1) A *fuzzy subsemigroup* (FSSG) ω of $\mathcal{S}$ if $\omega(trk) \geq \omega(t) \wedge \omega(r) \wedge \omega(k)$ for all $t, r, k \in \mathcal{S}$.
(2) A *fuzzy left ideal* (FLID) ω of $\mathcal{S}$ if $\omega(trk) \geq \omega(k)$ for all $t, r \in \mathcal{S}$.
(3) A *fuzzy middle ideal* (FMID) ω of $\mathcal{S}$ if $\omega(trk) \geq \omega(r)$ for all $t, r \in \mathcal{S}$.
(4) A *fuzzy right ideal* (FRID) ω of $\mathcal{S}$ if $\omega(trk) \geq \omega(t)$ for all $t, r \in \mathcal{S}$.
(5) A *fuzzy ideal* (FID) ω of $\mathcal{S}$ if $\omega(trk) \geq \omega(t) \vee \omega(r) \vee \omega(k)$ for all $t, r, k \in \mathcal{S}$.

The characteristic function $\chi_{\mathcal{I}}$ of a subset $\mathcal{I}$ of a non-empty set $\mathfrak{T}$ is a FS of $\mathcal{S}$

$$\chi_{\mathcal{I}}(t) = \begin{cases} 1 & \text{if} \quad t \in \mathcal{I}, \\ 0 & \text{if} \quad t \notin \mathcal{I} \end{cases}$$

for all $t \in \mathfrak{T}$.

The following theorems are true.

Theorem 2.1. *[14] Let $\mathcal{S}$ be a ternary semigroup. Then $\mathcal{I}$ is an SSG (LID, MID, RID, ID) of $\mathcal{S}$ if and only if the characteristic function $\chi_{\mathcal{I}}$ is an FSSG (FLID, FMID, FRID, FID) of $\mathcal{S}$.*

Theorem 2.2. *[14] Let ω be a nonzero FS of a ternary semigroup $\mathcal{S}$. Then ω is an FSSG (FLID, FMID, FRID, FID) of $\mathcal{S}$ if and only if* $\operatorname{supp}(\omega)$ *is an SSG (LID, MID, RID, ID) of $\mathcal{S}$.*

Next, we will review essential ideals and fuzzy essential ideals in a ternary semigroup and their properties.

Definition 2.3. *[13] An* essential ideal *(EID) $\mathcal{I}$ of a ternary semigroup $\mathcal{S}$ if $\mathcal{I}$ is an ID of $\mathcal{S}$ and $\mathcal{I} \cap \ddot{\mathcal{J}} \neq \emptyset$ for every ID $\ddot{\mathcal{J}}$ of $\mathcal{S}$.*

Theorem 2.3. *[13] Let $\mathcal{I}$ be an EID of a ternary semigroup $\mathcal{S}$. If $\mathcal{I}_1$ is an ID of $\mathcal{S}$ containing $\mathcal{I}$, then $\mathcal{I}_1$ is also, an EID of $\mathcal{S}$.*

Theorem 2.4. *[13] Let $\mathcal{I}$ and $\ddot{\mathcal{J}}$ be EIDs of a ternary semigroup $\mathcal{S}$. Then $\mathcal{I} \cup \ddot{\mathcal{J}}$ and $\mathcal{I} \cap \ddot{\mathcal{J}}$ are EIDs of $\mathcal{S}$.*

Definition 2.4. *[13] An* essential fuzzy ideal *(EFID) ω of an ternary semigroup $\mathcal{S}$ if ρ is a nonzero FID of $\mathcal{S}$ and $\rho \wedge \nu \neq 0$ for every nonzero FID ν of $\mathcal{S}$.*

Theorem 2.5. *Let ρ be an EFID of a ternary semigroup $\mathcal{S}$. If ρ_1 is a FID of $\mathcal{S}$ such that $\rho \leq \rho_1$, then ρ_1 is also an EFID of $\mathcal{S}$.*

Theorem 2.6. *Let ρ_1 and ρ_2 be EFIDs of a ternary semigroup $\mathcal{S}$. Then $\rho_1 \vee \rho_2$ and $\rho_1 \wedge \rho_2$ are EFIDs of $\mathcal{S}$.*

Theorem 2.7. *[13] Let $\mathcal{I}$ be an ID of a ternary semigroup $\mathcal{S}$. Then $\mathcal{I}$ is an EID of $\mathcal{S}$ if and only if $\chi_{\mathcal{I}}$ is an EFID of $\mathcal{S}$.*

Theorem 2.8. *[13] Let ω be a nonzero FID of a ternary semigroup $\mathcal{S}$. Then ω is an EFID of $\mathcal{S}$ if and only if* $\mathrm{supp}(\omega)$ *is an EID of $\mathcal{S}$.*

Now, we review interval valued fuzzy set.

Let $C[0,1]$ be the set of all closed subintervals of $[0,1]$, i.e.,

$$C[0,1] = \{\breve{\mathfrak{p}} = [\mathfrak{p}^-, \mathfrak{p}^+] \mid 0 \leq \mathfrak{p}^- \leq \mathfrak{p}^+ \leq 1\}.$$

We note that $[\mathfrak{p}, \mathfrak{p}] = \{\mathfrak{p}\}$ for all $\mathfrak{p} \in [0,1]$. For $\mathfrak{p} = 0$ or 1 we shall denote $[0,0]$ by $\breve{0}$ and $[1,1]$ by $\breve{1}$.

Let $\breve{\mathfrak{p}} = [\mathfrak{p}^-, \mathfrak{p}^+]$ and $\breve{\mathfrak{q}} = [\mathfrak{q}^-, \mathfrak{q}^+] \in C[0,1]$. Define the operations $\preceq$, $=$, $\curlywedge$ and $\curlyvee$ as follows:

(1) $\breve{\mathfrak{p}} \preceq \breve{\mathfrak{q}}$ if and only if $\mathfrak{p}^- \leq \mathfrak{q}^-$ and $\mathfrak{p}^+ \leq \mathfrak{q}^+$

(2) $\breve{\mathfrak{p}} = \breve{\mathfrak{q}}$ if and only if $\mathfrak{p}^- = \mathfrak{q}^-$ and $\mathfrak{p}^+ = \mathfrak{q}^+$

(3) $\breve{\mathfrak{p}} \curlywedge \breve{\mathfrak{q}} = [(\mathfrak{p}^- \wedge \mathfrak{q}^-), (\mathfrak{p}^+ \wedge \mathfrak{q}^+)]$

(4) $\breve{\mathfrak{p}} \curlyvee \breve{\mathfrak{q}} = [(\mathfrak{p}^- \vee \mathfrak{q}^-), (\mathfrak{p}^+ \vee \mathfrak{q}^+)]$.

If $\breve{\mathfrak{p}} \succeq \breve{\mathfrak{q}}$, we mean $\breve{\mathfrak{q}} \preceq \breve{\mathfrak{p}}$.

For each interval $\breve{\mathfrak{p}}_i = [\mathfrak{p}_i^-, \mathfrak{p}_i^+] \in C[0,1]$, $i \in \mathcal{A}$ where $\mathcal{A}$ is an index set, we define

$$\underset{i\in\mathcal{A}}{\curlywedge} \breve{\mathfrak{p}}_i = [\underset{i\in\mathcal{A}}{\wedge} \mathfrak{p}_i^-, \underset{i\in\mathcal{A}}{\wedge} \mathfrak{p}_i^+] \quad \text{and} \quad \underset{i\in\mathcal{A}}{\curlyvee} \breve{\mathfrak{p}}_i = [\underset{i\in\mathcal{A}}{\vee} \mathfrak{p}_i^-, \underset{i\in\mathcal{A}}{\vee} \mathfrak{p}_i^+].$$

Definition 2.5. *[15] Let $\mathfrak{T}$ be a non-empty set. Then the function $\breve{\eta} : \mathfrak{T} \to C[0,1]$ is called* interval valued fuzzy set *(shortly, IVF set) of $\mathfrak{T}$.*

Definition 2.6. *[15] Let $\mathcal{L}$ be a subset of a non-empty set $\mathfrak{T}$. An* interval valued characteristic function *of $\mathfrak{T}$ is defined to be a function $\breve{\chi}_{\mathcal{L}} : \mathfrak{T} \to C[0,1]$ by*

$$\breve{\chi}_{\mathcal{L}}(\mathfrak{u}) = \begin{cases} \breve{1} & \text{if} \quad \mathfrak{u} \in \mathcal{L}, \\ \breve{0} & \text{if} \quad \mathfrak{u} \notin \mathcal{L} \end{cases}$$

for all $u \in \mathfrak{T}$.

