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The Supply Security Problem and its Solution

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Abstract. This study introduces a mathematical model to address the supply security problem, a critical issue in today's global landscape. Furthermore, we present a method for efficiently identifying optimal solutions to security assignment problems, which holds significant implications for various sectors, including business and policy-making, particularly in the context of food security and supply security. Leveraging fundamental principles of convex optimization, such as proximity operators and projection onto convex sets, this study not only identifies instances where optimal solutions to security problems exist but also provides an explicit formula for the solution when it exists. Additionally, we establish the existence of pseudo-solutions in security assignment problems and demonstrate that they coincide with actual solutions when present. A key highlight of this paper is its pioneering application of convex optimization techniques to address security challenges.

1. Introduction

Food crises have emerged as a significant global challenge over time. According to the World Bank [1], there persists high domestic food price inflation on a global scale. This crisis has not been limited to food alone; it has also manifested as shortages in other essential commodities, exacerbated by our growing population amidst finite resources. Researchers worldwide have been actively seeking solutions to mitigate or, ideally, halt these shortages. A multitude of research articles on supply security have examined, assessed, or proposed strategies for various types of supplies [2–5]. Furthermore, mathematicians have contributed diverse studies employing various approaches to address supply security issues, including modeling [6,7] and programming [8].

In this paper, we introduce and develop a mathematical model applicable to general supply-related challenges. We formulate a comprehensive solution designed to mitigate shortages and

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prevent over-supply of resources, drawing on principles from convex analysis, proximity operators, and non-linear programming. We establish the existence of solutions to these problems or provide a pseudo-solution if a true solution proves elusive. Moreover, this paper incorporates real-life applications to illustrate our approach.

In the context of the supply security problem, we define Ω as the set of products, represented by $u_1, u_2, \ldots, u_n$. For each product u_i within Ω, we use $\tau(u_i)$ to denote the anticipated demand quantity for the upcoming month, and $\theta(u_i)$ to indicate the projected price (production cost) of the same product for that month. Additionally, we aim to satisfy the constraint conditions of set C, where C is a nonempty, closed, and convex subset of $\mathbb{R}^n$. The primary objective is to determine the optimal supply quantity, denoted as $s(u_i)$, for each product u_i, ensuring the prevention of shortages while minimizing over-supply.

To be more precise, we set $s(u_i) = r(u_i)\tau(u_i)$. Since our objective is to prevent shortages of the product u_i, we require that $r(u_i)\tau(u_i) \geq \tau(u_i)$, which translates to $r(u_i) \geq 1$. Furthermore, over-supply is minimized when $r(u_i)\tau(u_i)$ and $\tau(u_i)$ are close for each product u_i. Thus, the goal is to find the solution $r : \Omega \to [1, +\infty)$ to the problem

$$\min_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} |\tau(u_i) - t(u_i)\tau(u_i)|^p + \iota_C(x(t)) \right) \tag{1.1}$$

where $p > 1$, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))$, $x_i(t) = \tau(u_i) - \tau(u_i)t(u_i)$, and $\iota_C : \mathbb{R}^n \to \{0, +\infty\}$ is a function defined as $\iota_C(x) = 0$ if $x \in C$ and $+\infty$ otherwise.

Furthermore, we aim to minimize the total cost of production. In other words, we seek a function $r : \Omega \to [1, +\infty)$ that is the solution to the minimization problem:

$$\min_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} \tau(u_i)\theta(u_i)t(u_i) + \iota_C(x(t)) \right). \tag{1.2}$$

Lastly, in order to assist farmers or business owners, we want to maximize the supply. This means we are looking for a function $r : \Omega \to [1, +\infty)$ that is the solution to the maximization problem:

$$\max_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} \tau(u_i)t(u_i) - \iota_C(x(t)) \right). \tag{1.3}$$

In summary, this study aims to address the problem outlined in Definition 1.1 where $p = 2$.

