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An Integrated TOPSIS Model with Neutrosophic Vague Soft Expert Sets for Uncertain Decision Making

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ABSTRACT. In traditional decision making, data is usually seen as just true or false. However, soft expert set (SES) use the opinions of real experts. Unlike a standard soft set, SES includes 'agree' and 'disagree' options to capture the different views of human experts. In this study, we used the strength of SES combined with neutrosophic vague sets to handle uncertainty. Thus, this study proposes the integration of a neutrosophic vague soft expert set with the TOPSIS method (NVSES-TOPSIS) to handle uncertainty in decision making process. Alternatives are evaluated using NVSES while, unknown weights are obtained through maximizing deviation method. The ranking of alternatives is obtained using TOPSIS method. A Numerical example involving candidate selection for a vacancy validates the method. Finally, a comparative analysis illustrates how different distance measures influence the final ranking order. Future studies could integrate NVSES with other MCDM methods to increase computational ease without requiring aggregation operators for multiple experts.

1. Introduction

Real-world decision-making is plagued by uncertainty, imprecision and inconsistency. Various set theories have been proposed to solve uncertainty issues until now. This began with traditional

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fuzzy set (FS) introduced by Zadeh [1] with a single membership degree. Subsequently. Atanassov [2] developed IFS to extend FS by incorporating non-membership degree. Various set theories have been integrated with FS to improve its functionality to solve uncertainty issues such as Pythagorean fuzzy set (PFS) [3], Complex fuzzy set [4] and bipolar fuzzy set [5]. However, to deal with indeterminate and inconsistent information, a proper description of an object within uncertain and complex environments is required. Therefore, Smarandache [6] introduced the neutrosophic set (NS) by introducing an indeterminacy membership function as an independent component. However, NS cannot be used in real life application due to its non-standard constraints. Thus, Wang et al., [7] improved it by introducing a standard interval known as single valued neutrosophic set (SVNS). The emergence of SVNS has gained significant attention among scholars in various fields due to its ability to handle uncertainty and vagueness in decision-making problems. Many studies have applied SVNS in different research areas, including the development of aggregation operators [8-11] distance measures [12-15] and hybrid multi-criteria decision-making (MCDM) methods [16-20].

The Soft set (SS) is another well-known set theory introduced by Molodtsov [21]. SS is a flexible alternative to crisp sets and FS. Unlike FS, no membership is required in SS and it effectively handles incomplete data. Since the inception of SS by Molodtsov, various researchers proposed mathematical hybrid models such as fuzzy soft set [22], intuitionistic fuzzy soft set [23] and posibillity fuzzy soft set [24]. However, SS consider only a single expert, which leads to difficulties when collecting data via questionnaires. To overcome these challenges, Alkhazaleh and Saleh [25] introduced soft expert set (SES), which allows for the inclusion of multi experts' opinion without require any opinions. They also proposed he basic operation of SES and studied some properties and applied it in decision-making problems. Recently, many researchers have developed SES in various environments ranging from fuzzy set to neutrosphic sets. These developments include fuzzy parameterized soft expert set [26], intuitionistic fuzzy soft expert set [27], neutrosophic soft expert set [28], vague soft expert set [29] and neutrosophic vague soft expert set (NVSES) [30]. NVSES is introduced to improve the reasonability of decision making in reality.

The integration of SES with other set theories has been applied to Multi Criteria Decision Making (MCDM). For instance, Pramanik et al., [31] proposed SVNS approach combined with TOPSIS to solve the selection process for mathematics teachers. Besides that, other MCDM that has been combined with soft expert set is AHP such as study by Saeed et al., [32].

TOPSIS is well-known method as a powerful tool for ranking alternatives based on their distances from an ideal solution. However, traditional TOPSIS struggles to handle multiple experts at once. In a multi-criteria decision-making environment involving several experts, traditional TOPSIS requires an aggregation operator to synthesize individual judgments into a single collective value. Unlike TOPSIS, SES provides an organized way to collect data from many experts, while

TOPSIS provides best choice based on ranking the highest score [33-36]. This combination ensures that no expert's opinion is lost during the process.

Based on the above summary, the current set theories are significant in capturing the vagueness of the human thought, they are often restricted to a single expert representative. Conversely, existing SES can handle multiple experts, but frequently rely on simpler logic that fails to capture the full complexity of indeterminacy. To fill this gap, this study proposes a neutrosophic vague SES (NVSES) by incorporating with the TOPSIS method. The TOPSIS is utilized as the decision-making engine to rank the alternatives.

The objectives of this study are threefold: (1) to develop a decision-making framework by integrating NVSES with the TOPSIS method: (2) to calculate the criteria weights using the maximizing deviation method: and (3) to rank the alternatives based on their closeness to the ideal solution using the TOPSIS engine.

The research is organized as follows: Section 2 presents important definitions of NVSES and SES in the preliminaries. Decision making with NVSES-TOPSIS is presented in Section 3. Following this, a numerical example is provided to demonstrate the application of NVSES-TOPSIS in section 4. Finally, a comparative analysis is performed in Section 5.

