Int. J. Anal. Appl. (2026), 24:234

International Journal of Analysis and Applications

New Significant Contractions in the Frame of Gamma-k Distance Mapping With Application

Tariq Qawasmeh, Jamal Abu Najm*

Department of mathematics, College of Arts and Sciences, Amman Arab University, Amman 11953, Jordan

**Corresponding author:* 202411902@aau.edu.jo, abunajmjamal@gmail.com

Abstract. This manuscript is primarily concerned with the formulation of two new distinct contractions, namely $\gamma - k$ and $\mathscr{H} - \gamma(k)$ mixed power type contractions. These new contractions are based on the concept of $\gamma - k$ distance mapping. A significant application and examples are also provided to illustrate the importance of the findings.

1. Introduction

Fixed point theory is a key concept in mathematics and is important in many areas, such as nonlinear analysis, differential equations, optimization theory, and dynamical systems. The Banach contraction principle [1], first introduced in 1922, is one of the most significant theories in fixed point theory.

In recent decades, many researchers have suggested various generalizations of the Banach contraction principle. These are designed to broaden their applications to different types of mappings and generalized metric structures. Some notable generalizations include Kannan [2] contractions, Chatterjea [3] contractions, weak contractions, and several types of nonlinear contractive conditions such as [4–23, 25, 26].

Recently, the theory of generalized distance mappings has gained increased attention within fixed point theory. This theory provides a new and more flexible way to create different contractive inequalities in a generalized metric setting. The theory of γ-k distance mappings, which was formulated by Bataihah et. al. [27] as a refinement of the theory of γ distance mapping, which was introduced by Bataihah et.al. [28]. The theory of $\gamma - k$ offers a new method for studying the existence and uniqueness of a fixed point under different types of nonlinear contractive inequalities.

Received: Apr. 1, 2026.

2020 *Mathematics Subject Classification.* 47H10.

Key words and phrases. fixed point; $\gamma - k$ distance; $\gamma - k$ mixed power type contraction; $\mathscr{H}$ simulation function.

https://doi.org/10.28924/2291-8639-24-2026-234
ISSN: 2291-8639
© 2026 the author(s).

The notion of Mixed power type contraction formulated by Bataihah and Shatnawi [29] in 2025 and extended this notion to proved significant results.

In this subsequent discussion, $\mathscr{D}$ refers to the non empty set, $(\mathscr{D}, d)$ to the metric space, γ to $\gamma - k$ distance mappings, and $\mathscr{H}$ to H simulations, and $\mathscr{A}_f$ refers to the set of all fixed points of the self mapping $f : \mathscr{D} \to \mathscr{D}$.

2. Preliminary

The concept of $\gamma - k$ distance mapping was introduced by Bataihah et al. [27] and proved various fixed point theorems using this concept.

Definition 2.1. *[27] Let d be a metric on $\mathscr{D}$. A function $\gamma : [0, \infty) \times \mathscr{D} \times \mathscr{D} \to [0, \infty)$ is said to be γ-k distance over $(\mathscr{D}, d)$ if γ satisfying:*

(γ_1) *For all $t \geq 0$, $\gamma(t, x, v) \geq t^k d(x, v)$, $k \geq 1$.*
(γ_2) *for each sequences $(x_n), (v_n)$ in $\mathscr{B}$ and (t_n) in $(0, \infty)$, we have $d(x_n, v_n) \to \xi > 0$ and $t_n \to t > 0$ imply $\liminf_{n\to\infty} \gamma(t_n, x_n, v_n) > \xi t^k$.*

The following examples are $\gamma - k$ distance mappings [27].
In subsequent examples, the functions $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$ are established as mappings from the domain $[0, +\infty) \times \mathscr{D} \times \mathscr{D}$ to the co-domain $[0, +\infty)$.

