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A Displacement Functional Approach to Fixed Points of Strictly Contractive and Kannan Mappings

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Abstract. A minimizing sequence approach to the Banach fixed point theorem was recently introduced, based on minimizing the displacement functional $\phi(x) = d(x, Tx)$. In this paper, we clarify how this method relates to approximating fixed point sequences studied in the literature. We first provide an alternative proof of Edelstein's fixed point theorem for strictly contractive mappings on compact metric spaces by minimizing $\phi(x)$. The argument relies entirely on the minimization principle without the use of Picard iteration. We then obtain an alternative proof of Kannan's fixed point theorem using the same minimizing sequence technique, showing that the method applies in settings where continuity is not assumed. Finally, we establish a new fixed point theorem for strictly contractive mappings on proper metric spaces under a coercivity condition on the displacement functional. This result identifies attainment of the minimum of $\mathcal{A} = \{\phi(x) : x \in X\}$ as the fundamental mechanism of the method, rather than global compactness of the underlying space.

1. Introduction

The Banach fixed point theorem [1] is a cornerstone of analysis, guaranteeing the existence and uniqueness of fixed points for contraction mappings in complete metric spaces. Its classical proof uses Picard iteration, from an arbitrary starting point x_0, one defines $x_{n+1} = Tx_n$ and shows that $\{x_n\}$ converges to a unique fixed point. Fixed point theory has recently witnessed substantial progress across multiple metric frameworks, particularly in neutrosophic and fuzzy settings. [4] studied H-simulation functions with matrix equations, while [6] contributed to G-metric spaces. [2] introduced gamma distance mappings for fractional differential equations, and [5] examined Geraghty-type contractions under equivalent distances. [8] obtained results using T-distance spaces in complete b-metric spaces. Within neutrosophic frameworks, [3] studied nonlinear contractions, [7] proved ψ-quasi contraction results, [10] established quasi contraction

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theorems, [11] utilized Geraghty functions for common fixed points, and [12] employed simulation functions.

In a recent paper Bataihah [9] introduced an alternative approach based on *minimizing sequences*. Defining the displacement functional $\phi(x) = d(x, Tx)$ and the set $\mathcal{A} = \{\phi(x) : x \in X\}$, then constructed a sequence $\{x_n\}$ such that $\phi(x_n) \to \inf \mathcal{A}$. For a contraction mapping, it proved that $\inf \mathcal{A} = 0$, so $\{x_n\}$ satisfies $d(x_n, Tx_n) \to 0$. Completeness of X then yields convergence to a fixed point.

The aim of this paper is threefold. First, we demonstrate the application of our minimizing displacement method by providing an alternative proof of Edelstein's fixed point theorem for strictly contractive mappings on compact metric spaces, cf. [15]. Our proof is simpler and more concise than previous proofs.

Furthermore, we also prove that our minimizing displacement method is applicable to Kannan contractions, cf. [21]. This is relevant since Kannan contractions are not necessarily continuous, making our result interesting and relevant to this area of research.

Third, we prove a new fixed point theorem for strictly contractive mappings on proper metric spaces with coercive displacement. This result extends Edelstein's theorem to a non-compact setting and demonstrates that the key requirement is that the displacement functional attains its minimum.

Sequences with $d(x_n, Tx_n) \to 0$ are known in the literature as *approximating fixed point sequences*. Górnicki [14] studied them systematically and, citing Brouwer [13], noted their significance in constructive approaches to fixed point theory: *"only approximating fixed point sequences have a meaning for the intuitionist."* The minimizing sequence construction in [9] thus provides a natural method for generating such sequences.

By presenting this material, we hope to bridge the gap between the classical iterative approach and the approximating sequence perspective, while also correcting the omission of relevant references in our previous work.

2. Minimizing Sequences and Approximating Fixed Point Sequences

We recall two related concepts from the literature.

Definition 2.1 (Approximating Fixed Point Sequence). *Let (X, d) be a metric space, C a nonempty subset of X, and $T : C \to C$ a mapping. A sequence $\{x_n\} \subset C$ is called an* approximating fixed point sequence *for T if*

$$\lim_{n\to\infty} d(x_n, Tx_n) = 0.$$

Such sequences were studied systematically by Górnicki [14] in the context of Kannan-type mappings.

Definition 2.2 (Minimizing Sequence). *Let (X, d) be a metric space and $T : X \to X$ a mapping. Define*

$$\mathcal{A} = \{d(x, Tx) : x \in X\} \subseteq [0, \infty), \qquad \alpha = \inf \mathcal{A}.$$

A sequence $\{x_n\} \subset X$ *is called a* minimizing sequence *for* T *if*

$$\lim_{n\to\infty} d(x_n, Tx_n) = \alpha.$$

For a contraction mapping on a complete metric space, Bataihah [9] proved that $\alpha = 0$; consequently, any minimizing sequence is automatically an approximating fixed point sequence. Completeness of X then ensures that this sequence converges to a fixed point.