2. Preliminaries

In this part, we recall some definitions of neutrosophic vague set (NVS) and soft expert set (SES) as follows:

Definition 2.1: Neutrosophic vague set [37]

Over a given universe of discourse X, a neutrosophic vague set (NVS), denoted as A_{NV}. For each of set $x \in X$, the set is defined as follows:

$$A_{NV} = \left\{ x, \left[T^{-}(x), T^{+}(x) \right], \left[I^{-}(x), I^{+}(x) \right], \left[F^{-}(x), F^{+}(x) \right] \right\},$$

These values are constrained by conditions:

$$T^{+} = 1 - F^{-} \text{ and } F^{+} = 1 - T^{-},$$

These values are constrained by:

$$0 \leq T^{-} + I^{-} + F^{-} \leq 2 \text{ and } 0 \leq T^{+} + I^{+} + F^{+} \leq 2$$

Definition 2.2: Soft expert set [25]

Let X be an initial universal set, E be the set of parameters, Z be the set of experts and $O = \{agree = 1, disagree = 0\}$ be the set of opinions. Consider $W = E \times Z \times O$, $A \subseteq W$. Then, a pair (M, A) is called soft expert set over X, where M is a mapping given by $M : A \rightarrow P(X)$, repsent power set of X.

Definition 2.3: Agree-soft expert set [25]

An agree-soft expert set $(M,A)_1$ over X is a soft expert subset of (M,A) is defined as follows:

$$(M,A)_1=\{M(\beta):\beta\in E\times Z\times\{1\}\}$$

Definition 2.4: Disagree-soft expert set [25]

An disagree-soft expert set $(M,A)_0$ over X is a soft expert subset of (M,A) is defined as follows:

$$(M,A)_0=\{M(\beta):\beta\in E\times Z\times\{0\}\}$$

Definition 2.5: Neutrosophic vague soft expert set [30]

A pair (F,A) is called a neutrosophic vague soft expert set (NVSES) over U, where F is a mapping defined as $F:A\to NV^U$. Here, NV^U denotes the power neutrosophic vague set of U. Suppose $F:A\to NV^U$ is a function defined by $F(a)=F(a)(u)$, for all of $u_i\in U$. For each $a_i\in A$, $F(a_i)=F(a_i)(u)$, where $F(a_i)$ represents the degree of membership, indeterminacy, and non membership of element in U with respect to $F(a_i)$.

The NVSES (F,A) can be expressed as the following set of ordered pairs:

$$(F,A)=\left\{\left(a_i,\left\{\frac{u_j}{F(a_i)(u_j)}:u_j\in U\right\}\right):a_i\in A\right\},$$

where

$$F(a_i)(u_j)=\left(\left[T^-_{F(a_i)}(u_j),T^+_{F(a_i)}(u_j)\right],\left[I^-_{F(a_i)}(u_j),I^+_{F(a_i)}(u_j)\right],\left[F^-_{F(a_i)}(u_j),F^+_{F(a_i)}(u_j)\right]\right),$$

and

$$T^+_{F(a_i)}(u_j)=1-T^-_{F(a_i)}(u_j),F^+_{F(a_i)}(u_j)=1-F^-_{F(a_i)}(u_j).$$

Here, represent the $\left[T^-_{F(a_i)}(u_j),T^+_{F(a_i)}(u_j)\right]$ truth-membership, $\left[I^-_{F(a_i)}(u_j),I^+_{F(a_i)}(u_j)\right]$ indeterminacy membership and $\left[F^-_{F(a_i)}(u_j),F^+_{F(a_i)}(u_j)\right]$ falsity-membership functions of each element $u_i\in U$, respectively.

3. Decision-Making with NVSES-TOPSIS

Let $Q=\{q_1,q_2,q_3,...,q_m\},m\ge 2$ be a discrete set of m feasible alternative, $A=\{a_1,a_2,a_3,...,a_n\},n\ge 2$ be a set of parameters under consideration and $w=\{w_1,w_2,...,w_n\}$ be the unknown weight vector of the criteria, where $0\le w_j\le 1$ and $\sum_{j=1}^{n}w_j=1$. Let $P=\{p_1,p_2,...,p_t\}$ be a set of experts, where the weights of the experts are considered equal and let $R=\{agree=1, disagree=0\}$ be a set of opinions. The rating of performance values of alternatives $Q_i=(i=1,2,...,m)$ with respect to the parameters is presented by the experts, and they can be expressed in terms of neutrosophic vague soft expert set (NVSES). Therefore, steps for solving neutrosophic vague soft expert set MADM problem based on TOPSIS method is presented as follows:

Step 1: Formulation of the decision matrix

To construct the group of decision, we let the rating of alternatives $Qi(i=1,2,3,..,m)$ with respect to parameter by the experts $P(u,v)$ that can be presented in matrix form as follows:

$$A_{NVSES}=\langle aij\rangle_{m\times n}=\begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix} \tag{3.1}$$

where

$$a_{ij}=T_{ij}^{-},T_{ij}^{+},I_{ij}^{-},I_{ij}^{+},F_{ij}^{-},F_{ij}^{+},$$

$$0\le T_{ij}^{-},I_{ij}^{-},F_{ij}^{-}\le 2$$

and

$$0\le T_{ij}^{+},I_{ij}^{+},F_{ij}^{+}\le 2$$

$(i=1,2,\ldots,m;\ j=1,2,\ldots,n)$ denotes an NV value.

Step 2: Calculate the weights of the criteria

In the decision-making process, it is assumed that the relative importance (weights) of the attributes are not identical and are initially unknown. To address this, the maximizing deviation method proposed by Yang [38] is employed to calculate these weights. The steps to obtains weight are as follows:

1) The total deviation value for each attribute A_j is determined by summing the differences between all pairs of alternatives.
2) A maximizing deviation model is formulated to find weights w_j that maximize the total deviation across all attributes, subject to the constraint that the sum of the squared weights equals to 1.
3) The raw weights are obtained by solving the optimization model, which is result of a ratio specific attribute deviation to the square root of total squared deviations.
4) Finally, the weights are normalized so that their sum equals 1, ensuring a standard distribution of importance.