Example 2.1. *[28] On $(\mathscr{D}, d)$, let $\phi : [0, \infty) \to [0, \infty)$ is continuous function with $\phi(a) > 0$ for $a > 0$. Then, $\gamma_1, \gamma_2, \gamma_3, \gamma_4$* and *$\gamma_5$ are γ-k distance over $(\mathscr{D}, d)$.*

(1) $\gamma_1(t, x, v) = t^k\, d(x, v) + \phi(d(x, v))$,
(2) $\gamma_2(t, x, v) = t^k\, d(x, v) + \phi(t)$,
(3) $\gamma_3(t, x, v) = e^{tk} d(x, v)$,
(4) $\gamma_4(t, x, v) = \dfrac{(t + \varepsilon)^{2k} + [d(x, v)]^2}{2}$, $\varepsilon > 0$,
(5) $\gamma_6(t, x, v) = \dfrac{t^{2k} + 1}{2}\, d(x, v)$,

Bataihah et al. [30] introduced the notion of a H-simulation function and utilized this concept to prove several significant theorems of fixed points (uniqueness and existence). On the other hand, Pioneers extended this notion to introduced many fixed points theorems such as [31–33]

Definition 2.2. *[30] A function $\mathscr{H} : [1, +\infty) \times [1, +\infty) \to \mathbb{R}$ is called H-simulation function if $\mathscr{H}(t_1, t_2) \leq \frac{t_2}{t_1}\ \forall\ t_1, t_2 \in [1, +\infty)$.*

Remark 2.1. *[30] Let $\mathscr{H} \in H$. If (t_n) and (t'_n) are sequences in $[1, +\infty)$ with $1 \leq \lim_{n\to\infty} t'_n < \lim_{n\to\infty} t_n$, then $\limsup_{n\to\infty} \mathscr{H}(t_n, t'_n) < 1$.*

Now, we provide some examples of H-simulation functions.

Example 2.2. *[30] The following functions belong to H-simulation functions:*

(1) $\mathscr{H}_1(t_1, t_2) = \frac{k\, t_2^r}{t_1}$, $k, r \in (0, 1]$,

(2) $\mathscr{H}_2(t_1, t_2) = \frac{\min\{t_1, t_2\}}{\max\{t_1, t_2\}}$.

(3) $\mathscr{H}_3(t_1, t_2) = \frac{t_2}{t_1 + \ln\left(\frac{t_2}{t_1}\right)}$.

(4) $\mathscr{H}_4(t_1, t_2) = \frac{t_2}{t_1 + \sqrt{t_2}}$.

(5) $\mathscr{H}_5(t_1, t_2) = \frac{t_2}{1 + t_1 t_2}$.

(6) *Let* $f_1, f_2 : (0, +\infty) \to (0, +\infty)$ *be continuous functions such that* $f_1(r) < r$ *and* $f_2(r) \geq r$, *for each* $r \in (0, +\infty)$. *Define* $\mathscr{H}_6(t_1, t_2) = \frac{f_1(t_2)}{f_2(t_1)}$.

3. Fixed Point Results for $\gamma - k$ Mixed Power Type Contractions

Definition 3.1. *Suppose that there is* $\gamma - k$ *distance mapping* γ *over* $(\mathscr{D}, d)$. *A self-mapping* $f : \mathscr{D} \to \mathscr{D}$ *is said to be a* $\gamma - k$ *mixed power type contraction if* $\exists\, c_1, c_2, \alpha \in \mathbb{R}$ *with* $0 < c_1 + c_2 < 1,\ 0 < \alpha < 1$ *s.t.*

$$\gamma(d(fx, fy), fx, fy) < [c_1(d^{\alpha}(x, y) + c_2(d^{1-\alpha}(x, y)]^{k+1}, \text{ with } K \geq 1 \tag{3.1}$$

Remark 3.1. *If* $f : \mathscr{D} \to \mathscr{D}$ *is a* $\gamma - k$ *mixed power type contraction, then* $\forall\, t, s \in \mathscr{D}$, *we have:*

$$d(ft, fs) < c_1 d^{\alpha}(t, s) + c_2 d^{1-\alpha}(t, s) \tag{3.2}$$

Proof. By the definition of $\gamma - k$ distance mapping we have :

$$\gamma(d(ft, fs), ft, fs) \geq d^{k+1}(ft, fs).$$

Therefore,

$$d^{k+1}(ft, fs) \leq \gamma(d(ft, fs), ft, fs) < [c_1(d^{\alpha}(x, y) + c_2(d^{1-\alpha}(x, y)]^{k+1}$$

Hence, the desired result. □

Lemma 3.1. *Suppose that the self mapping* $f : \mathscr{D} \to \mathscr{D}$ *is a* $\gamma - k$ *mixed power type contraction, then* $\mathscr{A}_f$ *has at most one element.*