For two IVF sets $\breve{\eta}$ and $\breve{\nu}$ of a non-empty set $\mathfrak{T}$, define

(1) $\breve{\eta} \sqsubseteq \breve{\nu} \Leftrightarrow \breve{\eta}(\mathfrak{u}) \preceq \breve{\nu}(\mathfrak{u}) \quad$ for all $\mathfrak{u} \in \mathfrak{T}$,

(2) $\breve{\eta} = \breve{\nu} \Leftrightarrow \breve{\eta} \sqsubseteq \breve{\nu}$ and $\breve{\nu} \sqsubseteq \breve{\eta}$,

(3) $(\breve{\eta} \curlywedge \breve{\nu})(\mathfrak{u}) = \breve{\eta}(\mathfrak{u}) \curlywedge \breve{\nu}(\mathfrak{u}) \quad$ for all $\mathfrak{u} \in \mathfrak{T}$,

(4) $(\breve{\eta} \curlyvee \breve{\nu})(\mathfrak{u}) = \breve{\eta}(\mathfrak{u}) \curlyvee \breve{\nu}(\mathfrak{u}) \quad$ for all $\mathfrak{u} \in \mathfrak{T}$.

Definition 2.7. *[15] An IVF subset $\breve{\eta}$ of a ternary semigroup $\mathcal{S}$ is said to be an* IVF ideal *(IVFID) of $\mathcal{S}$ if $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \succeq \breve{\eta}(\mathfrak{u}) \curlyvee \breve{\eta}(\mathfrak{v}) \curlyvee \breve{\eta}(\mathfrak{z})$ for all $\mathfrak{u}, \mathfrak{v}, \mathfrak{z} \in \mathcal{S}$.*

The following theorems are true.

Theorem 2.9. *[15] Let $\mathcal{S}$ be a ternary semigroup and let $\mathcal{L}$ be non-empty subset of $\mathcal{S}$. Then $\mathfrak{L}$ is an ID of $\mathcal{S}$ if and only if the characteristic function $\breve{\chi}_{\mathcal{L}}$ is an IVFID of $\mathcal{S}$.*

Theorem 2.10. *[15] Let $\breve{\eta}$ and $\breve{\omega}$ be IVFIDs of $\mathcal{S}$. Then $\breve{\eta} \wedge \breve{\omega}$ is an IVFID of $\mathcal{S}$.*

Theorem 2.11. *[15] Let $\mathcal{L}$ and $\mathcal{J}$ be subsets of a non-empty set $\mathcal{S}$. Then $\breve{\chi}_{\mathcal{L}\cap\mathcal{J}} = \breve{\chi}_{\mathcal{L}} \wedge \breve{\chi}_{\mathcal{J}}$ and $\breve{\chi}_{\mathcal{L}} \circ \breve{\chi}_{\mathcal{J}} = \breve{\chi}_{\mathcal{L}\mathcal{J}}$.*

The *support* of an IVF set $\breve{\eta}$ of a set is defined by $\text{supp}(\breve{\eta}) = \{\mathfrak{u} \in \mathfrak{T} \mid \breve{\eta}(\mathfrak{u}) \neq \breve{0}\}$.

Theorem 2.12. *Let $\breve{\eta}$ be a nonzero IVF set of a ternary semigroup $\mathcal{S}$. Then $\breve{\eta}$ is an IVFID of $\mathcal{S}$ if and only if* $\text{supp}(\breve{\eta})$ *is an ID of $\mathcal{S}$.*

Proof. Supposet that $\breve{\eta}$ is an IVFID of $\mathcal{S}$ and let $\mathfrak{u}, \mathfrak{v}, \mathfrak{z} \in \mathcal{S}$ with $\mathfrak{u}, \mathfrak{v}, \mathfrak{z} \in \text{supp}(\breve{\eta})$. Then $\breve{\eta}(\mathfrak{u}) \neq \breve{0}$, $\breve{\eta}(\mathfrak{v}) \neq \breve{0}$ and $\breve{\eta}(\mathfrak{z}) \neq \breve{0}$. Since $\breve{\eta}$ is an IVFID of $\mathcal{S}$ we have $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \succeq \breve{\eta}(\mathfrak{u}) \vee \breve{\eta}(\mathfrak{v}) \vee \breve{\eta}(\mathfrak{z})$. Thus,, $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \neq \breve{0}$ So, $\mathfrak{u}\mathfrak{v}\mathfrak{z} \in \text{supp}(\breve{\eta})$. Hence, $\text{supp}(\breve{\eta})$ is an ID of $\mathcal{S}$.

Conversely, suppose that $\text{supp}(\breve{\eta})$ is an ID of $\mathcal{S}$ and let $\breve{\eta}$ is not an IVFID of $\mathcal{S}$. Then there exist $\mathfrak{u}, \mathfrak{v}, \mathfrak{z} \in \mathcal{S}$ such that $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \prec \breve{\eta}(\mathfrak{u}) \vee \breve{\eta}(\mathfrak{v}) \vee \breve{\eta}(\mathfrak{z})$. Since $\text{supp}(\breve{\eta})$ is an ID of $\mathcal{S}$ we have $\mathfrak{u}\mathfrak{v}, \mathfrak{z} \in \text{supp}(\breve{\eta})$. Thus, $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \neq \breve{0}$.

If $\mathfrak{u} \in \text{supp}(\breve{\eta})$ or $\mathfrak{v} \in \text{supp}(\breve{\eta})$ or $\mathfrak{z} \in \text{supp}(\breve{\eta})$, then $\breve{\eta}(\mathfrak{u}) \neq \breve{0}$ or $\breve{\eta}(\mathfrak{v}) \neq \breve{0}$ or $\breve{\eta}(\mathfrak{z}) \neq \breve{0}$ So, $\mathfrak{u}\mathfrak{v}\mathfrak{z} \in \text{supp}(\breve{\eta})$. Thus, $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \succeq \breve{\eta}(\mathfrak{u}) \wedge \breve{\eta}(\mathfrak{v}) \wedge \breve{\eta}(\mathfrak{z})$. It is a contradiction.

If $\mathfrak{u} \in \text{supp}(\breve{\eta})$, $\mathfrak{v} \in \text{supp}(\breve{\eta})$ and $\mathfrak{z} \in \text{supp}(\breve{\eta})$, then $\breve{\eta}(\mathfrak{u}) \neq \breve{0}$, $\breve{\eta}(\mathfrak{v}) \neq \breve{0}$ and $\breve{\eta}(\mathfrak{z}) \neq \breve{0}$ So, $\mathfrak{u}\mathfrak{v}\mathfrak{z} \in \text{supp}(\breve{\eta})$. Thus, $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \succeq \breve{\eta}(\mathfrak{u}) \wedge \breve{\eta}(\mathfrak{v}) \wedge \breve{\eta}(\mathfrak{z})$. It is a contradiction.