The normalized weight of the attribute is presented as follows:

$$w_j^{*}=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\left|a_{ij}-a_{kj}\right|}{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\left|a_{ij}-a_{kj}\right|\right)} \tag{3.2}$$

Step 3: Construct weighted decision matrix

By using the weight of criterion $\left(w^*\right)$ in step 2 and the neutrosophic vague decision matrix A_{NVSES} in Step 1. The aggregated weighted neutrosophic vague decision matrix A_{NVSES}^{w} is express as follows:

$$A^w = A_{NVSES} \otimes w_j^*$$
$$A_{ij}^w = \left\langle a_{ij} \right\rangle_{m\times n} \otimes w_j^* .$$

Where

$$A_{NVSES}^{w} = \left\langle a_{ij} \right\rangle_{m\times n}^{wj} = \begin{bmatrix} a_{11}^{w_1} & a_{12}^{w_2} & \cdots & a_{1n}^{w_n} \\ a_{21}^{w_1} & a_{22}^{w_2} & \cdots & a_{2n}^{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}^{w_1} & a_{m2}^{w_2} & \cdots & a_{mn}^{w_n} \end{bmatrix} \tag{3.3}$$

Where

$$T_{ij}^{-}, T_{ij}^{+}, I_{ij}^{-}, I_{ij}^{+}, F_{ij}^{-}, F_{ij}^{+} \in [0,1]$$

$0 \le T_{ij}^{-}, I_{ij}^{-}, F_{ij}^{-} \le 2,\ for\ i = 1,2,...,m\ and\ j = 1,2,...,n,$

$0 \le T_{ij}^{+}, I_{ij}^{+}, F_{ij}^{+} \le 2,\ for\ i = 1,2,...,m\ and\ j = 1,2,...,n.$

Step 4: Determine the NVSE-RPIS and NVSE-RNIS

In the TOPSIS method, the evaluation criteria can be categorized into two types, namely benefit-type attributes a_1 and cost-type attributes a_2. Let $R_{NVSE-RPIS}^{(w+)}$ and $R_{NVSE-RPIS}^{(w-)}$ denote the neutrosophic vague soft expert set relative positive ideal solution (NVSE-RPIS) and neutrosophic vague soft expert set relative negative ideal solution (NVSE-RNIS), respectively. Then, $R_{NVSE-RPIS}^{(w+)}$ and $R_{NVSE-RPIS}^{(w-)}$ can be defined as follows:

$$R_{NVSE-RPIS}^{(w+)} = \left\langle \begin{array}{c} \left\langle \left(T_1^{-}\right)^{w_1^{+}}, \left(T_1^{+}\right)^{w_1^{+}}, \left(I_1^{-}\right)^{w_1^{+}}, \left(I_1^{+}\right)^{w_1^{+}}, \left(F_1^{-}\right)^{w_1^{+}}, \left(F_1^{+}\right)^{w_1^{+}} \right\rangle \\ ,...,\left\langle \left(T_n^{-}\right)^{w_n^{+}}, \left(T_n^{+}\right)^{w_n^{+}}, \left(I_n^{-}\right)^{w_n^{+}}, \left(I_n^{+}\right)^{w_n^{+}}, \left(F_n^{-}\right)^{w_n^{+}}, \left(F_n^{+}\right)^{w_n^{+}} \right\rangle \end{array} \right\rangle \tag{3.4}$$

where

$$R^{w+} = \left\langle \left\{ \max\left(T_{ij}^{-}, T_{ij}^{+}\right) \mid j \in a_1 \right\}; \left\{ \min\left(I_{ij}^{-}, I_{ij}^{+}\right) \mid j \in a_2 \right\}; \left\{ \min\left(F_{ij}^{-}, F_{ij}^{+}\right) \mid j \in a_3 \right\} \right\rangle \tag{3.5}$$

$$R_{NVSE-RPIS}^{(w-)} = \left\langle \begin{array}{c} \left\langle \left(T_1^-\right)^{w_1^-}, \left(T_1^+\right)^{w_1^-}, \left(I_1^-\right)^{w_1^-}, \left(I_1^+\right)^{w_1^-}, \left(F_1^-\right)^{w_1^-}, \left(F_1^+\right)^{w_1^-} \right\rangle \\ ,\ldots,\left\langle \left(T_n^-\right)^{w_n^-}, \left(T_n^+\right)^{w_n^-}, \left(I_n^-\right)^{w_n^-}, \left(I_n^+\right)^{w_n^-}, \left(F_n^-\right)^{w_n^-}, \left(F_n^+\right)^{w_n^-} \right\rangle \end{array} \right\rangle \tag{3.6}$$

where

$$R^{w-} = \left\langle \left\{ \min\left(T_{ij}^-, T_{ij}^+\right) \mid j \in a_1 \right\}; \left\{ \max\left(I_{ij}^-, I_{ij}^+\right) \mid j \in a_2 \right\}; \left\{ \max\left(F_{ij}^-, F_{ij}^+\right) \mid j \in a_3 \right\} \right\rangle \tag{3.7}$$

Step 5: Calculate the traditional TOPSIS by using Euclidean distance measure of each alternative