Proof. Assume to contrary that $\exists t, s \in \mathscr{A}_f$. By using remark 3.1 we have

$$d(ft, fs) < c_1 d^{\alpha}(t, s) + c_2 d^{1-\alpha}(t, s)$$

If $r = \min\{\alpha, 1 - \alpha\}$ *and* $c_1 + c_2 = R$, then $r, R < 1$. Now,

$$\begin{aligned} d(t, s) &= d(ft, fs) \\ &< c_1 d^{\alpha}(t, s) + c_2 d^{1-\alpha}(t, s) \\ &\leq R d^{r}(t, s) \\ &\leq R(R d^{r}(t, s))^{R} \end{aligned}$$

$$
\begin{aligned}
&= R^{r+1} d^{r^2}(t,s) \\
&\vdots \\
&\leq R^{nr}.
\end{aligned}
$$

Taking the limit as $n \to \infty$. Hence, the desired result. $\square$

Lemma 3.2. *Assume that $f : \mathscr{D} \to \mathscr{D}$ is a $\gamma - k$ mixed power type contraction with diam$\mathscr{D} \leq 1$, and $t_0 \in \mathscr{D}$ be arbitrary element. Then the Picard sequence (t_n) generated by the function f at t_0, if for each $n \in \mathbb{N}$ we have $t_n \neq t_{n+1}$, then*

$$\lim_{n\to\infty} d(t_n, t_{n+1}) = 0.$$

Proof. By the definition of $\gamma - k$ distance mapping, we have

$$
\begin{aligned}
d^{k+1}(t_{n+1}, t_n) \quad &\leq \gamma(d(t_{n+1}, t_n), t_{n+1}, t_n) \\
&\gamma(d(ft_n, ft_{n-1}), ft_n+, ft_{n-1}) \\
&< [c_1 d^{\alpha}(t_n, t_{n-1} + c_2 d^{1-\alpha}(t_n, t_{n-1})]^{k+1} \\
&\leq R(Rd^r(t,s))^R \\
&= R^{r+1} d^{r^2}(t,s) \\
&\vdots \\
&\leq R^{nr}.
\end{aligned}
$$

$\square$

Theorem 3.1. *Suppose there exists a γ-k distance mapping γ over a complete metric space $(\mathscr{D}, d)$ with* diam$(\mathscr{D}) \leq 1$. *Assume that $f : \mathscr{D} \to \mathscr{D}$ is a γ-k mixed power type contraction. Then $\mathscr{A}_f$ has only one element.*

Proof. We construct a Picard sequence $\{t_n\}$ by choosing an arbitrary element $t_0 \in \mathscr{D}$ and defining $t_{n+1} = ft_n \qquad \forall n \in \mathbb{N}$.

Now assume for some $m \in \mathbb{N}$ that $t_{m+1} = t_m$. Then t_m is a fixed point.

So for all $n \in \mathbb{N}$ assume that $t_{n+1} \neq t_n$. By Condition 3.1 and Lemma 3.2, we obtain

$$d(t_n, t_{n+1}) \leq R^{nr}.$$

For $m \geq n \geq 0$, we have

$$
\begin{aligned}
d(t_n, t_m) \quad &\leq d(t_n, t_{n+1}) + d(t_{n+1}, t_m) \\
&\leq d(t_n, t_{n+1}) + d(t_{n+1}, t_{n+2}) + d(t_{n+2}, t_m) \\
&\leq R^{nr} + R^{(n+1)r} + \cdots + R^{mr} \\
&= R^{nr}\left[1 + R^r + R^{2r} + \cdots + R^{(m-n)r}\right].
\end{aligned}
$$

Taking the limit as $n \to \infty$, we get

$$\lim_{n,m\to\infty} d(t_n, t_m) = 0.$$

Hence, (t_n) is a Cauchy sequence. Since $\mathscr{D}$ is complete, there exists $t \in \mathscr{D}$ such that $t_n \to t$.
Now, by the triangle inequality we get

$$d(ft,t) \le d(ft,ft_n) + d(ft_n,t) < c_1 d(t,t_n)^{\alpha} + c_2 d(t,t_n)^{1-\alpha} + d(ft_n,t).$$

Taking the limit as $n \to \infty$, we obtain $d(ft,t) = 0 \implies ft = t$.
Lemma 3.1 ensures that $\mathscr{A}_f$ has exactly one element. □

In the next corollary, we may assume that $c = c_1 + c_2 \quad \text{and} \quad \beta = \max\{\alpha,\, 1-\alpha\}$.