Definition 1.1. *Let $\Omega = \{u_1, u_2, ..., u_n\}$, $\tau : \Omega \to (0, +\infty)$, $\theta : \Omega \to (0, +\infty)$, and $p > 1$. Consider C as a nonempty, closed, and convex subset of $\mathbb{R}^n$. Define*

$$x(t) = (\tau(u_1) - \tau(u_1)t(u_1), \tau(u_2) - \tau(u_2)t(u_2), ..., \tau(u_n) - \tau(u_n)t(u_n))$$

where $t : \Omega \to [1, +\infty)$ is a function. The $(\Omega, C, p)-$supply security problem involves finding a function $r : \Omega \to [1, +\infty)$ satisfies the following three conditions:

1. $\min\limits_{t:\Omega\to[1,+\infty)}\left(\sum\limits_{i=1}^{n}|\tau(u_i)-\tau(u_i)t(u_i)|^p+\iota_C(x(t))\right)=\sum\limits_{i=1}^{n}|\tau(u_i)-\tau(u_i)r(u_i)|^p$
2. $\min\limits_{t:\Omega\to[1,+\infty)}\left(\sum\limits_{i=1}^{n}\tau(u_i)\theta(u_i)t(u_i)+\iota_C(x(t))\right)=\sum\limits_{i=1}^{n}\tau(u_i)\theta(u_i)r(u_i)$
3. $\max\limits_{t:\Omega\to[1,+\infty)}\left(\sum\limits_{i=1}^{n}\tau(u_i)t(u_i)-\iota_C(x(t))\right)=\sum\limits_{i=1}^{n}\tau(u_i)r(u_i).$

In this study, $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$ are the standard norm and standard inner product in $\mathbb{R}^n$, respectively. Furthermore, we define

$$x(t)=(\tau(u_1)-\tau(u_1)t(u_1),\tau(u_2)-\tau(u_2)t(u_2),...,\tau(u_n)-\tau(u_n)t(u_n))$$

where $t:\Omega\to[1,+\infty)$ is a function.

2. Optimal Solution to the Supply Security Problem where $p=2$

In this section, we will solve the solution of the $(\Omega,C,2)$-supply security problem. The function $f:\mathbb{R}^n\to(-\infty,+\infty]$ is convex if and only if

$$f(\alpha x+(1-\alpha)y)\leq\alpha f(x)+(1-\alpha)f(y)$$

for all $x,y\in\mathbb{R}^n$ and for all $\alpha\in[0,1]$. We say that $f:\mathbb{R}^n\to(-\infty,+\infty]$ is proper if and only if $\mathrm{dom}f=\{x:f(x)<+\infty\}$ is non-empty. Throughout this paper, the function $f:\mathbb{R}^n\to(-\infty,+\infty]$ is consistently assumed to be convex, continuous, and proper function.

The following definition is taken from [9,10].

Definition 2.1. *Consider a function $f:\mathbb{R}^n\to(-\infty,+\infty]$ that is a convex, continuous, and proper function, and let $u\in\mathbb{R}^n$. The proximity of u in f is the unique point $\mathrm{Prox}_f(u)\in\mathbb{R}^n$ such that*

$$\mathrm{Prox}_f(u)=\arg\min_{x\in\mathbb{R}^n}\left(\frac{1}{2}\|x-u\|^2+f(x)\right).\tag{2.1}$$

In particular, if C is a nonempty, closed, and convex set, and $f=\iota_C$ where $\iota_C(x)=0$ if $x\in C$ and $+\infty$ otherwise, then $\mathrm{Prox}_f(u)=P_C(u)$ where $x_0=P_C(u)$ is a unique point in C such that $\inf\limits_{x\in C}\|x-u\|=\|x_0-u\|$.

It is well-known that if $g=f+\frac{\alpha}{2}\|\cdot-u\|^2+\langle\cdot,v\rangle+\beta$ where $\alpha\geq0$ and $\beta\in\mathbb{R}$, then $\mathrm{Prox}_g(x)=\mathrm{Prox}_{(\alpha+1)^{-1}f}\Big((\alpha+1)^{-1}(x+(\alpha u-v))\Big)$ (see Proposition 24.8 of [9]). Consequently, if $\alpha=0$, $f=\iota_C$, then $\mathrm{Prox}_g(x)=\mathrm{P}_C(x-v)$ where C is a nonempty, closed, and convex set. The following is vital in proving the main result of this study.

Lemma 2.1. *Let $r_1,r_2,...,r_n\in\mathbb{R}$, and let C be a closed and convex subset of $\mathbb{R}^n$. Consider the minimization problem:*

$$\min_{(x_1,x_2,...,x_n)\in C}\left(\sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}x_ir_i\right).\tag{2.2}$$

If C is nonempty, then the problem in (2.2) has exactly one solution. Furthermore, $P_C(-\frac{1}{2}v)$ is the unique solution to (2.2), where $v = (r_1, r_2, ..., r_n)$. If C is empty, then the problem in (2.2) has no solution.