The distance measure of each alternative is calculated based on the relative positive ideal solution (NVSE-RPIS) and the relative negative ideal solution (NVSE-RNIS) by using the normalized Euclidean distance given by:

$$q_i^+ = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left[\left|T_{ij}(x_j) - T_{ij}^+(x_j)\right|^2 + \left|I_{ij}(x_j) - I_{ij}^+(x_j)\right|^2 + \left|F_{ij}(x_j) - F_{ij}^+(x_j)\right|^2\right]},\ i = 1, 2, \ldots, m \tag{3.8}$$

$$q_i^- = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left[\left|T_{ij}(x_j) - T_{ij}^-(x_j)\right|^2 + \left|I_{ij}(x_j) - I_{ij}^-(x_j)\right|^2 + \left|F_{ij}(x_j) - F_{ij}^-(x_j)\right|^2\right]},\ \ i = 1, 2, \ldots, m \tag{3.9}$$

Step 6: Calculate the relative closeness coefficient

Compute the relative closeness coefficient of each alternative $q_i = (i = 1, 2, \ldots, m)$ is defined as follows:

$$\tau_i = \frac{q_i^-}{q_i^+ + q_i^-}$$

$$\text{where } 0 \le \tau_i \le 1,\ i = 1, 2, \ldots, m \tag{3.10}$$

Step 7: Rank the alternatives

Rank the alternatives based on values of τ_i. The highest τ_i indicate best choices.

4. Application with NVSES-TOPSIS

Assume that company Sky Forge is looking to hire a person to fill the job vacancy for a position in their company [39]. Out of all the people who applied for the position, three candidates were shortlisted and these three candidates form the universe of elements, $Q = \{q_1, q_2, q_3\}$. The hiring committee consists of the set P, where the elements are the hiring manager (u) and the head of department (v). While the set $R = \{1 = \text{agree}, \quad 0 = \text{disagree}\}$ represents the set of opinions of the hiring committee members. The hiring committee considers a set of parameters that represent the characteristics or qualities that the candidates are evaluated on, namely "training required" e_1 and "educational qualifications in the field" e_2, respectively. After interviewing the three candidates and reviewing their qualifications and other supporting documents, the hiring committee constructs the following neutrosophic vague soft expert set (NVSES) designed to select the best candidate.

The committee provides the evaluation of the candidates based on NVSESs as follows:

$$\left\{\begin{array}{l}
(e_1,u,1),\left\{\begin{array}{l} q_1:([0.3,0.7],[0.5,0.9],[0.3,0.7]) \\ q_2:([0.6,0.9],[0.2,0.5],[0.1,0.4]) \\ q_3:([0.5,0.7],[0.2,0.2],[0.1,0.4]) \end{array}\right\} \\
(e_2,u,1),\left\{\begin{array}{l} q_1:([0.5,0.7],[0.1,0.2],[0.3,0.5]) \\ q_2:([0.2,0.3],[0.8,0.9],[0.7,0.8]) \\ q_3:([0.4,0.5],[0.9,0.5],[0.5,0.6]) \end{array}\right\} \\
(e_1,v,1),\left\{\begin{array}{l} q_1:([0.4,0.8],[0.5,0.8],[0.2,0.6]) \\ q_2:([0.5,0.6],[0.4,0.5],[0.4,0.5]) \\ q_3:([0.1,0.7],[0.4,0.4],[0.3,0.6]) \end{array}\right\} \\
(e_2,v,1),\left\{\begin{array}{l} q_1:([0.7,0.8],[0.4,0.6],[0.2,0.3]) \\ q_2:([0.2,0.3],[0.4,0.7],[0.7,0.8]) \\ q_3:([0.1,0.7],[0.4,0.4],[0.3,0.6]) \end{array}\right\}
\end{array}\right\}$$

$$\left\{\begin{array}{l}
(e_1,u,0),\left\{\begin{array}{l} q_1:([0.3,0.7],[0.1,0.5],[0.3,0.7]) \\ q_2:([0.1,0.4],[0.5,0.8],[0.6,0.9]) \\ q_3:([0.5,0.5],[0.7,0.8],[0.5,0.7]) \end{array}\right\} \\
(e_2,u,0),\left\{\begin{array}{l} q_1:([0.3,0.5],[0.8,0.9],[0.5,0.7]) \\ q_2:([0.7,0.8],[0.1,0.6],[0.2,0.3]) \\ q_3:([0.1,0.3],[0.3,0.5],[0.7,0.9]) \end{array}\right\} \\
(e_1,v,0),\left\{\begin{array}{l} q_1:([0.2,0.6],[0.2,0.5],[0.4,0.8]) \\ q_2:([0.4,0.5],[0.5,0.5],[0.5,0.6]) \\ q_3:([0.1,0.6],[0.4,0.5],[0.4,0.9]) \end{array}\right\} \\
(e_2,v,0),\left\{\begin{array}{l} q_1:([0.2,0.3],[0.4,0.6],[0.7,0.8]) \\ q_2:([0.7,0.8],[0.3,0.6],[0.2,0.1]) \\ q_3:([0.3,0.6],[0.5,0.6],[0.4,0.7]) \end{array}\right\}
\end{array}\right\}$$

Step 1: Formulation of the decision matrix

The NVSESs are organized in tabular form, as shown Table 1 and Table 2 as follows:

Table 1. A_{NVSES} for Agree = 1

AGREE												
Candidates (Q_i)	e_1u						e_1v					
	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.3	0.7	0.5	0.9	0.3	0.7	0.4	0.8	0.5	0.8	0.2	0.6
q_2	0.6	0.9	0.2	0.5	0.1	0.4	0.5	0.6	0.5	0.5	0.4	0.5
q_3	0.5	0.7	0.2	0.2	0.3	0.5	0.4	0.9	0.5	0.6	0.1	0.6