Corollary 3.1. *Suppose there exists a γ-k distance mapping γ over a complete metric space $(\mathscr{D}, d)$ with* $\operatorname{diam}(\mathscr{D}) \le 1$. *If $f : \mathscr{D} \to \mathscr{D}$ satisfies the condition that $c, \beta \in \mathbb{R}$ such that $c = c_1 + c_2$ and $\beta =$* $\max\{\alpha, 1-\alpha\}$, *and*

$$x \neq y \Longrightarrow \gamma(d(fx,fy), fx, fy) < \left[c\, d^{\beta}(x,y)\right]^{k+1}, \qquad k \ge 1,$$

then $\mathscr{A}_f$ has exactly one element.

Proof. Since

$$\gamma(d(fx,fy),fx,fy) < \left[c\, d^{\beta}(x,y)\right]^{k+1} = c^{k+1} d^{\beta(k+1)}(x,y) \le \left[c_1 d^{\alpha}(x,y) + c_2 d^{1-\alpha}(x,y)\right]^{k+1}.$$

Hence, the result follows from Theorem 3.1. □

In the following significant example, we will show the novelty of Theorem 3.1

Example 3.1. *Suppose that $\mathscr{D} = [0,1]$ (diam$\mathscr{D} = 1$), the function $f : [0,1] \to [0,1]$ be defined as:*

$$fx = \frac{1-x^m}{\Gamma + x^m}\ \text{, where } \Gamma \ge 3 \text{ and } \frac{2}{3} \le m < 1.$$

Then $\mathscr{A}_f$ has only one element on $[0,1]$.
To show this, define the following mappings:
$d : \mathscr{D} \times \mathscr{D} \to [0,\infty)$ By $d(x,y) = |x-y|$ the Euclidean metric. It is obvious that $(\mathscr{D}, d)$ is a complete metric space. Assume that $\gamma(t,x,y) = \frac{1}{2}\left(d^2(x,y)+1\right)d(x,y)$. Then,$\gamma$ is a $\gamma-k$-distance mapping (with $k=1$) over $(\mathscr{D}, d)$.
We claim that $\forall\ x,y \in \mathscr{D}$ with $x \neq y$ we have:

$$d(fx,fy) < \frac{\Gamma+1}{\Gamma^2} d^m(x,y). \tag{3.3}$$

To prove this claim, $\forall\ x,y \in \mathscr{D}$ with $x \neq y$ we have

$$\begin{aligned}
|fx - fy| &= \left|\frac{1-x^m}{\Gamma + x^m} - \frac{1-x^m}{\Gamma + x^m}\right| \\
&\leq \frac{1}{(\Gamma + x^m)(\Gamma + y^m)}(|x^m - y^m| + \Gamma|x^m - y^m|) \\
&< \frac{\Gamma+1}{\Gamma^2}|x^m - y^m| \\
&\leq \frac{\Gamma+1}{\Gamma^2}|x-y|^m \\
&= \frac{\Gamma+1}{\Gamma^2}d^m(x,y).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\gamma(d(fx,fy),fx,fy) &= \frac{1}{2}\left(|fx-fy|^2+1\right)|fx-fy| \\
&< \frac{1}{2}\left[\left(\frac{\Gamma+1}{\Gamma^2}\left|x-y\right|^m\right)^2+1\right]\frac{\Gamma+1}{\Gamma^2}\left|x-y\right|^m \\
&= \frac{1}{2}\left[\left(\frac{(\Gamma+1)^2}{\Gamma^4}\left|x-y\right|^{2m}\right)+1\right]\frac{\Gamma+1}{\Gamma^2}\left|x-y\right|^m \\
&= \frac{1}{2}\left[\frac{(\Gamma+1)^3}{\Gamma^5}\left|x-y\right|^{3m}+\frac{\Gamma+1}{\Gamma^2}\left|x-y\right|^m\right] \\
&\leq \frac{1}{2}[(\sqrt{\frac{(\Gamma+1)^3}{\Gamma^5}}\left|x-y|^m\right)^2+\frac{\Gamma+1}{\Gamma^2}\left|x-y\right|^m] \\
&\leq \left[(\sqrt{\frac{(\Gamma+1)^3}{2\Gamma^5}}\left|x-y|^m\right)+\sqrt{\frac{\Gamma+1}{2\Gamma^2}}\left|x-y\right|^{1-m}\right]^2
\end{aligned}$$