Hence, $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{z}) \succeq \breve{\eta}(\mathfrak{u}) \wedge \breve{\eta}(\mathfrak{v}) \wedge \breve{\eta}(\mathfrak{z})$. Therefore, $\breve{\eta}$ is an IVFID of $\mathcal{S}$. □

3. Essential Interval Valued Fuzzy Ideals in Ternary Semigroups

Definition 3.1. *A nonzero IVF ideal $\breve{\eta}$ of a ternary semigroup $\mathcal{S}$ is called an* essential IVF ideal *(EIVFID) of $\mathcal{S}$ if $\breve{\eta} \wedge \breve{\nu} \neq \breve{0}$ for every nonzero IVF ideal $\breve{\nu}$ of $\mathcal{S}$.*

Theorem 3.1. *Let $\mathcal{I}$ be an ideal of a ternary semigroup $\mathcal{S}$. Then $\mathcal{I}$ is an EID of $\mathcal{S}$ if and only if $\breve{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$.*

Proof. Suppose that $\mathcal{I}$ is an EID of $\mathcal{S}$ and let $\breve{\nu}$ be a nonzero IVFID of $\mathcal{S}$. Then by Theorem 2.12 $\text{supp}(\breve{\nu})$ is an ID of $\mathcal{S}$. Since $\mathcal{I}$ is an EID of $\mathcal{S}$ we have $\mathcal{I}$ is an ID of $\mathcal{S}$. Thus, $\mathcal{I} \cap \text{supp}(\breve{\nu}) \neq \emptyset$. So, there exists $\mathfrak{u} \in \mathcal{I} \cap \text{supp}(\breve{\nu})$. Since $\mathcal{I}$ is an ID of $\mathcal{S}$ we have $\breve{\chi}_{\mathcal{I}}$ is an IVFID of $\mathcal{S}$. Since $\breve{\nu}$ is a nonzero IVF ideal of $\mathcal{S}$ we have $(\breve{\chi}_{\mathcal{I}} \wedge \breve{\nu})(\mathfrak{u}) \neq \breve{0}$. Thus, $\breve{\chi}_{\mathcal{I}} \wedge \breve{\nu} \neq \breve{0}$. Therefore, $\breve{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$.

Conversely, assume that $\breve{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$ and let $\mathcal{J}$ be an ideal of $\mathcal{S}$. Then $\breve{\chi}_{\mathcal{J}}$ is a nonzero IVFID of $\mathcal{S}$. Since $\breve{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$ we have $\breve{\chi}_{\mathcal{I}}$ is an IVF ideal of $\mathcal{S}$. Thus, $\breve{\chi}_{\mathcal{I}} \wedge \breve{\chi}_{\mathcal{J}} \neq \breve{0}$. So, by Theorem 2.11, $\breve{\chi}_{\mathcal{I}\cap\mathcal{J}} \neq \breve{0}$. Hence, $\mathcal{I} \cap \mathcal{J} \neq \emptyset$. Therefore, $\mathcal{I}$ is an EID of $\mathcal{S}$. □

Theorem 3.2. *Let $\breve{\eta}$ be a nonzero IVFID of a ternary semigroup $\mathcal{S}$. Then $\breve{\eta}$ is an EIVFID of $\mathcal{S}$ if and only if* $\text{supp}(\breve{\eta})$ *is an EID of $\mathcal{S}$.*

Proof. Assume that $\breve{\eta}$ is an EIVFID of $\mathcal{S}$ and let $\mathcal{J}$ be an ID of $\mathcal{S}$. Then by Theorem 2.9, $\breve{\chi}_{\mathcal{J}}$ is an IVFID of $\mathcal{S}$. Since $\breve{\eta}$ is an EIVFID of $\mathcal{S}$ we have $\breve{\eta}$ is an IVF ideal of $\mathcal{S}$. Thus, $\breve{\eta} \wedge \breve{\chi}_{\mathcal{J}} \neq \breve{0}$. So, there exists $\mathfrak{u} \in \mathcal{S}$ such that $(\breve{\eta} \wedge \breve{\chi}_{\mathcal{J}})(\mathfrak{u}) \neq \breve{0}$. It implies that $\breve{\eta}(\mathfrak{u}) \neq 0$ and $\breve{\chi}_{\mathcal{J}}(\mathfrak{u}) \neq 0$. Hence, $\mathfrak{u} \in \mathrm{supp}(\breve{\eta}) \cap \mathcal{J}$ So, $\mathrm{supp}(\breve{\eta}) \cap \mathcal{J} \neq \emptyset$. Therefore, $\mathrm{supp}(\breve{\eta})$ is an EID of $\mathcal{S}$.

Conversely, assume that $\mathrm{supp}(\breve{\eta})$ is an EID of $\mathcal{S}$ and let $\breve{\nu}$ be a nonzero IVFID of $\mathcal{S}$. Then by Theorem 2.12 $\mathrm{supp}(\breve{\nu})$ is an ID of $\mathcal{S}$. Since $\mathrm{supp}(\breve{\eta})$ is an EID of $\mathcal{S}$ we have $\mathrm{supp}(\breve{\eta})$ is an ID of $\mathcal{S}$. Thus, $\mathrm{supp}(\breve{\eta}) \cap \mathrm{supp}(\breve{\nu}) \neq \emptyset$. So, there exists $\mathfrak{u} \in \mathrm{supp}(\breve{\eta}) \cap \mathrm{supp}(\breve{\nu})$. It implies that $\breve{\eta}(\mathfrak{u}) \neq \breve{0}$ and $\breve{\nu}(\mathfrak{u}) \neq \breve{0}$ for all $\mathfrak{u} \in \mathcal{S}$. Hence, $(\breve{\eta} \wedge \breve{\nu})(\mathfrak{u}) \neq \breve{0}$ for all $\mathfrak{u} \in \mathcal{S}$. Therefore, $\breve{\eta} \wedge \breve{\nu} \neq \breve{0}$. We conclude that $\breve{\eta}$ is an EIVFID of $\mathcal{S}$. □

Theorem 3.3. *Let $\breve{\eta}$ be an EIVFID of a ternary semigroup $\mathcal{S}$. If $\breve{\nu}$ is an IVFID of $\mathcal{S}$ such that $\breve{\eta} \sqsubseteq \breve{\nu}$, then $\breve{\nu}$ is also an EIVFID of $\mathcal{S}$.*

Proof. Let $\breve{\nu}$ be an IVFID of $\mathcal{S}$ such that $\breve{\eta} \sqsubseteq \breve{\nu}$ and let $\breve{\mu}$ be any IVFID of $\mathcal{S}$. Since $\breve{\eta}$ is an EIVFID of $\mathcal{S}$ we have $\breve{\eta}$ is an IVFID of $\mathcal{S}$. Thus, $\breve{\eta} \wedge \breve{\mu} \neq \breve{0}$. So, $\breve{0} \neq \breve{\eta} \wedge \breve{\mu} \sqsubseteq \breve{\nu} \wedge \breve{\mu} \neq \breve{0}$. It implies that $\breve{\nu} \wedge \breve{\mu} \neq \breve{0}$. Hcene $\breve{\nu}$ is an EIVFID of $\mathcal{S}$. □

Theorem 3.4. *Let $\breve{\eta}_1$ and $\breve{\eta}_2$ be EIVFIDs of a ternary semigroup $\mathcal{S}$. Then $\breve{\eta}_1 \vee \breve{\eta}_2$ and $\breve{\eta}_1 \wedge \breve{\eta}_2$ are EIVFIDs of $\mathcal{S}$.*

Proof. Since $\breve{\eta}_2 \sqsubseteq \breve{\eta}_1 \vee \breve{\eta}_2$ we have $\breve{\eta}_1 \vee \breve{\eta}_2$ is an EIVFID of $\mathcal{S}$.