AGREE												
Candidates (Q_i)	e_2u						e_2v					
	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.5	0.7	0.1	0.2	0.3	0.5	0.7	0.8	0.4	0.6	0.2	0.3
q_2	0.2	0.3	0.8	0.9	0.7	0.8	0.2	0.3	0.4	0.7	0.7	0.8
q_3	0.4	0.5	0.7	0.9	0.5	0.6	0.4	0.7	0.4	0.4	0.3	0.6

Table 2. A_{NVSES} for Disagree = 0

DISAGREE												
Candidates (Q_i)	e_1u						e_1v					
	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.3	0.7	0.1	0.5	0.3	0.7	0.2	0.6	0.2	0.5	0.4	0.8
q_2	0.1	0.4	0.5	0.8	0.6	0.9	0.4	0.5	0.5	0.5	0.5	0.6
q_3	0.3	0.5	0.8	0.8	0.5	0.7	0.1	0.6	0.4	0.5	0.4	0.9

DISAGREE												
Candidates (Q_i)	e_2u						e_2v					
	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.3	0.5	0.8	0.9	0.5	0.7	0.2	0.3	0.4	0.6	0.7	0.8
q_2	0.7	0.8	0.1	0.2	0.2	0.3	0.7	0.8	0.3	0.6	0.2	0.3
q_3	0.5	0.6	0.1	0.3	0.4	0.5	0.3	0.6	0.6	0.6	0.4	0.7

Step 2: Calculate the weights of the criteria

The weights of the attribute are obtained using the Equation 3.2 as follows:

$$\langle q_1, q_2 \rangle = \left| \left(\left| T_{12}^{-} - T_{21}^{-} \right| \right) + \left(\left| T_{12}^{+} - T_{22}^{+} \right| \right) + \left(\left| I_{13}^{-} - I_{32}^{-} \right| \right) \right| + \left(\left| I_{14}^{+} - I_{42}^{+} \right| \right) + \left(\left| F_{15}^{+} - F_{52}^{+} \right| \right) + \left(\left| F_{16}^{+} - F_{62}^{+} \right| \right)$$

$$= \left\| 0.3 - 0.6 \right| + \left| 0.7 - 0.9 \right| + \left| 0.5 - 0.2 \right| + \left| 0.9 - 0.5 \right| + \left| 0.3 - 0.1 \right| + \left| 0.7 - 0.4 \right\|$$

$$= 1.7$$

$$\langle q_1, q_2 \rangle = \sum_{i=1}^{m} \sum_{j=1}^{n} \left| a_{ij} + a_{kj} \right| = 1.7 + 1.4 + 0.9 = 4$$

Therefore, the total summation of all pairwise comparison values between alternatives is:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \langle q_i, q_j \rangle = 32.8$$

Applying Equation 3.2, yields:

$$w_1^{*} = \frac{4}{32.8} = 0.12$$

Following a similar calculation, the overall weights of attributes are determined as follows:

$$w_1^{*} = 0.12, w_2^{*} = 0.07, w_3^{*} = 0.17, w_4^{*} = 0.14, w_5^{*} = 0.12, w_6^{*} = 0.07, w_7^{*} = 0.17, w_8^{*} = 0.14$$

Step 3: Construct weighted decision matrix

In this section, the weighted decision matrix is calculated by multiplying the weights as in Equation 3.3, w^{*} with the decision matrix, A_{NVSES}^{w}. The weighted decision matrix is shown in Tables 3 and 4.

Table 3. A_{NVSES}^{*} for Agree = 1

AGREE												
Candidates	e_1u						e_1v					
(Q_i)	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.043	0.137	0.919	0.987	0.863	0.957	0.034	0.102	0.919	0.985	0.898	0.966
q_2	0.106	0.245	0.822	0.919	0.755	0.894	0.045	0.060	0.919	0.955	0.940	0.955
q_3	0.081	0.137	0.822	0.822	0.863	0.919	0.034	0.143	0.919	0.966	0.857	0.966

AGREE												
Candidates	e_2u						e_2v					
(Q_i)	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.112	0.186	0.675	0.760	0.814	0.888	0.155	0.202	0.879	0.931	0.798	0.845
q_2	0.037	0.059	0.963	0.982	0.941	0.963	0.031	0.049	0.879	0.951	0.951	0.969
q_3	0.084	0.112	0.941	0.982	0.888	0.916	0.069	0.155	0.879	0.879	0.845	0.931

Table 4. A^*_{NVSES} for disagree = 0

DISAGREE												
Candidates	e_1u						e_1v					
(Q_i)	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.043	0.137	0.755	0.919	0.863	0.957	0.015	0.060	0.898	0.955	0.940	0.985
q_2	0.013	0.060	0.919	0.973	0.940	0.987	0.034	0.045	0.955	0.955	0.966	0.966
q_3	0.043	0.081	0.973	0.973	0.919	0.957	0.007	0.060	0.940	0.955	0.940	0.993

DISAGREE												
Candidates	e_2u						e_2v					
(Q_i)	T^-	T^+	I^-	I^+	F^-	F^+	T^-	T^+	I^-	I^+	F^-	F^+
q_1	0.059	0.112	0.963	0.982	0.888	0.941	0.031	0.049	0.879	0.931	0.931	0.879
q_2	0.186	0.240	0.675	0.760	0.760	0.814	0.155	0.202	0.845	0.931	0.798	0.845
q_3	0.112	0.145	0.675	0.814	0.855	0.888	0.049	0.121	0.931	0.931	0.879	0.951