Proof. If C is empty, then the problem in (2.2) has no solution. Assume C is nonempty. Let $x = (x_1, x_2, ..., x_n)$ and $v = (r_1, r_2, ..., r_n)$. Then, the problem in (2.2) can be expressed as

$$\min_{x \in \mathbb{R}^n} \left(\frac{1}{2} \|x\|^2 + \frac{1}{2} \langle x, v \rangle + \iota_C(x) \right). \tag{2.3}$$

The definition (2.1) guarantees that the problem in (2.3) has a unique solution

$$\mathrm{Prox}_g(0) = \arg\min_{x \in \mathbb{R}^n} \left(\frac{1}{2} \|x\|^2 + \langle x, \frac{1}{2} v \rangle + \iota_C(x) \right) \tag{2.4}$$

where $g(x) = \frac{1}{2}\langle x, v \rangle + \iota_C(x)$. Applying Proposition 24.5 of [1] by setting $\alpha = 0$ and $f = \iota_C$, we obtain

$$\mathrm{Prox}_g(0) = \mathrm{Prox}_{\iota_C}\left(-\frac{1}{2} v \right) = P_C\left(-\frac{1}{2} v \right).$$

□

The following theorem is the main result of this study.

Theorem 2.1. *The $(\Omega, C, 2)$-supply security problem has at most one solution.*

Proof. Let C be a nonempty, closed, and convex subset of $\mathbb{R}^n$. We will show that the $(\Omega, C, 2)$-supply security problem has at most one solution. Indeed, let $r : \Omega \to [1, +\infty)$ be a solution to the $(\Omega, C, 2)$-supply security problem. Then $r : \Omega \to [1, +\infty)$ is also a solution to the minimization problem

$$\min_{t : \Omega \to [1, +\infty)} \left(\sum_{i=1}^{n} (\tau(u_i) - \tau(u_i) t(u_i))^2 + \sum_{i=1}^{n} \tau(u_i) \theta(u_i) t(u_i) - \sum_{i=1}^{n} \tau(u_i) t(u_i) \right) \tag{2.5}$$

subject to $x(t) \in C$. Set $x_i = \tau(u_i) - \tau(u_i) t(u_i)$. Then, the minimization problem in (2.5) will becomes

$$\min_{(x_1, x_2, ..., x_n) \in C} \left(\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} x_i (\theta(u_i) - 1) \right). \tag{2.6}$$

Therefore, the solution of the $(\Omega, C, 2)$-supply security problem is uniquely determined by the solution of the minimization problem in (2.6). However, from Lemma 2.1, the minimization problem in (2.6) has a unique solution. Therefore, the $(\Omega, C, 2)$-supply security problem has at most one solution. □

The following results delve into the formula for the solution to the $(\Omega, C, 2)$-supply security problem.

Corollary 2.1. *If the solution of the $(\Omega, C, 2)$-supply security problem exists, then the explicit formula for the solution is given by*

$$r(u_i) = \frac{\tau(u_i) - \langle P_C(-\frac{1}{2} v), e_i \rangle}{\tau(u_i)} \tag{2.7}$$

where $v = (\theta(u_1) - 1, \theta(u_2) - 1, ..., \theta(u_n) - 1)$ *and* $e_i = (0, 0, ..., 1..., 0, 0)$ *is the vector where the ith coordinate is 1 and 0 elsewhere. Furthermore,* $P_C(-\frac{1}{2}v) = (x_1, x_2, ..., x_n)$ *if and only if* $(x_1, x_2, ..., x_n)$ *is the unique solution to the minimization problem in (2.6).*

Proof. Recall from the proof of Theorem 2.1 that $r : \Omega \to [1, +\infty)$ is the solution to the $(\Omega, C, 2)$-supply security problem if and only if $(x_1, ..., x_n)$ is the solution to the problem in (2.6), where $x_i = \tau(u_i) - \tau(u_i)r(u_i)$. By invoking Lemma 2.1, $(x_1, ..., x_n) = P_C(-\frac{1}{2}v)$, where $v = (\theta(u_1) - 1, \theta(u_2) - 1, ..., \theta(u_n) - 1)$. Therefore, $\langle P_C(-\frac{1}{2}v), e_i\rangle = \tau(u_i) - \tau(u_i)r(u_i)$, as desired. □

In view of Lemma 2.1, we have the following definition.