The relationship between the two concepts is straightforward:

Proposition 2.1. *Let* (X, d) *be a metric space and* $T : X \to X$ *a mapping.*

(1) *If* $\{x_n\}$ *is a minimizing sequence for* T *with* $\alpha = 0$, *then* $\{x_n\}$ *is an approximating fixed point sequence.*
(2) *Conversely, every approximating fixed point sequence* $\{x_n\}$ *(satisfying* $d(x_n, Tx_n) \to 0$*) is a minimizing sequence for* $\alpha = 0$, *since* 0 *is the infimum of* $\mathcal{A}$.

Therefore, when the minimal displacement is zero, minimizing sequences and approximating fixed point sequences coincide. The minimizing sequence method hence offers a variational interpretation for the approximating fixed point theory and ties the theory in [9] with the general literature, as seen in the survey by Górnicki [14].

3. Edelstein's Theorem for Strictly Contractive Mappings

In 1962, Edelstein [15] proved that a strictly contractive self-map on a compact metric space admits a unique fixed point.

Definition 3.1. *Let* (X, d) *be a metric space. A mapping* $T : X \to X$ *is called* strictly contractive *(or shortly contractive) if*

$$d(Tx, Ty) < d(x, y) \quad \text{for all } x, y \in X,\ x \neq y.$$

Clearly, if $T : X \to X$ is contractive, then T is continuous. The study of contractive mappings (satisfying $d(Tx, Ty) < d(x, y)$ for $x \neq y$) was pioneered by Edelstein [15] and Rakotch [19] in the early 1960s. Rakotch [19] proved fixed point theorems for contractive mappings under various compactness conditions and, significantly, introduced the idea of replacing the constant contraction factor with a function $\alpha(x, y) = \alpha(d(x, y))$ that is monotonically decreasing and satisfies $0 \le \alpha(\rho) < 1$ for all $\rho > 0$ [19, pp. 459-460]. This innovation anticipated the more general control functions later studied by Geraghty [20].

Theorem 3.1 (Edelstein)**.** *Let* (X, d) *be a compact metric space and let* $T : X \to X$ *be strictly contractive. Then* T *has a unique fixed point.*

Proof. Define the displacement functional

$$\phi(x) = d(x, Tx), \qquad x \in X.$$

Since the metric d is continuous, and T is continuous (strict contractivity implies continuity), hence ϕ is continuous on X. Since X is compact, ϕ attains its minimum. Thus there exists $u \in X$ such that

$$\phi(u) = \min_{x \in X} \phi(x).$$

Now, suppose $u \neq Tu$. Then $\phi(u) = d(u, Tu) > 0$. Applying strict contractivity to the pair (u, Tu) gives

$$d(Tu, T^2u) < d(u, Tu).$$

But $\phi(Tu) = d(Tu, T^2u)$, so $\phi(Tu) < \phi(u)$, contradicting the minimality of u. Hence $Tu = u$.

Finally, if v is another fixed point with $v \neq u$, then

$$d(u, v) = d(Tu, Tv) < d(u, v),$$

which is impossible. Therefore the fixed point is unique. □

The proof of Edelstein's theorem given above differs significantly from classical arguments in the literature [15, 19]. It uses only the compactness of X and the strict contraction condition, without Picard iteration, subsequence argument, or explicit use of Cauchy sequences. The idea is to minimize the "displacement" functional, which was not used in earlier proofs.

Edelstein's original theorem assumes that there is a point for which there is a convergent subsequence in the orbit. Compactness of X is a sufficient condition that assumes this property for all points in X, and we use this property directly. Rakotch [19, Theorem 1] proved fixed point theorems for contractive mappings by applying Edelstein's theorem, showing that under certain conditions, the orbit of a point remains in a set M which maps into a compact subset of X. This argument, as well as Edelstein's, depends on analyzing the sequence of iterates.

The use of compactness in our proof is essential and non-constructive in a technical sense because it relies on the Extreme Value Theorem; for a more constructive approach, readers are referred to [16].

4. A Minimizing Sequence Proof for Kannan Contractions

We now demonstrate the flexibility of the minimizing sequence method by applying it to Kannan contractions, an important class of mappings that do not satisfy the Banach contraction principle.