Step 4: Determine the NVSE-RPIS and NVSE-RNIS

In this step, *NVSE-RPIS* and *NVSE-RNIS* are determined to represent the most favorable and least favorable solutions, respectively, for the decision-making problem. Tables 3 and 4 are used to obtain *NVSE-RPIS* and *NVSE-RNIS* as follows:

The *NVSE-RPIS* for e_1u calculated using Equation (4) and (5) as follows:

$$=\begin{pmatrix} \max(0.043, 0.106, 0.081), \max(0.137, 0.245, 0.137), \min(0.919, 0.822, 0.822), \\ \min(0.987, 0.919, 0.822), \min(0.863, 0.755, 0.863), \min(0.957, 0.894, 0.919) \end{pmatrix}$$
$$=(0.106, 0.245, 0.822, 0.822, 0.755, 0.894)$$

Thus, the overall *NVSE-RPIS* is:

$$NVSE-RPIS=$$
$$\langle(0.106, 0.245, 0.822, 0.822, 0.755, 0.894), (0.045, 0.143, 0.919, 0.955, 0.857, 0.955),$$
$$(0.112, 0.186, 0.675, 0.760, 0.814, 0.888), (0.155, 0.202, 0.879, 0.879, 0.798, 0.845),$$
$$(0.043, 0.137, 0.755, 0.919, 0.863, 0.957), (0.034, 0.060, 0.898, 0.955, 0.940, 0.966),$$
$$(0.186, 0.240, 0.675, 0.760, 0.760, 0.814), (0.155, 0.202, 0.845, 0.931, 0.798, 0.845)\rangle$$

The NVSE-RNIS for e_1u is calculated as in Equation (3.6) an (3.7) as follows:

$$=\begin{pmatrix}\min(0.043,0.106,0.081),\min(0.137,0.245,0.137),\max(0.919,0.822,0.822),\\ \max(0.987,0.919,0.822),\max(0.863,0.755,0.863),\max(0.957,0.894,0.919)\end{pmatrix}$$
$$=(0.043,0.137,0.919,0.987,0.863,0.957)$$

Thus, the overall *NVSE-RNIS* is:

$$\begin{aligned}&NVSE-RNIS=\\&\langle(0.043,0.137,0.919,0.987,0.863,0.957),(0.034,0.060,0.919,0.985,0.940,0.966),\\&(0.037,0.059,0.963,0.982,0.941,0.963),(0.031,0.049,0.879,0.951,0.951,0.969),\\&(0.013,0.060,0.973,0.973,0.940,0.987),(0.007,0.045,0.955,0.955,0.955,0.993),\\&(0.059,0.112,0.963,0.982,0.888,0.914),(0.031,0.049,0.931,0.931,0.951,0.969)\rangle\end{aligned}$$

Hence, the same calculation is applied for the remaining alternatives.
Using Equation 8, the normalized Euclidean distance for each alternative from the NVSE-RPIS is calculated as follows:

$$q_1^+=\begin{pmatrix}\frac{1}{3}\left(|0.043-0.106|^2+|0.137-0.245|^2+|0.919-0.822|^2+|0.987-0.822|^2\right)+\ldots+\\ |0.879-0.845|^2+|0.931-0.931|^2+|0.951-0.798|^2+|0.969-0.845|^2\end{pmatrix}^{\frac{1}{2}}$$
$$=0.3429$$

Following the same calculation, the values for q_2^+ and q_3^+ are:

$q_2^+=0.3310$, $q_3^+=0.3228$

$$q_1^-=\begin{pmatrix}\frac{1}{3}\left(|0.043-0.043|^2+|0.137-0.137|^2+|0.919-0.919|^2+|0.987-0.987|^2\right)+\ldots+\\ |0.879-0.931|^2+|0.931-0.931|^2+|0.951-0.951|^2+|0.969-0.969|^2\end{pmatrix}^{\frac{1}{2}}$$
$$=0.3305$$

Similarly, Equation 3.9 is used to determine the normalized Euclidean distance of each alternative from the NVSE-RNIS.

Following the same calculation, the values for q_2^- and q_3^- are:

$q_2^+=0.3337$, $q_3^+=0.2757$

Step 6: Calculation of the relative closeness coefficient

The relative closeness coefficient $\tau_i\,(i=1,2,3)$ is calculated using Equation 3.10 as follows:

For $i=1$;

$$\tau_1 = \frac{0.3305}{0.3429+0.3305} = 0.4908$$

The relative closeness coefficients for the remaining alternatives $i=2,3$ are calculated as follows:

$$\tau_2 = 0.5020\,,\ \tau_2 = 0.4660$$

Step 7: Rank the alternatives

The alternatives are rank accordingly to their closeness coefficients as follows:

The ranking is:

$$\tau_2 \succ \tau_1 \succ \tau_3$$

The closeness coefficient indicates the relative closeness of each candidate to the ideal solution. Therefore, the ranking for the candidate is

$$q_2 \succ q_1 \succ q_3$$

Based on the final ranking, q_2 is identified as the best candidate to fill the vacancy.

5. Comparative Analysis

This study conducts a comparative analysis among different distance measures, namely: normalized Euclidean, normalized Manhattan and normalized Minkowski distances. Table 5 shows the final ranking obtained from each distances measure.

Table 5: Closeness coefficient values for each alternative.