Definition 2.2. *The pseudo-solution of the* $(\Omega, C, 2)-$*supply security problem is given by*

$$r^*(u_i) = \frac{\tau(u_i) - \langle P_C(-\frac{1}{2}v), e_i\rangle}{\tau(u_i)} \tag{2.8}$$

where $v = (\theta(u_1) - 1, \theta(u_2) - 1, ..., \theta(u_n) - 1)$ *and* $e_i = (0, 0, .., 1, .., 0, 0)$ *is the vector where the ith coordinate is 1 and 0 elsewhere.*

Remark 2.1. *If the* $(\Omega, C, 2)$*-supply security problem has a solution* $r : \Omega \to [1, +\infty)$*, then it is unique, and its solution is its pseudo-solution. Consequently, the* $(\Omega, C, 2)$*-supply security problem has a solution if and only if the pseudo-solution of* $(\Omega, C, 2)$*-supply security problem is its solution.*

3. Real Life Applications

This section is devoted to the discussion of the basic application of the $(\Omega, C, 2) = (\Omega, C)$-supply security problem.

3.1. **Application to Basic Supply Problem without Budget Restriction.**

Let $\Omega = \{u_1, u_2, \ldots, u_n\}$ be the set of n products. Define $\tau(u_i)$ as the forecasted demand quantity, and $\theta(u_i)$ as the forecasted cost of production per unit of product u_i. Suppose the forecasted supply of product u_i falls within the interval $[m_i, M_i]$. Consider the cross-product of intervals defined by:

$$C = \prod_{i=1}^{n} [\tau(u_i) - M_i, \tau(u_i) - \max\{m_i, \tau(u_i)\}] \subset \mathbb{R}^n. \tag{3.1}$$

Note that C is a closed and convex subset of $\mathbb{R}^n$. Furthermore, if C is nonempty, then the (Ω, C)-supply security problem has at most one solution. The (Ω, C)-supply security problem aims to determine the optimal supply quantity $\tau(u_i)r(u_i)$ for each product u_i to prevent shortages, meaning that the supply quantity must be greater than or equal to the forecasted demand. This is expressed as $\tau(u_i)r(u_i) \geq \tau(u_i)$, which implies $r(u_i) \geq 1$ for all $u_i \in \Omega$. Thus, we require that r is a function from Ω to $[1, +\infty)$. Given that $r(u_i) \geq 1$ and $\tau(u_i)r(u_i) \in [m_i, M_i]$, we can deduce that $\tau(u_i) - \tau(u_i)r(u_i) \in [\tau(u_i) - M_i, \tau(u_i) - \max\{m_i, \tau(u_i)\}]$, leading to $x(r) \in C$, where

$$x(r) = (\tau(u_1) - \tau(u_1)r(u_1), \tau(u_2) - \tau(u_2)r(u_2), ..., \tau(u_n) - \tau(u_n)r(u_n)).$$

Now, our goal is to find $r : \Omega \to [1, +\infty)$ to minimize oversupply. Thus, $r : \Omega \to [1, +\infty)$ is the solution to the minimization problem:

$$\min_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} (\tau(u_i) - \tau(u_i)t(u_i))^2 + \iota_C(x(t)) \right). \tag{3.2}$$

Additionally, we aim to minimize the total cost of production, meaning that $r : \Omega \to [1, +\infty)$ is the solution to the minimization problem:

$$\min_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} \tau(u_i)\theta(u_i)t(u_i) + \iota_C(x(t)) \right). \tag{3.3}$$

Finally, we seek to maximize supply, making $r : \Omega \to [1, +\infty)$ the solution to the minimization problem:

$$\max_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} \tau(u_i)t(u_i) - \iota_C(x(t)) \right). \tag{3.4}$$

Summarizing equation (3.2) to (3.4), then we recover the problem in Definition 1.1. If the (Ω, C)-supply security problem has no solution, then the strategic solution is given by:

$$r(u_i) = \frac{\tau(u_i) - \langle P_C(-\frac{1}{2}v), e_i \rangle}{\tau(u_i)} \tag{3.5}$$

where $v = (\theta(u_1) - 1, \theta(u_2) - 1, \dots, \theta(u_n) - 1)$, and $e_i = (0, 0, \dots, 1, \dots, 0, 0)$ is the vector with the ith coordinate as 1 and 0 elsewhere. To calculate $\langle P_C(-\frac{1}{2}v), e_i \rangle$, set $a_i = \tau(u_i) - M_i$ and $b_i = \tau(u_i) - \max\{m_i, \tau(u_i)\}$. Since C is a cross-product of intervals defined in (3.1), then

$$\left\langle P_C\left(-\frac{1}{2}v\right), e_i \right\rangle = P_{[a_i,b_i]}\left(-\frac{1}{2}(\theta(u_i) - 1)\right). \tag{3.6}$$