Definition 4.1. *A mapping $T : X \to X$ on a metric space (X, d) is called a Kannan contraction if there exists $k \in [0, \frac{1}{2})$ such that for all $x, y \in X$,*

$$d(Tx, Ty) \leq k\Big[d(x, Tx) + d(y, Ty)\Big].$$

Theorem 4.1. *[21][Fixed point for Kannan contractions] Let (X, d) be a complete metric space and $T : X \to X$ a Kannan contraction with constant $k \in [0, \frac{1}{2})$. Then T has a unique fixed point $x^* \in X$.*

Proof. Define $\phi(x) = d(x, Tx)$ and let $\mathcal{A} = \{\phi(x) : x \in X\}$ with $\alpha = \inf \mathcal{A}$. Choose a minimizing sequence $\{x_n\} \subseteq X$ such that $\phi(x_n) \to \alpha$.

First, we show that $\alpha = 0$. Apply the Kannan condition to x_n and Tx_n:

$$d(Tx_n, T^2x_n) \leq k\Big[d(x_n, Tx_n) + d(Tx_n, T^2x_n)\Big] = k\Big[\phi(x_n) + \phi(Tx_n)\Big].$$

Thus,

$$\phi(Tx_n) \leq k\phi(x_n) + k\phi(Tx_n).$$

Rearranging,

$$(1-k)\phi(Tx_n) \leq k\phi(x_n) \quad \Rightarrow \quad \phi(Tx_n) \leq \frac{k}{1-k}\phi(x_n).$$

Since $\phi(Tx_n) \in \mathcal{A}$, we have $\alpha \leq \phi(Tx_n)$ for all n. Taking the limit inferior as $n \to \infty$ yields

$$\alpha \leq \liminf_{n\to\infty} \phi(Tx_n) \leq \frac{k}{1-k} \liminf_{n\to\infty} \phi(x_n) = \frac{k}{1-k}\alpha.$$

Thus

$$\alpha \leq \frac{k}{1-k}\alpha.$$

Because $0 \leq \frac{k}{1-k} < 1$ for $k \in [0, \frac{1}{2})$, this forces $\alpha = 0$. Hence

$$\lim_{n\to\infty} \phi(x_n) = 0. \tag{1}$$

For $m, n \in \mathbb{N}$, apply the Kannan condition to x_n and x_m:

$$d(Tx_n, Tx_m) \leq k\Big[\phi(x_n) + \phi(x_m)\Big].$$

By the triangle inequality,

$$\begin{aligned} d(x_n, x_m) &\leq \phi(x_n) + d(Tx_n, Tx_m) + \phi(x_m) \\ &\leq \phi(x_n) + k[\phi(x_n) + \phi(x_m)] + \phi(x_m). \end{aligned}$$

Thus,

$$d(x_n, x_m) \leq (1+k)[\phi(x_n) + \phi(x_m)].$$

Taking $n, m \to \infty$ and using (1), we obtain $d(x_n, x_m) \to 0$. Therefore, $\{x_n\}$ is Cauchy.

By completeness, $x_n \to x^*$ for some $x^* \in X$. Now,

$$d(x^*, Tx^*) \leq d(x^*, x_n) + \phi(x_n) + d(Tx_n, Tx^*).$$

Apply the Kannan condition to x_n and x^*:

$$d(Tx_n, Tx^*) \leq k\Big[\phi(x_n) + d(x^*, Tx^*)\Big].$$

Substituting,

$$d(x^*, Tx^*) \leq d(x^*, x_n) + \phi(x_n) + k\phi(x_n) + kd(x^*, Tx^*).$$

Rearranging,

$$(1-k)d(x^*, Tx^*) \leq d(x^*, x_n) + (1+k)\phi(x_n).$$

Taking the limit as $n \to \infty$, the right-hand side tends to 0, so $d(x^*, Tx^*) = 0$. Hence $Tx^* = x^*$.

The uniqueness follows easily from the Kannan condition. This completes the proof. □

Remark 4.1. *Kannan contractions were introduced by Kannan [21] and have been extensively studied; see Górnicki [14] for a comprehensive survey. The proof above demonstrates that the minimizing sequence method applies naturally to this class of mappings. Note also that if the displacement functional $\phi(x) = d(x, Tx)$ attains its minimum at some point $x^* \in X$, then x^* must be a fixed point regardless of completeness, since otherwise $\phi(Tx^*) < \phi(x^*)$ would contradict minimality. This observation, however, does not require the full minimizing sequence machinery.*

5. A Proper Metric Space Version

A metric space (X, d) is called *proper* (see [18]) if every closed bounded subset of X is compact. For instance every finite metric space is proper and $\mathbb{R}^n$ with the Euclidean metric is also proper.