If $c_1 = \sqrt{\frac{(\Gamma+1)^3}{2\Gamma^5}}$ *and* $c_2 = \sqrt{\frac{\Gamma+1}{2\Gamma^2}}$ *and* $B \geq 3$*, then* $c_1 + c_2 < 1$. *and we get*

$$\gamma(d(fx,fy),fx,fy) < [c_1 d^m(x,y) + c_2 d^{1-m} d(x,y)]^2 \tag{3.4}$$

Consequently, all conditions of $\gamma - k$ *mixed power type contraction are met by the mapping* f. *Theorem 3.1 confirms that* $\mathscr{A}_f$ *has exactly one element.*

4. Fixed Point Results for $\mathscr{H} - \gamma(k)$ Mixed Power Type Contractions

Definition 4.1. *Suppose that there is* $\gamma - k$ *distance mapping* γ *over* $(\mathscr{D}, d)$ *and there is* $\mathscr{H}$ *simulation function* H. *A self-mapping* $f : \mathscr{D} \to \mathscr{D}$ *is said to be a* $\mathscr{H} - \gamma(k)$ *mixed power type contraction if*

$\exists\, c_1, c_2, \alpha \in \mathbb{R}$ *with* $0 < c_1 + c_2 < 1,\ 0 < \alpha < 1$ *s.t.*

$$\mathscr{H}\left(\gamma(d(fx, fy), fx, fy), [c_1(d^{\alpha}(x,y) + c_2(d^{1-\alpha}(x,y))]^{k+1}\right) > 1, \text{ with } K \geq 1 \tag{4.1}$$

Remark 4.1. *Assume that $f : \mathscr{D} \to \mathscr{D}$ is a $\mathscr{H} - \gamma(k)$ mixed power type contraction. Then for each $\forall\, t_1, t_2 \in \mathscr{D}$, we have:*

$$d(ft_1, ft_2) < c_1 d^{\alpha}(t_1, t_2) + c_2 d^{1-\alpha}(t_1, t_2) \tag{4.2}$$

Proof. Using the definition of H simulation function, we get:

$$1 < \frac{[c_1(d^{\alpha}(x,y) + c_2(d^{1-\alpha}(x,y))]^{k+1}}{\gamma(d(fx, fy), fx, fy)}.$$

This implies that

$$\gamma(d(fx, fy), fx, fy) < c_1(d^{\alpha}(x,y) + c_2(d^{1-\alpha}(x,y))]^{k+1}$$

Hence, the desired result we get by Remark 3.1. □

Lemma 4.1. *Assume that the self-mapping $f : \mathscr{D} \to \mathscr{D}$ is an $\mathscr{H} - \gamma(k)$ mixed power type contraction. Then $\mathscr{A}_f$ has at most one element.*

Proof. The proof follows from the definition of H simulation function and Remark 4.1. □

Lemma 4.2. *Assume that $f : \mathscr{D} \to \mathscr{D}$ is a a $\mathscr{H} - \gamma(k)$ mixed power type contraction with diam$\mathscr{D} \leq 1$, and $t_0 \in \mathscr{D}$ be arbitrary element. Then the Picard sequence (t_n) generated by the function f at t_0, if for each $n \in \mathbb{N}$ we have $t_n \neq t_{n+1}$, then*

$$\lim_{n\to\infty} d(t_n, t_{n+1}) = 0.$$

Proof. Since $t_n \neq t_{n+1}$, by Condition 4.3

$$1 < \mathscr{H}\left(\gamma(d(ft_{n-1}, f_n), ft_{n-1}, ft_n), [c_1(d^{\alpha}(t_{n-1}, t_n) + c_2(d^{1-\alpha}(t_{n-1}, t_n))]^{k+1}\right).$$

This implies that

$$\gamma(d(ft_{n-1}, f_n), ft_{n-1}, ft_n) < [c_1(d^{\alpha}(t_{n-1}, t_n) + c_2(d^{1-\alpha}(t_{n-1}, t_n))]^{k+1}$$

From Remark 4.1, we have

$$d(ft_{n-1}, f_n) < [c_1(d^{\alpha}(t_{n-1}, t_n) + c_2(d^{1-\alpha}(t_{n-1}, t_n))].$$