Distance	q_1	q_2	q_3	Ranking
Normalized Euclidean [40]	0.4908	0.5020	0.4606	$q_2 \succ q_1 \succ q_3$
Normalized Manhattan [40]	0.4751	0.5015	0.4421	$q_2 \succ q_1 \succ q_3$
Normalized Minkowski [40]	0.5033	0.4998	0.4882	$q_1 \succ q_2 \succ q_3$

Table 5 presents the closeness coefficients values of each alternative, calculated using three different distance measures applied to the same example. The findings indicate that q_2 is the best candidate when using the Euclidean and Manhattan distances, however q_1 emerges as the best candidate under Minkowski distance. Notably, q_3 is consistently identified as the least preferred candidate across all three distances.

6. Conclusion

This study proposes integration of NVSES and TOPSIS method (NVSES-TOPSIS) to solve decision-making problems. The SES uses 'agree' and 'disagree' options to capture how expert think, rather than just using 'true' and 'false'. Besides, the soft expert set assists experts in the decision-making process without increasing computational complexity. Experts evaluate the alternatives using NVSES, and weight of attributes are calculated through maximization deviation method. Finally, TOPSIS is used to perform final ranking. The NVSES is applied to the selection of candidate to fill a job vacancy. The finding shows that q_2 is the best candidate, while q_3 is the least preferred. Comparison analysis across various distance measure shows a consistent ranking order, particularly when the parameter is set to while $p=\frac{1}{2}$. For future studies, the proposed method can be integrated with other set theory particularly different types of soft expert sets and implemented in various case studies.

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References

[1] L.A. Zadeh, Fuzzy Sets, Inf. Control 8 (1965), 338-353. https://doi.org/10.1016/s0019-9958(65)90241-x.
[2] K.T. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets Syst. 20 (1986), 87–96.
[3] R.R. Yager, Pythagorean Fuzzy Subsets, in: 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), IEEE, pp. 57-61, 2013. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375.
[4] D. Ramot, R. Milo, M. Friedman, A. Kandel, Complex Fuzzy Sets, IEEE Trans. Fuzzy Syst. 10 (2002), 171-186. https://doi.org/10.1109/91.995119.
[5] K.M. Lee, Bipolar Valued Fuzzy Sets and Their Applications, in: Proceedings of International Conference on Intelligent Technologies, pp. 307-312, 2000.
[6] F. Smarandache, A Unifying Field of Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, American Research Press, Rehoboth, 1998.
[7] H.B. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, Single Valued Neutrosophic Sets, in: F. Smarandache, (ed) Multispace & Multistructure. Neutrosophic Transdisciplinarity, Vol IV, North-European Scientific Publishers, Hanko, Finland, 410-413, (2010).
[8] S. Yaacob, H. Hashim, N. Awang, N. Sulaiman, A. Al-Quran, et al., Bipolar Neutrosophic Dombi-Based Heronian Mean Operators and Their Application in Multi-Criteria Decision-Making Problems, Int. J. Comput. Intell. Syst. 17 (2024), 181. https://doi.org/10.1007/s44196-024-00544-2.

[9] S. Özlü, Bipolar-Valued Complex Hesitant Fuzzy Dombi Aggregating Operators Based on Multi-Criteria Decision-Making Problems, Int. J. Fuzzy Syst. 27 (2024), 162-189. https://doi.org/10.1007/s40815-024-01770-8.

[10] L. Shi, Y. Yuan, Hybrid Weighted Arithmetic and Geometric Aggregation Operator of Neutrosophic Cubic Sets for MADM, Symmetry 11 (2019), 278. https://doi.org/10.3390/sym11020278.

[11] A.U. Rehman, M. Gulistan, M. Ali, M.M. Al-Shamiri, S. Abdulla, Development of Neutrosophic Cubic Hesitant Fuzzy Exponential Aggregation Operators with Application in Environmental Protection Problems, Sci. Rep. 13 (2023), 5262. https://doi.org/10.1038/s41598-022-22399-3.

[12] V. Ulucay, A. Kilic, M. Sahin, H. Deniz, A New Hybrid Distance-Based Similarity Measure for Refined Neutrosophic Sets and Its Application in Medical Diagnosis, MATEMATIKA 35 (2019), 83-94. https://doi.org/10.11113/matematika.v35.n1.1063.

[13] D. Sarkar, S. Ghosh, Hausdorff Similarity Measure on Neutrosophic Soft Set with Its Applicability in Decision Making, Fuzzy Inf. Eng. 16 (2024), 194-206. https://doi.org/10.26599/fie.2024.9270041.

[14] T.J. Puzhakkara, S. Jose, New Distance Measure between Fermatean Neutrosophic Sets and Its Application in Crop Farming, South East Asian J. Math. Math. Sci. 20 (2024), 233-246. https://doi.org/10.56827/SEAJMMS.2024.2001.18.

[15] R.T. Ngan, F. Smarandache, S. Broumi, H-Max Distance Measure of Bipolar Neutrosophic Sets and an Application to Medical Diagnosis, Neutrosoph. Sets Syst. 45 (2021), 444-458.

[16] Y. Tian, K. Zhang, Bipolar Neutrosophic WINGS for Green Technology Innovation, Sci. Rep. 13 (2023), 19159. https://doi.org/10.1038/s41598-023-46699-4.