Since $\breve{\eta}_1$, $\breve{\eta}_2$ are EIVFIDs of $\mathcal{S}$ we have $\breve{\eta}_1$ and $\breve{\eta}_2$ are IVFIDs of $\mathcal{S}$. Thus, $\breve{\eta}_1 \wedge \breve{\eta}_2$ is an IVFID of $\mathcal{S}$. Let $\breve{\mu}$ be an IVFID of $\mathcal{S}$. Then $\breve{\eta}_1 \wedge \breve{\mu} \neq \breve{0}$. Thus, there exists $\mathfrak{u} \in \mathcal{S}$ such that $(\breve{\eta}_1 \wedge \breve{\mu})(\mathfrak{u}) \neq \breve{0}$. So, $\breve{\eta}_1(\mathfrak{u}) \neq \breve{0}$ and $\breve{\mu}(\mathfrak{u}) \neq \breve{0}$. Since $\breve{\eta}_2 \neq 0$, let $\mathfrak{v} \in \mathcal{S}$ such that $\breve{\eta}_2(\mathfrak{v}) \neq \breve{0}$. Since $\breve{\eta}_1$ and $\breve{\eta}_2$ are IVFIDs of $\mathcal{S}$ we have $\breve{\eta}_1(\mathfrak{u}\mathfrak{u}\mathfrak{v}) \geq \breve{\eta}_1(\mathfrak{u}) \vee \breve{\eta}_1(\mathfrak{v}) \geq \breve{\eta}_1(\mathfrak{u}) \geq \breve{0}$ and $\breve{\eta}_2(\mathfrak{u}\mathfrak{u}\mathfrak{v}) \geq \breve{\eta}_2(\mathfrak{u}) \vee \breve{\eta}_2(\mathfrak{v}) \geq \breve{\eta}_2(\mathfrak{u}) \geq \breve{0}$. Thus, $(\breve{\eta}_1 \wedge \breve{\eta}_2)(\mathfrak{u}\mathfrak{u}\mathfrak{v}) = \breve{\eta}_1(\mathfrak{u}\mathfrak{u}\mathfrak{v}) \wedge \breve{\eta}_2(\mathfrak{u}\mathfrak{u}\mathfrak{v}) \neq \breve{0}$. Since $\breve{\mu}$ is an IVF ideal of $\mathcal{S}$ and $\breve{\mu}(\mathfrak{u}) \neq \breve{0}$ we have $\breve{\mu}(\mathfrak{u}\mathfrak{u}\mathfrak{v}) \neq \breve{0}$ for all $\mathfrak{u}, \mathfrak{v} \in \mathcal{S}$. Thus, $((\breve{\eta}_1 \wedge \breve{\eta}_2) \wedge \breve{\mu})(\mathfrak{u}\mathfrak{u}\mathfrak{v}) \neq \breve{0}$. Hence, $((\breve{\eta}_1 \wedge \breve{\eta}_2) \wedge \breve{\mu}) \neq \breve{0}$. Therefore, $\breve{\eta}_1 \wedge \breve{\eta}_2$ is an EIVFID of $\mathcal{S}$. □

Definition 3.2. *[13] An EID $\mathcal{I}$ of a ternary semigroup $\mathcal{S}$ is called*

(1) minimal *if for every EID of $\mathcal{J}$ of $\mathcal{S}$ such that $\mathcal{J} \subseteq \mathcal{I}$, we have $\mathcal{J} = \mathcal{I}$,*
(2) prime *if $\mathfrak{u}\mathfrak{v}\mathfrak{k} \in \mathcal{I}$ implies $\mathfrak{u} \in \mathcal{I}$ or $\mathfrak{v} \in \mathcal{I}$ or $\mathfrak{k} \in \mathcal{I}$, for all $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$,*
(3) semiprime *if $\mathfrak{u}^3 \in \mathcal{I}$ implies $\mathfrak{u} \in \mathcal{I}$, for all $\mathfrak{u} \in \mathcal{S}$.*

Example 3.1. *[13] Let $\mathcal{S}$ be a ternary semigroup with zero. Then $\{0\}$ is a unique minimal EID of $\mathcal{S}$, since $\{0\}$ is an EID of $\mathcal{S}$.*

Definition 3.3. *An EIVFID $\breve{\eta}$ of a ternary semigroup $\mathcal{S}$ is called*

(1) minimal *if for every EIVFID of $\breve{\nu}$ of $\mathcal{S}$ such that $\breve{\nu} \sqsubseteq \breve{\eta}$, we have $\mathrm{supp}(\breve{\eta}) = \mathrm{supp}(\breve{\nu})$,*
(2) prime *if $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) \preceq \breve{\eta}(\mathfrak{u}) \vee \breve{\eta}(\mathfrak{v}) \vee \breve{\eta}(\mathfrak{k})$, for all $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$,*
(3) semiprime *if $\breve{\eta}(\mathfrak{u}^3) \preceq \breve{\eta}(\mathfrak{u})$, for all $\mathfrak{u} \in \mathcal{S}$.*

Theorem 3.5. *Let $\mathcal{I}$ be a non-empty subset of a ternary semigroup $\mathcal{S}$. Then the following statement holds.*

(1) *$\mathcal{I}$ is a minimal EID of $\mathcal{S}$ if and only if $\breve{\chi}_{\mathcal{I}}$ is a minimal EIVFID of $\mathcal{S}$,*

(2) *$\mathcal{I}$ is a prime EID of $\mathcal{S}$ if and only if $\check{\chi}_{\mathcal{I}}$ is a prime EIVFID of $\mathcal{S}$,*

(3) *$\mathcal{I}$ is a semiprime EID of S if and only if $\check{\chi}_{\mathcal{I}}$ is a semiprime EIVFID of $\mathcal{S}$.*

Proof. (1) Suppose that $\mathcal{I}$ is a minimal EID of $\mathcal{S}$. Then $\mathcal{I}$ is an EID of $\mathcal{S}$. By Theorem 3.1, $\check{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$. Let $\check{\eta}$ be an EIVFID of $\mathcal{S}$ such that $\check{\eta} \sqsubseteq \check{\chi}_{\mathcal{I}}$. Then by Theorem 3.2, $\operatorname{supp}(\check{\chi}_{\mathcal{I}})$ and $\operatorname{supp}(\check{\eta})$ are EIDs of $\mathcal{S}$ such that $\operatorname{supp}(\check{\eta}) \subseteq \operatorname{supp}(\check{\chi}_{\mathcal{I}})$. Thus, $\operatorname{supp}(\check{\eta}) \subseteq \operatorname{supp}(\check{\chi}_{\mathcal{I}}) = \mathcal{I}$. Hence, $\operatorname{supp}(\check{\eta}) \subseteq \mathcal{I}$. By assumption, $\operatorname{supp}(\check{\eta}) = \operatorname{supp}(\check{\chi}_{\mathcal{I}}) = \mathcal{I}$. Hence, $\check{\chi}_{\mathcal{I}}$ is a minimal EIVFID of $\mathcal{S}$.

Conversely, $\check{\chi}_{\mathcal{I}}$ is a minimal EIVFID of $\mathcal{S}$. $\check{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$. Let $\mathcal{B}$ be an EID of $\mathcal{S}$ such that $\mathcal{B} \subseteq \mathcal{I}$. Then by Theorem 3.1, $\check{\chi}_{\mathcal{B}}$ is an EIVFID of $\mathcal{S}$ such that $\check{\chi}_{\mathcal{B}} \sqsubseteq \check{\chi}_{\mathcal{I}}$. Thus, by Theorem 3.2, $\operatorname{supp}(\check{\chi}_{\mathcal{B}})$ and $\operatorname{supp}(\check{\chi}_{\mathcal{I}})$ are EIDs of $\mathcal{S}$ such that $\operatorname{supp}(\check{\chi}_{\mathcal{B}}) \subseteq \operatorname{supp}(\check{\chi}_{\mathcal{I}})$. So, $\mathcal{B} = \operatorname{supp}(\check{\chi}_{\mathcal{B}}) \subseteq \operatorname{supp}(\check{\chi}_{\mathcal{I}}) = \mathcal{I}$. By assumption, $\operatorname{supp}(\check{\chi}_{\mathcal{B}}) = \operatorname{supp}(\check{\chi}_{\mathcal{I}})$ Hence, $\mathcal{B} = \operatorname{supp}(\check{\chi}_{\mathcal{B}}) = \operatorname{supp}(\check{\chi}_{\mathcal{I}}) = \mathcal{I}$. Therefore, $\mathcal{I}$ is a minimal EID of $\mathcal{S}$.