Theorem 5.1. *Let (X, d) be a proper metric space and let $T : X \to X$ be strictly contractive, i.e.,*

$$d(Tx, Ty) < d(x, y) \quad \text{for all } x \neq y.$$

Assume that the displacement functional $\phi(x) = d(x, Tx)$ is coercive in the sense that for any $x_0 \in X$,

$$\sup_x d(x, x_0) = \infty \;\Rightarrow \sup_x \phi(x) = \infty.$$

Then T has a unique fixed point in X.

Proof. Let $\{x_n\}$ be a minimizing sequence for ϕ, i.e.,

$$\phi(x_n) \to \inf_{x \in X} \phi(x) = \alpha.$$

Suppose $\{x_n\}$ were unbounded. Then there exists a subsequence (it may also denoted $\{x_n\}$) such that $d(x_n, x_0) \to \infty$ for any fixed $x_0 \in X$. By coercivity, $\phi(x_n) \to \infty$, contradicting $\phi(x_n) \to \alpha < \infty$. Hence $\{x_n\}$ is bounded.

Since $\{x_n\}$ is bounded, the set $A = \{x_n : n \in \mathbb{N}\}$ is bounded. Its closure $\overline{A}$ is closed and bounded, hence compact by properness of X. Therefore $\{x_n\}$ has a convergent subsequence $x_{n_k} \to u \in X$.

Note that ϕ is continuous (hence lower semicontinuous) because d and T are continuous. Thus, we obtain

$$\phi(u) \leq \liminf_{k \to \infty} \phi(x_{n_k}) = \alpha.$$

Thus $\phi(u) = \alpha$; i.e., u minimizes ϕ.

Now, if $u \neq Tu$, then strict contractivity gives

$$d(Tu, T^2 u) < d(u, Tu),$$

so $\phi(Tu) < \phi(u)$, contradicting minimality. Hence $Tu = u$.

Finally, if v is another fixed point with $v \neq u$, then

$$d(u, v) = d(Tu, Tv) < d(u, v),$$

impossible. Therefore the fixed point is unique. □

Example 5.1 (Rakotch's counterexample)**.** *Let $X = \mathbb{R}$ be equipped with the usual metric $d(x,y) = |x-y|$. Define $T : \mathbb{R} \to \mathbb{R}$ by*

$$T(x) = \ln(1+e^x).$$

Note that the derivative is $T'(x) = \frac{e^x}{1+e^x} = \frac{1}{1+e^{-x}}$. For all $x \in \mathbb{R}$, we have $0 < T'(x) < 1$. By the Mean Value Theorem, for any $x \neq y$,

$$|T(x) - T(y)| = |T'(\xi)|\,|x-y| < |x-y|.$$

Thus T satisfies Edelstein's condition [15].

By solving $T(x) = x$ gives $\ln(1+e^x) = x \Rightarrow 1+e^x = e^x \Rightarrow 1 = 0$, impossible. Hence T has no fixed point.

The displacement functional is

$$\phi(x) = |x - T(x)| = |x - \ln(1+e^x)|.$$

As $x \to +\infty$, $\ln(1+e^x) = x + \ln(1+e^{-x}) \sim x$, so $x - \ln(1+e^x) \to 0$ and $\phi(x) \to 0$. As $x \to -\infty$, $\ln(1+e^x) \sim e^x \to 0$, so $\phi(x) \sim |x| \to \infty$. Since $\phi(x) \not\to \infty$ as $|x| \to \infty$, coercivity fails.

This example, due to Rakotch [19, Remark 2], shows that a strictly contractive mapping need not be coercive, and may lack a fixed point even on a proper metric space such as $\mathbb{R}$. Thus the coercivity condition in Theorem 5.1 is essential.

Next, we construct a concrete mapping on a non-compact, proper metric space that is strictly contractive and has a coercive displacement functional. This demonstrates the applicability of the proper metric theorem to guarantee a unique fixed point.

Example 5.2. *Let $X = \mathbb{R}$ be equipped with the usual metric $d(x,y) = |x-y|$. The space $(\mathbb{R}, d)$ is complete, proper (by the Heine–Borel theorem, every closed and bounded set is compact), and non-compact (as $\mathbb{R}$ itself is unbounded).*

Define $T : \mathbb{R} \to \mathbb{R}$ by

$$T(x) = \frac{x}{2} + \frac{\tanh x}{4}.$$

The function T is $C^1(\mathbb{R})$. Its derivative is

$$T'(x) = \frac{1}{2} + \frac{\operatorname{sech}^2 x}{4}.$$

Since $0 < \operatorname{sech}^2 x \leq 1$ for all $x \in \mathbb{R}$, we have the estimate

$$\frac{1}{2} < T'(x) \leq \frac{1}{2} + \frac{1}{4} = \frac{3}{4}.$$

Consequently, $\sup_{x\in\mathbb{R}} |T'(x)| = \frac{3}{4} < 1$. By the Mean Value Theorem, for any $x \neq y$,

$$|T(x) - T(y)| = |T'(\xi)|\,|x-y| \leq \frac{3}{4}|x-y| < |x-y|.$$

Thus, T is a strict contraction, and therefore strictly contractive with Lipschitz constant $L = 3/4$.