[17] A. Nafei, C.Y. Huang, A. Javadpour, H. Garg, S. Azizi, et al., Neutrosophic Fuzzy Decision-Making Using TOPSIS and Autocratic Methodology for Machine Selection in an Industrial Factory, Int. J. Fuzzy Syst. 26 (2024), 860-886. https://doi.org/10.1007/s40815-023-01640-9.

[18] J. Rachel, E. Devarasan, A. Razzaque, S. Selvakumar, Enhancing Biometric System Selection: A Hybrid AHP-Neutrosophic Fuzzy TOPSIS Approach, BMC Res. Notes 17 (2024), 263. https://doi.org/10.1186/s13104-024-06903-8.

[19] A. Karasan, E. Ilbahar, S. Cebi, C. Kahraman, Customer-Oriented Product Design Using an Integrated Neutrosophic AHP & DEMATEL & QFD Methodology, Appl. Soft Comput. 118 (2022), 108445. https://doi.org/10.1016/j.asoc.2022.108445.

[20] N.A. Awang, L. Abdullah, H. Hashim, M.S. Yang, Effective Decision Making with Hesitant Bipolar Neutrosophic DEMATEL Approach for Comprehensive Coastal Erosion Evaluation, Soft Comput. (2025). https://doi.org/10.1007/s00500-025-10684-y.

[21] D. Molodtsov, Soft Set Theory—First Results, Comput. Math. Appl. 37 (1999), 19-31. https://doi.org/10.1016/s0898-1221(99)00056-5.

[22] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy Soft Sets, Comput. Math. Appl. 45 (2003), 555-562.

[23] P.K. Maji, A.R. Roy, R. Biswas, Intuitionistic Fuzzy Soft Sets, Comput. Math. Appl. 45 (2003), 555–562.

[24] S. Alkhazaleh, A.R. Salleh, N. Hassan, Possibility Fuzzy Soft Set, Adv. Decis. Sci. 2011 (2011), 479756. https://doi.org/10.1155/2011/479756.

[25] S. Alkhazaleh, A.R. Salleh, Soft Expert Sets, Adv. Decis. Sci. 2011 (2011), 757868. https://doi.org/10.1155/2011/757868.

[26] M. Bashir, A.R. Salleh, Fuzzy Parameterized Soft Expert Set, Abstr. Appl. Anal. 2012 (2012), 258361. https://doi.org/10.1155/2012/258361.

[27] S. Broumi, F. Smarandache, Intuitionistic Fuzzy Soft Expert Sets and its Application in Decision Making, J. New Theory 1 (2015), 89–105.

[28] M. Sahin, S. Alkhazaleh, V. Ulucay, Neutrosophic Soft Expert Sets, Appl. Math. 06 (2015), 116-127. https://doi.org/10.4236/am.2015.61012.

[29] N. Hassan, K. Alhazaymeh, Vague Soft Expert Set Theory, AIP Conf. Proc. 1522 (2013), 953-958. https://doi.org/10.1063/1.4801233.

[30] A. Al-Quran, N. Hassan, Neutrosophic Vague Soft Expert Set Theory, J. Intell. Fuzzy Syst. 30 (2016), 3691-3702. https://doi.org/10.3233/ifs-162118.

[31] S. Pramanik, P.P. Dey, B.C. Giri, TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-Attribute Decision Making Problems, Neutrosophic Sets Syst. 10 (2015), 88–95.

[32] M. Saeed, S. Mahmood, H. Taufeeq, Hybridization of Soft Expert Set with AHP in Decision Making, Int. J. Comput. Appl. 179 (2017), 24-29. https://doi.org/10.5120/ijca2017915920.

[33] J. Su, Y. Sun, An Improved TOPSIS Model Based on Cumulative Prospect Theory: Application to ESG Performance Evaluation of State-Owned Mining Enterprises, Sustainability 15 (2023), 10046. https://doi.org/10.3390/su151310046.

[34] D. Karabašević, D. Stanujkić, E.K. Zavadskas, P. Stanimirović, G. Popović, et al., A Novel Extension of the TOPSIS Method Adapted for the Use of Single-Valued Neutrosophic Sets and Hamming Distance for E-Commerce Development Strategies Selection, Symmetry 12 (2020), 1263. https://doi.org/10.3390/sym12081263.

[35] R. Bardhan, D. Chauhan, M. Sahni, A Neutrosophic TOPSIS Framework for Multi-Criteria Selection of Functionally Graded Materials, Procedia Struct. Integr. 72 (2025), 507-519. https://doi.org/10.1016/j.prostr.2025.08.133.

[36] M. Sun, X. Ding, Y. Geng, J. Zhao, TOPSIS Method Based on Entropy Weight for Interval-Valued Neutrosophic Numbers and Its Application in Multi-Attribute Group Decision-Making, in: 2024 IEEE International Conference on Systems, Man, and Cybernetics (SMC), IEEE, 2024, pp. 149-154. https://doi.org/10.1109/smc54092.2024.11169723.

[37] S. Alkhazaleh, Neutrosophic Vague Set Theory, Crit. Rev. 10 (2015), 29–39.

[38] Y.M. Yang, Using the Method of Maximizing Deviations to make Decision for Multi-Indices, Syst. Eng. Electron. 7 (1998), 24-26.

[39] A. Al-Quran, Fuzzy Parameterized Neutrosophic Soft Expert Set and its Applications, Ph.D. Thesis, Universiti Kebangsaan Malaysia, 2021.

[40] N. Gayathri, M. Helen, P. Mounika, On Neutrosophic Vague Measure Using Python, AIP Conf. Proc. 2274 (2020), 030036. https://doi.org/10.1063/5.0017191.