The rest of the proof is typical to Lemma 3.2. □

Theorem 4.1. *Suppose that there exists a γ-k distance mapping γ and H simulation function $\mathscr{H}$ over a complete metric space $(\mathscr{D}, d)$ with* diam$(\mathscr{D}) \leq 1$. *Assume that $f : \mathscr{D} \to \mathscr{D}$ is a $\mathscr{H} - \gamma(k)$ mixed power type contraction. Then A_f has exactly one element.*

Proof. First, we construct a Picard sequence $\{t_n\}$ by choosing an arbitrary element $t_0 \in \mathscr{D}$ and defining $t_{n+1} = ft_n \qquad \forall n \in \mathbb{N}$.

Now assume for some $m \in \mathbb{N}$ that $t_{m+1} = t_m$. Then t_m is a fixed point.

So for all $n \in \mathbb{N}$ assume that $t_{n+1} \neq t_n$. we have:

$$\mathscr{H}\left(\gamma(d(ft_n, ft_{n-1}), ft_n, ft_{n-1}), [c_1(d^{\alpha}(t_n, t_{n-1}) + c_2(d^{1-\alpha}(t_n, t_{n-1})]^{k+1}\right) > 1, \ with\ K \geq 1 \tag{4.3}$$

Using Remark 4.1, we obtain:

$$d(t_{n+1,t_n}) = d(ft_n, ft_{n-1}) < c_1 d^{\alpha}(t_n, t_{n-1}) + c_2 d^{1-\alpha}(t_n, t_{n-1}) \tag{4.4}$$

By condition 4.1 and Lemma 4.2, we get $\lim_{n\to\infty} d(t_{n+1}, t_n) = 0$.

For $m \geq n \geq 0$, we have $d(t_n, t_m) \leq d(t_n, t_{n+1}) + d(t_{n+1}, t_m) \leq d(t_n, t_{n+1}) + d(t_{n+1}, t_{n+2}) + d(t_{n+2}, t_m) \leq R^{nr} d(t_0, t_1) + R^{(n+1)r} d(t_0, t_1) + \cdots + R^{mr} d(t_0, t_1) \leq R^{nr} + R^{(n+1)r} + \cdots + R^{mr} = R^{nr}\left[1 + R^r + R^{2r} + \cdots + R^{(m-n)r}\right]$.

Taking the limit as $n \to \infty$, we get $\lim_{n\to\infty} d(t_n, t_m) = 0$.

Hence (t_n) is a Cauchy sequence. Since $\mathscr{D}$ is complete, there exists $t \in \mathscr{D}$ such that $t_n \to t$.

Now, by the triangle inequality, we get $d(ft, t) \leq d(ft, ft_n) + d(ft_n, t) < c_1 d(t, t_n)^{\alpha} + c_2 d(t, t_n)^{1-\alpha} + d(ft_n, t)$.

Taking the limit as $n \to \infty$, we obtain $d(ft, t) = 0 \implies ft = t$. □

5. Application

Our application refer to Example 3.1. Let $\frac{2}{3} \leq m < 1, \Gamma \in \mathbb{R}\ with\ \Gamma \geq 3$. The following equation on the unit interval $[0, 1]$:

$$1 - t^{m+1} - t^m - \Gamma t = 0, \ where\ \frac{2}{3} \leq m < 1, \Gamma \in \mathbb{R}\ with\ \Gamma \geq 3. \tag{5.1}$$

has a unique solution.

It is typical to show that the following mapping f has a unique fixed point within the unit interval [0,1].

$$ft = \frac{1 - t^m}{\Gamma + t^m}\ ,\ where\ \frac{2}{3} \leq m < 1, \Gamma \in \mathbb{R}\ with\ \Gamma \geq 3.$$

Example 3.1 guarantee that $\mathscr{A}_f$ has one element 0n [0,1]. Consequently, equation 5.1 has a unique solution.

Conclusion

In this study, we formulated several innovative contractions that are noteworthy. These contractions are based on the frame of $\gamma - k$ distance mapping. Moreover, to highlight the novelty of our findings, we formulated a significant application along with a numerical example.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] S. Banach, Sur les Opérations dans les Ensembles Abstraits et Leur Application aux Équations Intégrales, Fundam. Math. 3 (1922), 133–181. https://doi.org/10.4064/fm-3-1-133-181.