(2) Suppose that $\mathcal{I}$ is a prime EID of $\mathcal{S}$. Then $\mathcal{I}$ is an EID of $\mathcal{S}$. Thus, by Theorem 3.1, $\check{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$. Let $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$.

If $\mathfrak{u}\mathfrak{v}\mathfrak{k} \in \mathcal{I}$, then $\mathfrak{u} \in \mathcal{I}$ or $\mathfrak{v} \in \mathcal{I}$ or $\mathfrak{k} \in \mathcal{I}$. Thus, $\check{\chi}_{\mathcal{I}}(\mathfrak{u}) = \check{1} = \check{\chi}_{\mathcal{I}}(\mathfrak{v}) = \check{\chi}_{\mathcal{I}}(\mathfrak{k}) = \check{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k})$. Hence, $\check{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \check{\chi}_{\mathcal{I}}(\mathfrak{v}) = \check{1} \geq \check{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k})$.

If $\mathfrak{u}\mathfrak{v}\mathfrak{k} \notin \mathcal{I}$, then $\check{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) \preceq \check{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \check{\chi}_{\mathcal{I}}(\mathfrak{v}) \vee \check{\chi}_{\mathcal{I}}(\mathfrak{k})$.

Thus, $\check{\chi}_{\mathcal{I}}$ is a prime EIVFID of $\mathcal{S}$.

Conversely, suppose that $\check{\chi}_{\mathcal{I}}$ is a prime EIVFID of $\mathcal{S}$. Then $\check{\chi}_{\mathcal{I}}$ is an EIVFID. Thus, by Theorem 3.1, $\mathcal{I}$ is an EID of $\mathcal{S}$. Let $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$. If $\mathfrak{u}\mathfrak{v}\mathfrak{k} \in \mathcal{I}$ with $\mathfrak{u} \notin \mathcal{I}$ and $\mathfrak{v} \notin \mathcal{I}$ and $\mathfrak{k} \notin \mathcal{I}$, then $\check{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) = \check{1}$ and $\check{\chi}_{\mathcal{I}}(\mathfrak{u}) = \check{0} = \check{\chi}_{\mathcal{I}}(\mathfrak{v}) = \check{\chi}_{\mathcal{I}}(\mathfrak{k})$. By assumption, $\check{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) \preceq \check{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \check{\chi}_{\mathcal{I}}(\mathfrak{v}) \vee \check{\chi}_{\mathcal{I}}(\mathfrak{k})$. Thus, $\check{1} = \check{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) \preceq \check{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \check{\chi}_{\mathcal{I}}(\mathfrak{v}) \vee \check{\chi}_{\mathcal{I}}(\mathfrak{k}) = \check{0}$. It is a contradiction. So, $\mathfrak{u} \in \mathcal{I}$ or $\mathfrak{v} \in \mathcal{I}$ or $\mathfrak{k} \in \mathcal{I}$. Hence, $\mathcal{I}$ is a prime EID of $\mathcal{S}$.

(3) Suppose that $\mathcal{I}$ is a semiprime EID of $\mathcal{S}$. Then $\mathcal{I}$ is an EID of $\mathcal{S}$. Thus, by Theorem 3.1, $\check{\chi}_{\mathcal{I}}$ is an EIVFID of $\mathcal{S}$. Let $\mathfrak{u} \in \mathcal{S}$.

If $\mathfrak{u}^3 \in \mathcal{I}$, then $\mathfrak{u} \in \mathcal{I}$ Thus, $\check{\chi}_{\mathcal{I}}(\mathfrak{u}) = \check{\chi}_{\mathcal{I}}(\mathfrak{u}^3) = \check{1}$. Hence, $\check{\chi}_{\mathcal{I}}(\mathfrak{u}^3) \preceq \check{\chi}_{\mathcal{I}}(\mathfrak{u})$.

If $\mathfrak{u}^3 \notin \mathcal{I}$, then $\check{\chi}_{\mathcal{I}}(\mathfrak{u}^3) = 0 \preceq \check{\chi}_{\mathcal{I}}(\mathfrak{u})$.

Thus, $\check{\chi}_{\mathcal{I}}$ is a semiprime EIVFID of $\mathcal{S}$.

Conversely, suppose that $\check{\chi}_{\mathcal{I}}$ is a semiprime EIVFID of $\mathcal{S}$. Then $\check{\chi}_{\mathcal{I}}$ is an EIVFID. Thus, by Theorem 3.1, $\mathcal{I}$ is an EID of $\mathcal{S}$. Let $\mathfrak{u} \in \mathcal{S}$ with $\mathfrak{u}^3 \in \mathcal{I}$ and $\mathfrak{u} \notin \mathcal{I}$. Then $\check{\chi}_{\mathcal{I}}(\mathfrak{u}^3) = \check{1}$ and $\check{\chi}_{\mathcal{I}}(\mathfrak{u}) = \check{0}$. By assumption, $\check{\chi}_{\mathcal{I}}(\mathfrak{u}^3) \preceq \check{\chi}_{\mathcal{I}}(\mathfrak{u})$. Thus, $\check{1} = \check{\chi}_{\mathcal{I}}(\mathfrak{u}^3) \preceq \check{\chi}_{\mathcal{I}}(\mathfrak{u}) = \check{0}$. It is a contradiction. So, $\mathfrak{u} \in \mathcal{I}$. Hence, $\mathcal{I}$ is a semiprime EID of $\mathcal{S}$.

□

4. 0-Essential Interval Valued Ideal in Ternary Semigroups

In this section, we let $\mathcal{S}$ be a ternary semigroup with zero. begin we review the definition 0-essential ideal of $\mathcal{S}$ as follows:

Definition 4.1. *[1] A nonzero ID $\mathcal{I}$ of a ternary semigroup with zero $\mathcal{S}$ is called a* 0-essential ideal *(0-EID) of $\mathcal{S}$ if $\mathcal{I} \cap \mathcal{J} \neq \{0\}$ for every nonzero ID of $\mathcal{J}$ of $\mathcal{S}$.*

Example 4.1. *[1] Let $(\mathbb{Z}_{12}, +)$ be a semigroup. Then $\{0,2,4,6,8,10\}$ and $\mathbb{Z}_{12}$ are 0-EID of $\mathbb{Z}_{12}$.*

Definition 4.2. *An IVF ideal $\breve{\eta}$ of a ternary semigroup with zero $\mathcal{S}$ is called a* nontrivial IVFID *of $\mathcal{S}$ if there exists a nonzero element $\mathfrak{u} \in \mathcal{S}$ such that $\breve{\eta}(\mathfrak{u}) \neq \breve{0}$.*

We define the definition of 0-essential IVF ideals of a ternary semigroup with zero as follows:

Definition 4.3. *A* 0-essential IVF ideal *(0-EIVFID) $\breve{\eta}$ of a ternary semigroup with zero $\mathcal{S}$ if $\breve{\eta}$ is a nonzero IVFID of $\mathcal{S}$ and* $\mathrm{supp}(\breve{\eta} \wedge \breve{\nu}) \neq \{0\}$ *for every nonzero IVFID $\breve{\nu}$ of $\mathcal{S}$.*