The displacement functional $\phi(x) = d(x, T(x)) = |x - T(x)|$ *is*

$$\phi(x) = \left| x - \frac{x}{2} - \frac{\tanh x}{4} \right| = \left| \frac{x}{2} - \frac{\tanh x}{4} \right|.$$

Observe that

$$\lim_{|x|\to\infty} \frac{\phi(x)}{|x|} = \lim_{|x|\to\infty} \left| \frac{1}{2} - \frac{\tanh x}{4x} \right| = \frac{1}{2},$$

since $\tanh x \to 1$ *and* $\frac{1}{x} \to 0$. *Hence* $\phi(x) \to \infty$ *as* $|x| \to \infty$, *so* ϕ *is coercive.*

We have verified all the hypotheses:

- $X = \mathbb{R}$ *is a proper metric space.*
- $T : X \to X$ *is strictly contractive.*
- *The functional* $\phi(x) = d(x, Tx)$ *is coercive.*

Therefore, by the proper metric theorem, T *possesses a unique fixed point in* $\mathbb{R}$.

To find the fixed point, solve $T(x) = x$:

$$\frac{x}{2} + \frac{\tanh x}{4} = x \quad \Longrightarrow \quad \frac{\tanh x}{4} = \frac{x}{2} \quad \Longrightarrow \quad \tanh x = 2x.$$

For $x \neq 0$, *the left-hand side satisfies* $|\tanh x| < 1$, *while the right-hand side* $|2x| \geq 2$ *for* $|x| \geq 1$. *For* $0 < |x| < 1$, *one can check that* $|\tanh x| < |x| < 2|x|$. *The only solution is* $x = 0$. *Indeed,* $T(0) = 0 + \frac{0}{4} = 0$, *confirming that* 0 *is the unique fixed point.*

Remark 5.1. *This example is particularly instructive because it satisfies the conditions of the classical Banach contraction principle (since* T *is a contraction on a complete space) and thus guarantees a unique fixed point by that theorem as well. However, the construction successfully illustrates the core ideas of the proper metric theorem: it is set on a non-compact but proper space, and it is the coercivity of* ϕ *that provides the essential compactness argument (a minimizing sequence for* ϕ *is bounded and thus lies in a compact set) to prove the existence of a fixed point without requiring the global compactness of* X.

Conclusion

The minimizing sequence method introduced in [9] provides an alternative to Picard iteration for proving fixed point theorems. The method focuses on the infimum of the set of distances between points and their images under the mapping, namely $\inf\{d(x, Tx) : x \in X\}$, and constructs a sequence that attains this infimum in the limit. In this paper, we have demonstrated the versatility of this method by applying it to three distinct settings.

First, we provided a brief proof of Edelstein's theorem for strictly contractive mappings in compact metric spaces. This proof is optimal in the sense that it minimizes the displacement functional $\phi(x) = d(x, Tx)$ and highlights the simplicity and elegance of the approach.

Second, we used the approach to prove the existence result for Kannan contractions in complete metric spaces. This is particularly interesting since Kannan mappings may fail to be continuous, but the minimizing sequence works well in this situation anyway. As a corollary, we proved that if the minimizing displacement functional is attained in an incomplete metric space, then the fixed point exists without the completeness of the space, which may have important implications in applications where the completeness of the space is not obvious.

Third, we generalized the method beyond the compact case by proving a fixed point theorem for proper metric spaces with a coercive displacement. This result shows that the main condition for the method to work is that the displacement functional ϕ has a minimum; global compactness of the spaces is not necessary, but sufficient for the method to work.

These results prove the flexibility of the minimizing sequence method, which opens new directions for further research, such as relaxing the contractivity conditions for mappings that are not Lipschitz, such as Chatterjea mappings, Zamfirescu mappings, etc.; extending the method to multivalued mappings; examining if the minimizing sequence method can be used to derive stability rates; and examining the relationship between the minimizing sequence method and variational principles.

The connection between minimizing sequences and the approximating fixed point sequences studied in [14] became clear during the preparation of this manuscript and is made explicit here.

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