[2] R. Kannan, Some Results on Fixed Points, Bull. Calcutta Math. Soc. 60 (1968), 71–76.

[3] S.K. Chatterjea, Fixed Point Theorems for a Sequence of Mappings with Contractive Iterates, Publ. Inst. Math. 14 (1972), 15–18.

[4] T. Qawasmeh, A. Bataihah, A.A. Hazaymeh, R. Hatamleh, R. Abdelrahim, et al., New Fixed Point Results for Gamma Interpolative Contractions through Gamma Distance Mappings, WSEAS Trans. Math. 24 (2025), 424–430. https://doi.org/10.37394/23206.2025.24.40.

[5] T. Qawasmeh, A. Tallafha, W. Shatanawi, Fixed and Common Fixed Point Theorems through Modified ω-Distance Mappings, Nonlinear Funct. Anal. Appl. 24 (2019), 221–239. http://nfaa.kyungnam.ac.kr/journal-nfaa.

[6] K. Abodayeh, T. Qawasmeh, W. Shatanawi, A. Tallafha, ϵ_ϕ-Contraction and Some Fixed Point Results via Modified ω-Distance Mappings in the Frame of Complete Quasi Metric Spaces and Applications, Int. J. Electr. Comput. Eng. 10 (2020), 3839. https://doi.org/10.11591/ijece.v10i4.pp3839-3853.

[7] A.A. Hazaymeh, I.B. Abdullah, Z.T. Balkhi, R.I. Ibrahim, Generalized Fuzzy Soft Expert Set, J. Appl. Math. 2012 (2012), 328195. https://doi.org/10.1155/2012/328195.

[8] H. Aydi, E. Karapinar, M. Postolache, Tripled Coincidence Point Theorems for Weak ϕ-Contractions in Partially Ordered Metric Spaces, Fixed Point Theory Appl. 2012 (2012), 44. https://doi.org/10.1186/1687-1812-2012-44.

[9] A. Bataihah, Some Fixed Point Results with Application to Fractional Differential Equation via New Type of Distance Spaces, Results Nonlinear Anal. 7 (2024), 202–208. https://doi.org/10.31838/rna/2024.07.03.015.

[10] A. Bataihah, T. Qawasmeh, A New Type of Distance Spaces and Fixed Point Results, J. Math. Anal. 15 (2024), 81–90. https://doi.org/10.54379/jma-2024-4-5..

[11] I. Abu-Irwaq, I. Nuseir, A. Bataihah, Common Fixed Point Theorems in G-Metric Spaces with Ω-Distance, J. Math. Anal. 8 (2017), 120–129.

[12] A.M. Malkawi, A.M. Rabaiah, Fixed Point Theorems in Neutrosophic MR-Metric Spaces with Measure-Theoretic Convergence, Int. J. Anal. Appl. 24 (2026), 61. https://doi.org/10.28924/2291-8639-24-2026-61.

[13] T. Qawasmeh, A.A. Malkawi, Fixed Point Theory in MR-Metric Spaces: Fundamental Theorems and Applications to Integral Equations and Neutron Transport, Eur. J. Pure Appl. Math. 18 (2025), 6440. https://doi.org/10.29020/nybg.ejpam.v18i3.6440.

[14] W. Shatanawi, A. Bataihah, A. Pitea, Fixed and Common Fixed Point Results for Cyclic Mappings of Ω-Distance, J. Nonlinear Sci. Appl. 09 (2016), 727–735. https://doi.org/10.22436/jnsa.009.03.02.

[15] A. Anwar, Fixed Point Results of Geraghty Type Contractions with Equivalent Distance, Int. J. Neutrosophic Sci. 25 (2025), 177–186. https://doi.org/10.54216/IJNS.250316.

[16] W. Shatanawi, G. Maniu, A. Bataihah, F. Bani Ahmad, Common Fixed Points for Mappings of Cyclic Form Satisfying Linear Contractive Conditions with Omega-Distance, U.P.B. Sci. Bull., Ser. A. 79 (2017), 11–20.

[17] A.A. Hazaymeh, A. Bataihah, Neutrosophic Fuzzy Metric Spaces and Fixed Points for Contractions of Nonlinear Type, Neutrosophic Sets Syst. 77 (2025), 96–112.