Theorem 4.1. *Let $\mathcal{I}$ be a nonzero ID of a ternary semigroup with zero $\mathcal{S}$. Then $\mathcal{I}$ is a 0-EID of $\mathcal{S}$ if and only if $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID of $\mathcal{S}$.*

Proof. Suppose that $\mathcal{I}$ is a 0-EID of $\mathcal{S}$ and let $\breve{\nu}$ be a nontrivial IVFID of $\mathcal{S}$. Then by Theorem 2.12, $\mathrm{supp}(\breve{\nu})$ is a nonzero ID of $\mathcal{S}$. Since $\mathcal{I}$ is a 0-EID of $\mathcal{S}$ we have $\mathcal{I}$ is a nonzero ID of $\mathcal{S}$. Thus, $\mathcal{I} \cap \mathrm{supp}(\breve{\nu}) \neq \{0\}$. So, there exists $\mathfrak{u} \in \mathcal{I} \cap \mathrm{supp}(\breve{\nu})$. Since $\mathcal{I}$ is a nonzero ID of $\mathcal{S}$ we have $\breve{\chi}_{\mathcal{I}}$ is a nonzero IVF ideal of $\mathcal{S}$. Thus, $\breve{\chi}_{\mathcal{I}} \wedge \breve{\nu}$ is IVFID of $\mathcal{S}$ So, $\mathrm{supp}(\breve{\chi}_{\mathcal{I}} \wedge \breve{\nu})$ is an ID of $\mathcal{S}$. It implies that, $(\breve{\chi}_{\mathcal{I}} \wedge \breve{\nu})(\mathfrak{u}) \neq \breve{0}$. Hence, $\mathrm{supp}(\breve{\chi}_{\mathcal{I}} \wedge \breve{\nu}) \neq \{0\}$. Therefore, $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID of $\mathcal{S}$.

Conversely, assume that $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID of $\mathcal{S}$ and let $\mathcal{J}$ be a nonzero ID of $\mathcal{S}$. Then $\breve{\chi}_{\mathcal{J}}$ is a nonzero IVF ID of $\mathcal{S}$. Sicne $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID of $\mathcal{S}$ we have $\breve{\chi}_{\mathcal{I}}$ is a nontrivial IVFID of $\mathcal{S}$. Thus, $\mathrm{supp}(\breve{\chi}_{\mathcal{I}} \wedge \breve{\chi}_{\mathcal{J}}) \neq \{\breve{0}\}$. So, by Theorem 2.11, $\breve{\chi}_{\mathfrak{L} \cap \mathfrak{J}} \neq \breve{0}$. Hence, $\mathcal{I} \cap \mathcal{J} \neq \{0\}$. Therefore, $\mathcal{I}$ is a 0-EID of $\mathcal{S}$. □

Theorem 4.2. *Let $\breve{\eta}$ be a nonzero IVFID of a ternary semigroup with zero $\mathcal{S}$. Then $\breve{\eta}$ is a 0-EIVFID of $\mathcal{S}$ if and only if* $\mathrm{supp}(\breve{\eta})$ *is a 0-EID of $\mathcal{S}$.*

Proof. Assume that $\breve{\eta}$ is a 0-EIVFID of $\mathcal{S}$ and let $\mathcal{J}$ be a nontrivial ID of $\mathcal{S}$. Then by Theorem 2.9, $\breve{\chi}_{\mathcal{J}}$ is a nonzero IVFID of $\mathcal{S}$. Since $\breve{\eta}$ is a 0-EIVFID of $\mathcal{S}$ we have $\breve{\eta}$ is a nonzero IVFID of $\mathcal{S}$. Thus, $\mathrm{supp}(\breve{\eta})$ is an ID of $\mathcal{S}$ and $\breve{\eta} \wedge \breve{\chi}_{\mathcal{J}} \neq \breve{0}$. So, there exists a nonzero element $\mathfrak{u} \in \mathcal{S}$ such that $(\breve{\eta} \wedge \breve{\chi}_{\mathcal{J}})(\mathfrak{u}) \neq \breve{0}$. It implies that $\breve{\eta}(\mathfrak{u}) \neq \breve{0}$ and $\breve{\chi}_{\mathcal{J}}(\mathfrak{u}) \neq \breve{0}$. Hence, $\mathfrak{u} \in \mathrm{supp}(\breve{\eta}) \cap \mathcal{J}$ So, $\mathrm{supp}(\breve{\eta}) \cap \mathcal{J} \neq \{0\}$. Therefore, $\mathrm{supp}(\breve{\eta})$ is a 0-EID of $\mathcal{S}$.

Conversely, assume that $\mathrm{supp}(\breve{\eta})$ is a 0-EID of $\mathcal{S}$ and let $\breve{\nu}$ be a nonzero IVF ideal of $\mathcal{S}$. Then by Theorem 2.12 $\mathrm{sup}(\breve{\nu})$ is a nontrivial zero ideal of $\mathcal{S}$. Since $\mathrm{supp}(\breve{\eta})$ is a 0-EID of $\mathcal{S}$ we have $\mathrm{supp}(\breve{\eta})$ is a nonzero ID of $\mathcal{S}$. Thus, $\mathrm{supp}(\breve{\eta}) \cap \mathrm{supp}(\breve{\nu}) \neq \{0\}$. So, there exists $\mathfrak{u} \in \mathrm{supp}(\breve{\eta}) \cap \mathrm{supp}(\breve{\nu})$, this implies that $\breve{\eta}(\mathfrak{u}) \neq \breve{0}$ and $\breve{\nu}(\mathfrak{u}) \neq \breve{0}$ for all $\mathfrak{u} \in \mathcal{S}$. Hence, $(\breve{\eta} \wedge \breve{\nu})(\mathfrak{u}) \neq \breve{0}$ for all $\mathfrak{u} \in \mathcal{S}$. Therefore, $\mathrm{supp}(\breve{\eta} \wedge \breve{\nu}) \neq \{0\}$. We conclude that $\breve{\eta}$ is a 0-EIVFID of $\mathcal{S}$. □

Theorem 4.3. *Let $\mathcal{S}$ be a ternary semigroup with zero and $\breve{\eta}$ be a 0-EIVFID of $\mathcal{S}$. If $\breve{\nu}$ is an IVFID of $\mathcal{S}$ such that $\breve{\eta} \sqsubseteq \breve{\nu}$, then $\breve{\nu}$ is also, a 0-EIVFID of $\mathcal{S}$.*

Proof. Let $\breve{\nu}$ be an IVFID of $\mathcal{S}$ such that $\breve{\eta} \sqsubseteq \breve{\nu}$ and let $\breve{\mu}$ be any nonzero IVFID of $\mathcal{S}$. Since $\breve{\eta}$ is a 0-EIVFID of $\mathcal{S}$ we have $\breve{\eta}$ is an IVFID of $\mathcal{S}$. Thus, $\breve{\eta} \wedge \breve{\mu}$ is an IVFID of $\mathcal{S}$. So, $\mathrm{supp}(\breve{\eta} \wedge \breve{\mu})$ is an ID of $\mathcal{S}$. Thus, $\mathrm{supp}(\breve{\eta} \wedge \breve{\mu}) \neq \{0\}$. So, $\{0\} \neq \mathrm{supp}(\breve{\eta} \wedge \breve{\mu}) \subseteq \mathrm{supp}(\breve{\nu} \wedge \breve{\mu})$. It implies that, $\mathrm{supp}(\breve{\nu} \wedge \breve{\mu}) \neq \{0\}$. Hence, $\breve{\nu}$ is a 0-EIVFID of $\mathcal{S}$. □

Theorem 4.4. *Let $\breve{\eta}_1$ and $\breve{\eta}_2$ be 0-EIVFIDs of a ternary semigroup with zero $\mathcal{S}$. Then $\breve{\eta}_1 \vee \breve{\eta}_2$ and $\breve{\eta}_1 \wedge \breve{\eta}_2$ are 0-EIVFIDs of $\mathcal{S}$.*

Proof. Since $\breve{\eta}_2 \sqsubseteq \breve{\eta}_1 \vee \breve{\eta}_2$ we have $\breve{\eta}_1 \vee \breve{\eta}_2$ is a 0-EIVFID of $\mathcal{S}$.