Thus, from the definition of the projection operator in Definition 2.1, we have

$$\left\langle P_C\left(-\frac{1}{2}v\right), e_i \right\rangle = \begin{cases} a_i & \text{if } \frac{1}{2}(1 - \theta(u_i)) \le a_i \\ b_i & \text{if } \frac{1}{2}(1 - \theta(u_i)) \ge b_i \\ \frac{1}{2}(1 - \theta(u_i)) & \text{otherwise.} \end{cases} \tag{3.7}$$

In view of Equation (3.5) and (3.7), we will have the following Rule for the Basic Supply Problem without Budget Restriction:

Rule 1. If twice the difference between the forecasted maximum supply (M_i) and the forecasted demand ($\tau(u_i)$) is less than or equal to the difference between the forecasted cost ($\theta(u_i)$) and 1, then the proposed supply quantity ($r(u_i)\tau(u_i)$) equals the forecasted maximum supply. More precisely,

$$2(M_i - \tau(u_i)) \le \theta(u_i) - 1 \implies r(u_i)\tau(u_i) = M_i.$$

Rule 2. If two times the difference between the larger of the forecasted minimum supply (m_i) and the forecasted demand ($\tau(u_i)$) and the forecasted demand is greater than or equal to the difference between the forecasted cost ($\theta(u_i)$) and 1, then the proposed supply quantity ($r(u_i)\tau(u_i)$) equals the forecasted minimum supply. More precisely,

$$2(\max\{\tau(u_i), m_i\} - \tau(u_i)) \geq \theta(u_i) - 1 \implies r(u_i)\tau(u_i) = m_i.$$

Rule 3. If the condition in Rule 1 and Rule 2 does not hold, then the proposed supply quantity ($r(u_i)\tau(u_i)$) is equal to $\tau(u_i) + 0.5\theta(u_i) - 0.5$.

Example 1. The following table summarizes a list of products along with their forecasted demand, maximum supply, minimum supply, and production cost in a certain city for one month. Determine the optimal proposed supply quantity for each product using the Rule for the Basic Supply Problem without budget restrictions.

Products (u_i)	Forecasted Demand	Cost of Production per kg	m_i	M_i
Corn	1980 kg	Php 10	1000 kg	2000 kg
Rice	150000 kg	Php 25	125000 kg	175000 kg
Potato	1322 kg	Php 20	1400 kg	5000 kg
Onion	1060 kg	Php 50	500 kg	1100 kg
Garlic	1050 kg	Php 40	600 kg	1100 kg

Applying the policy in Rule 1-3, we obtain the following.

Products (u_i)	Forecasted Demand	Cost of Production	Proposed Supply Quantity
Corn	1980 kg	Php 10	1984.5
Rice	150000 kg	Php 25	150012
Potato	1322 kg	Php 20	1400
Onion	1060 kg	Php 50	1084.5
Garlic	1050 kg	Php 40	1069.5

3.2. **Application to Supply Problem with Budget Restriction.**

With the same information provided in section 3.1, suppose that the total budget is M, and therefore, the total cost of production is at most M. Set

$$D = \left\{(x_1, x_2, ..., x_n) : \sum_{i=1}^{n} x_i\theta(u_i) \geq \sum_{i=1}^{n} \tau(u_i)\theta(u_i) - M\right\}.$$

Let $E = C \cap D$, which is closed and convex since both C and D are closed and convex. Similar to section 3.1, we want to find $r : \Omega \to [1, +\infty)$ so that the oversupply is minimized. In other words, $r : \Omega \to [1, +\infty)$ is the solution to the minimization problem

$$\min_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} (\tau(u_i) - \tau(u_i)t(u_i))^2 + \iota_E(x(t))\right). \tag{3.8}$$

Furthermore, we want to minimize the total cost of production. This means that $r : \Omega \to [1, +\infty)$ is the solution to the minimization problem

$$\min_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} \tau(u_i)\theta(u_i)t(u_i) + \iota_E(x(t)) \right). \tag{3.9}$$

Lastly, we want to maximize the supply. In other words, $r : \Omega \to [1, +\infty)$ is the solution to the minimization problem

$$\max_{t:\Omega\to[1,+\infty)} \left(\sum_{i=1}^{n} \tau(u_i)t(u_i) - \iota_E(x(t)) \right). \tag{3.10}$$

Summarizing equations (3.8) to (3.10), we return to the problem defined in Definition 1.1. If the (Ω, E)-supply security problem has no solution, then the strategic solution is given by

$$r(u_i) = \frac{\tau(u_i) - \langle P_E(-\frac{1}{2}v), e_i\rangle}{\tau(u_i)} \tag{3.11}$$

where $v = (\theta(u_1) - 1, \theta(u_2) - 1, ..., \theta(u_n) - 1)$ and $e_i = (0, 0, ..1, .., 0, 0)$ is the vector where the ith coordinate is 1 and 0 elsewhere.