[18] A. Bataihah, T. Qawasmeh, M. Shatnawi, Discussion on b-Metric Spaces and Related Results in Metric and G-Metric Spaces, Nonlinear Funct. Anal. Appl. 27 (2022), 233–247. https://doi.org/10.22771/nfaa.2022.27.02.02.

[19] F. Uddin, U. Ishtiaq, A. Hussain, K. Javed, H. Al Sulami, et al., Neutrosophic Double Controlled Metric Spaces and Related Results with Application, Fractal Fract. 6 (2022), 318. https://doi.org/10.3390/fractalfract6060318.

[20] H. Ahmad, M. Younis, A.A.N. Abdou, Bipolar b-Metric Spaces in Graph Setting and Related Fixed Points, Symmetry 15 (2023), 1227. https://doi.org/10.3390/sym15061227.

[21] M.U. Ali, T. Kamran, M. Postolache, Fixed Point Theorems for Multivalued G-Contractions in Hausdorff b-Gauge Spaces, J. Nonlinear Sci. Appl. 08 (2015), 847–855. https://doi.org/10.22436/jnsa.008.05.34.

[22] A. Bejenaru, M. Postolache, On Suzuki Mappings in Modular Spaces, Symmetry 11 (2019), 319. https://doi.org/10.3390/sym11030319.

[23] S. Chandok, M. Postolache, Fixed Point Theorem for Weakly Chatterjea-Type Cyclic Contractions, Fixed Point Theory Appl. 2013 (2013), 28. https://doi.org/10.1186/1687-1812-2013-28.

[24] A. Bataihah, A. Hazaymeh, Quasi Contractions and Fixed Point Theorems in the Context of Neutrosophic Fuzzy Metric Spaces, Eur. J. Pure Appl. Math. 18 (2025), 5785. https://doi.org/10.29020/nybg.ejpam.v18i1.5785.

[25] M. Jleli, B. Samet, A New Generalization of the Banach Contraction Principle, J. Inequal. Appl. 2014 (2014), 38. https://doi.org/10.1186/1029-242x-2014-38.

[26] T. Kamran, M. Samreen, Q. UL Ain, A Generalization of B-Metric Space and Some Fixed Point Theorems, Mathematics 5 (2017), 19. https://doi.org/10.3390/math5020019.

[27] A. Bataihah, A.H. Ansari, S.H.J. Petroudi, T. Qawasmeh, A. Hazaymeh, Gamma-K Distance and Fixed Point Theorems, Gulf J. Math. 20 (2025), 275–287. https://doi.org/10.56947/gjom.v20i2.3169.

[28] A. Bataihah, T. Qawasmeh, I.M. Batiha, I.H. Jebril, T. Abdeljawad, Gamma Distance Mappings with Application to Fractional Boundary Differential Equation, J. Math. Anal. 15 (2024), 99–106. https://doi.org/10.54379/jma-2024-5-7.

[29] A. Bataihah, M. Shatnawi, Fixed Point Results for Mixed Power-Type Contractions on Complete Metric Spaces, Nonlinear Funct. Anal. Appl. 30 (2025), 1223–1236. https://doi.org/10.22771/nfaa.2025.30.04.15.

[30] A. Bataihah, W. Shatanawi, T. Qawasmeh, R. Hatamleh, On H-Simulation Functions and Fixed Point Results in the Setting of ω_t-Distance Mappings with Application on Matrix Equations, Mathematics 8 (2020), 837. https://doi.org/10.3390/math8050837.

[31] T. Qawasmeh, (H, Ω_b)-Interpolative Contractions in Ω_b-Distance Mappings with Application, Eur. J. Pure Appl. Math. 16 (2023), 1717–1730. https://doi.org/10.29020/nybg.ejpam.v16i3.4819.

[32] T. Qawasmeh, h-Simulation Functions and Ω_b-Distance Mappings in the Setting of G_b-Metric Spaces and Application, Nonlinear Funct. Anal. Appl. 28 (2023), 557–570. https://doi.org/10.22771/nfaa.2023.28.02.14.

[33] T. Qawasmeh, R. Hatamleh, A New Contraction Based on H-Simulation Functions in the Frame of Extended b-Metric Spaces and Application, Int. J. Electr. Comput. Eng. 13 (2023), 4212. https://doi.org/10.11591/ijece.v13i4.pp4212-4221.