Since $\breve{\eta}_1$ and $\breve{\eta}_2$ are 0-EIVFIDs of $\mathcal{S}$ we have $\breve{\eta}_1$ and $\breve{\eta}_2$ are IVFIDs of $\mathcal{S}$. Thus, $\breve{\eta}_1 \wedge \breve{\eta}_2$ is an IVFID of $\mathcal{S}$. Let $\breve{\nu}$ be a nontrivial IVFID of $\mathcal{S}$. Since $\breve{\eta}_1$ is an IVFID of $\mathcal{S}$ we have $\mathrm{supp}(\breve{\eta}_1)$ is an ID of $\mathcal{S}$. Thus, $\mathrm{supp}(\breve{\eta}_1 \wedge \breve{\nu}) \neq \{0\}$. So, there exists a nonzero element $\mathfrak{u} \in \mathcal{S}$ such that $(\breve{\eta}_1 \wedge \breve{\nu})(\mathfrak{u}) \neq \breve{0}$. Since $\breve{\eta}_2$ is a 0-EIVFID of $\mathcal{S}$ we have $\mathrm{supp}(\breve{\eta}_2)$ is a 0-EIVFID of $\mathcal{S}$. Thus, $\mathrm{supp}(\breve{\eta}_2) \cap (\mathfrak{u})_i \neq \{0\}$. So, there exists a nonzero element $\mathfrak{v} \in \mathrm{supp}(\breve{\eta}_2) \cap (\mathfrak{u})_i$ implies $(\breve{\eta}_2)(\mathfrak{y}) \neq \breve{0}$. Since $\breve{\eta}_1$ and $\breve{\nu}$ are IVFIDs of $\mathcal{S}$ we have $\breve{\eta}_1(\mathfrak{v}) \geq \breve{\eta}_1(\mathfrak{u})$ and $\breve{\nu}(\mathfrak{v}) \geq \breve{\nu}(\mathfrak{u})$. So, $((\breve{\eta}_1 \wedge \breve{\eta}_2) \wedge \breve{\nu})(\mathfrak{v}) \neq \breve{0}$. Thus, $\mathrm{supp}((\breve{\eta}_1 \wedge \breve{\eta}_2) \wedge \breve{\nu}) \neq \{0\}$. Therefore, $\breve{\eta}_1 \wedge \breve{\eta}_2$ is a 0-EIVFIDs of $\mathcal{S}$. □

Definition 4.4. *[1] Let $\mathcal{I}$ be a 0-essential ideal of a ternary semigroup with zero $\mathcal{S}$. Then $\mathcal{I}$ is called*

(1) minimal *if for every 0-EID of $\mathcal{J}$ of $\mathcal{S}$ such that $\mathcal{J} \subseteq \mathcal{I}$, we have $\mathcal{J} = \mathcal{I}$,*
(2) prime *if $\mathfrak{u}\mathfrak{v}\mathfrak{k} \in \mathcal{I}$ implies $\mathfrak{u} \in \mathcal{I}$ or $\mathfrak{v} \in \mathcal{I}$ or $\mathfrak{k} \in \mathcal{I}$ for all $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$.,*
(3) semiprime *if $\mathfrak{u}^3 \in \mathcal{I}$ implies $\mathfrak{u} \in \mathcal{I}$, for all $\mathfrak{u} \in \mathcal{S}$.*

Example 4.2. *Let $(\mathbb{Z}_{12}, +)$ be a ternary semigroup with zero. Then $\{0, 2, 4, 6, 8, 10\}$ is a minimal 0-EID of $\mathcal{S}$.*

Definition 4.5. *Let $\breve{\eta}$ be a 0-EIVFID of a ternary semigroup with zero $\mathcal{S}$. Then $\breve{\eta}$ is called*

(1) minimal *if for every 0-EIVFID of $\breve{\nu}$ of $\mathcal{S}$ such that $\breve{\nu} \sqsubseteq \breve{\eta}$, we have $\mathrm{supp}(\breve{\eta}) = \mathrm{supp}(\breve{\nu})$,*
(2) prime *if $\breve{\eta}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) \preceq \breve{\eta}(\mathfrak{u}) \vee \breve{\eta}(\mathfrak{v}) \vee \breve{\eta}(\mathfrak{k})$ for all $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$,*
(3) semiprime *if $\breve{\eta}(\mathfrak{u}^3) \preceq \breve{\eta}(\mathfrak{u})$, for all $\mathfrak{u} \in \mathcal{S}$.*

Theorem 4.5. *Let $\mathcal{I}$ be a non-empty subset of a ternary semigroup with zero $\mathcal{S}$. Then the following statement holds.*

(1) *$\mathcal{I}$ is a minimal 0-EID of $\mathcal{S}$ if and only if $\breve{\chi}_{\mathcal{I}}$ is a minimal 0-EIVFID of $\mathcal{S}$,*
(2) *$\mathcal{I}$ is a prime 0-EID of $\mathcal{S}$ if and only if $\breve{\chi}_{\mathcal{I}}$ is a prime 0-EIVFID of $\mathcal{S}$,*
(3) *$\mathcal{I}$ is a semiprime 0-EID of $\mathcal{S}$ if and only if $\breve{\chi}_{\mathcal{I}}$ is a semiprime 0-EIVFID of $\mathcal{S}$.*

Proof. (1) Suppose that $\mathcal{I}$ is a minimal 0-EID of $\mathcal{S}$. Then $\mathcal{I}$ is a 0-EID of $\mathcal{S}$. By Theorem 4.1, $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID of $\mathcal{S}$. Let $\breve{\eta}$ be a 0-EIVFID of $\mathcal{S}$ such that $\breve{\eta} \sqsubseteq \breve{\chi}_{\mathcal{I}}$. Then by Theorem 4.2, $\mathrm{supp}(\breve{\eta})$ and $\mathrm{supp}(\breve{\chi}_{\mathcal{I}})$ are 0-EIDs of $\mathcal{S}$ such that $\mathrm{supp}(\breve{\eta}) \subseteq \mathrm{supp}(\breve{\chi}_{\mathcal{I}})$. Thus, $\mathrm{supp}(\breve{\eta}) \subseteq \mathrm{supp}(\breve{\chi}_{\mathcal{I}}) = \mathcal{I}$. So, $\mathrm{supp}(\breve{\eta}) \subseteq \mathcal{I}$. By assumption, $\mathrm{supp}(\breve{\eta}) = \mathcal{I} = \mathrm{supp}(\breve{\chi}_{\mathcal{I}})$. Hence, $\breve{\chi}_{\mathcal{I}}$ is a minimal 0-EIVFID of $\mathcal{S}$.

Conversely, $\breve{\chi}_{\mathcal{I}}$ is a minimal 0-EIVFID of $\mathcal{S}$. $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID of $\mathcal{S}$. Let $\mathcal{B}$ be a 0-EID of $\mathcal{S}$ such that $\mathcal{B} \subseteq \mathcal{I}$. Then by Theorem 4.1, $\breve{\chi}_{\mathcal{B}}$ is a 0-EIVFID of $\mathcal{S}$ such that $\breve{\chi}_{\mathcal{B}} \sqsubseteq \breve{\chi}_{\mathcal{I}}$. Thus, by Theorem 4.2, $\mathrm{supp}(\breve{\chi}_{\mathcal{B}})$ and $\mathrm{supp}(\breve{\chi}_{\mathcal{I}})$ are 0-EIDs of $\mathcal{S}$ such that $\mathrm{supp}(\breve{\chi}_{\mathcal{B}}) \subseteq \mathrm{supp}(\breve{\chi}_{\mathcal{I}})$. So, $\mathcal{B} = \mathrm{supp}(\breve{\chi}_{\mathcal{B}}) \subseteq \mathrm{supp}(\breve{\chi}_{\mathcal{I}}) = \mathcal{I}$. By assumption, $\mathrm{supp}(\breve{\chi}_{\mathcal{B}}) = \mathrm{supp}(\breve{\chi}_{\mathcal{I}})$ Hence, $\mathcal{B} = \mathrm{supp}(\breve{\chi}_{\mathcal{B}}) = \mathrm{supp}(\breve{\chi}_{\mathcal{I}}) = \mathcal{I}$. Therefore, $\mathcal{I}$ is a minimal 0-EID of $\mathcal{S}$.

(2) Suppose that $\mathcal{I}$ is a prime 0-essential ideal of $\mathcal{S}$. Then $\mathcal{I}$ is a 0-essential ideal of $\mathcal{S}$. Thus, by Theorem 3.1 $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID of $\mathcal{S}$. Let $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$.

If $\mathfrak{u}\mathfrak{v}\mathfrak{k} \in \mathcal{I}$, then $\mathfrak{u} \in \mathcal{I}$ or $\mathfrak{v} \in \mathcal{I}$ or $\mathfrak{k} \in \mathcal{I}$. Thus, $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}) = \breve{1} = \breve{\chi}_{\mathcal{I}}(\mathfrak{v}) = \breve{\chi}_{\mathcal{I}}(\mathfrak{k}) = \breve{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k})$. Hence, $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \breve{\chi}_{\mathcal{I}}(\mathfrak{v}) = \breve{1} \geq \breve{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k})$.

If $\mathfrak{u}\mathfrak{v} \notin \mathcal{I}$, then $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}) \leq \breve{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \breve{\chi}_{\mathcal{I}}(\mathfrak{v})$.

Thus, $\breve{\chi}_{\mathcal{I}}$ is a prime 0-EIVFID of $\mathcal{S}$.

Conversely, suppose that $\breve{\chi}_{\mathcal{I}}$ is a prime 0-EIVFID of $\mathcal{S}$. Then $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID. Thus, by Theorem 4.1, $\mathcal{I}$ is a 0-EID of $\mathcal{S}$. Let $\mathfrak{u}, \mathfrak{v}, \mathfrak{k} \in \mathcal{S}$.

If $\mathfrak{u}\mathfrak{v}\mathfrak{k} \in \mathcal{I}$ with $\mathfrak{u} \notin \mathcal{I}$ and $\mathfrak{v} \notin \mathcal{I}$ and $\mathfrak{k} \notin \mathcal{I}$, then $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) = \breve{1}$ and $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}) = \breve{0} = \breve{\chi}_{\mathcal{I}}(\mathfrak{v}) = \breve{\chi}_{\mathcal{I}}(\mathfrak{k})$. By assumption, $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) \leq \breve{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \breve{\chi}_{\mathcal{I}}(\mathfrak{v}) \vee \breve{\chi}_{\mathcal{I}}(\mathfrak{k})$. Thus, $\breve{1} = \breve{\chi}_{\mathcal{I}}(\mathfrak{u}\mathfrak{v}\mathfrak{k}) \leq \breve{\chi}_{\mathcal{I}}(\mathfrak{u}) \vee \breve{\chi}_{\mathcal{I}}(\mathfrak{v}) \vee \breve{\chi}_{\mathcal{I}}(\mathfrak{k}) = \breve{0}$. It is a contradiction. So, $\mathfrak{u} \in \mathcal{I}$ or $\mathfrak{v} \in \mathcal{I}$ or $\mathfrak{k} \in \mathcal{I}$. Hence, $\mathcal{I}$ is a prime 0-SID of $\mathcal{S}$.

(3) Suppose that $\mathcal{I}$ is a semiprime 0-EID of $\mathcal{S}$. Then $\mathcal{I}$ is an essential ideal of $\mathcal{S}$. Thus, by Theorem 4.1, $\breve{\chi}_{\mathcal{I}}$ is an 0-EIVFID of $\mathcal{S}$. Let $\mathfrak{u} \in \mathcal{S}$.

If $\mathfrak{u}^3 \in \mathcal{I}$, then $\mathfrak{u} \in \mathcal{I}$ Thus, $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}) = \breve{\chi}_{\mathcal{I}}(\mathfrak{u}^3) = \breve{1}$. Hence, $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}^3) \leq \breve{\chi}_{\mathcal{I}}(\mathfrak{u})$.

If $\mathfrak{u}^3 \notin \mathcal{I}$, then $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}^3) = 0 \leq \breve{\chi}_{\mathcal{I}}(\mathfrak{u})$.

Thus, $\breve{\chi}_{\mathcal{I}}$ is a semiprime 0-EIVFID of $\mathcal{S}$.

Conversely, suppose that $\breve{\chi}_{\mathcal{I}}$ is a semiprime 0-EIVFID of $\mathcal{S}$. Then $\breve{\chi}_{\mathcal{I}}$ is a 0-EIVFID. Thus, by Theorem 4.1, $\mathcal{I}$ is a 0-EID of $\mathcal{S}$. Let $\mathfrak{u} \in \mathcal{S}$ with $\mathfrak{u}^3 \in \mathcal{I}$ and $\mathfrak{u} \notin \mathcal{I}$. Then $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}^3) = \breve{1}$ and $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}) = \breve{0}$. By assumption, $\breve{\chi}_{\mathcal{I}}(\mathfrak{u}^3) \leq \breve{\chi}_{\mathcal{I}}(\mathfrak{u})$. Thus, $\breve{1} = \breve{\chi}_{\mathcal{I}}(\mathfrak{u}^3) \leq \breve{\chi}_{\mathcal{I}}(\mathfrak{u}) = \breve{0}$. It is a contradiction. So, $\mathfrak{u} \in \mathcal{I}$. Hence, $\mathcal{I}$ is a semiprime 0-EID of $\mathcal{S}$.

□

5. Conclusion

In this work, we established that both the union and the intersection of essential ideals namely essential ideals, 0-essential ideals, essential interval-valued fuzzy ideals, and 0-essential interval-valued fuzzy ideals of a ternary semigroup are themselves essential ideals of the corresponding type. Furthermore, we proved that an ideal I of a ternary semigroup $\mathcal{S}$ is essential if and only if the associated fuzzy set $\breve{\chi}_{\mathcal{I}}$ is an essential fuzzy ideal of $\mathcal{S}$. It was also shown that a nonzero fuzzy ideal $\breve{\eta}$ of $\mathcal{S}$ is essential precisely when its support $\mathrm{supp}(\breve{\eta})$ is an essential ideal. Similarly, for a ternary semigroup $\mathcal{S}$ with zero, we demonstrated that a nonzero ideal $\mathcal{I}$ is 0-essential if and only if $\breve{\chi}_{\mathcal{I}}$ is a 0-essential interval-valued fuzzy ideal of $\mathcal{S}$. In addition, we established that an interval-valued fuzzy ideal $\breve{\eta}$ of $\mathcal{S}$ is 0-essential if and only if its support is a 0-essential ideal.

Moreover, this study clarified the connections between essential ideals and essential interval-valued fuzzy ideals in the context of ternary semigroups. As a direction for future research, similar concepts may be explored in other algebraic structures. For instance, one may introduce and investigate essential hyperideals and essential interval-valued fuzzy hyperideals within semihypergroups.

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