Example 2. The following table summarizes the list of products with the forecast number of demand, maximum supply, minimum supply, and cost of production of a certain city in one month. Suppose that the total budget is Php $3,894,000.00$. Determine the optimal proposed supply quantity by applying the formula in (3.11).

Products (u_i)	Forecast Demand	Cost of Production per kg	m_i	M_i
Corn	1980 kg	Php 10	1000 kg	2000 kg
Rice	150000 kg	Php 25	125000 kg	175000 kg
Potato	1322 kg	Php 20	1400 kg	5000 kg
Onion	1060 kg	Php 50	500 kg	1100 kg
Garlic	1050 kg	Php 40	600 kg	1100 kg

Solution: From (3.11), $r(u_i) = \frac{\tau(u_i) - \langle P_E(-\frac{1}{2}v), e_i\rangle}{\tau(u_i)}$. Applying Corollary 2.1, $P_E(-\frac{1}{2}v) = (x_1, x_2, ..., x_5)$ is the solution to the minimization problem

$$\min_{(x_1,x_2,...,x_5)\in\mathbb{R}^5} \left(\sum_{i=1}^{5} x_i^2 + \sum_{i=1}^{5} x_i(\theta(u_i) - 1) \right)$$

subject to
1. $x_i \in [\tau(u_i) - M_i, \tau(u_i) - \max\{m_i, \tau(u_i)\}]$
2. $10x_1 + 25x_2 + 20x_3 + 50x_4 + 40x_5 \geq -2760.$

Applying the Lagrange multiplier, we obtain the approximate solution $P_E(-\frac{1}{2}v) \approx$ (-2.11658, -6.04145, -78, -12.5829, -9.96632). The following table summarizes the optimal proposed supply for each product by applying the formula in (3.11).

Products (u_i)	Forecasted Demand	Cost of Production	**Proposed Supply Quantity**
Corn	1980 kg	Php 10 per kg	1982.11658 kg
Rice	150000 kg	Php 25 per kg	150006.04145 kg
Potato	1322 kg	Php 20 per kg	1400 kg
Onion	1060 kg	Php 50 per kg	1072.5829 kg
Garlic	1050 kg	Php 40 per kg	1059.96632 kg

The proposed total cost of production is $1982.11658(10) + 150006.04145(25) + 1400(20) + 1072.5829(50) + 1059.96632(40) = 3894000$, as desired.

4. Comment

Naturally, one might consider the following optimization problems:

$$\min_{t:\Omega\to[0,+\infty),t\geq\tau}\left(\sum_{i=1}^{n}(\tau(u_i)-t(u_i))^2+\iota_C(u(t))\right)$$
$$\min_{t:\Omega\to[0,+\infty),t\geq\tau}\left(\sum_{i=1}^{n}t(u_i)\theta(u_i)+\iota_C(u(t))\right)$$
$$\max_{t:\Omega\to[0,+\infty),t\geq\tau}\left(\sum_{i=1}^{n}t(u_i)-\iota_C(u(t))\right).$$

Here, $u(t) = (\tau(u_1) - t(u_1), \tau(u_2) - t(u_2), \dots, \tau(u_n) - t(u_n))$, and it is used to solve the supply security problem. The solution to the problem above is given by $s : \Omega \to [0, +\infty)$, where $s(u) = r(u)\tau(u)$, and $r : \Omega \to [1, +\infty)$ is the solution to the problem defined in Definition 1. Definition 1 offers the distinct advantage of allowing us to explicitly calculate the supply-demand ratio. The definition of the projection operator implies that we can calculate the function $r : \Omega \to [1, +\infty)$ using the formula

$$r^*(u_i) = \frac{\tau(u_i) - x_i}{\tau(u_i)}, \tag{4.1}$$

where $(x_1, x_2, ..., x_n)$ is the unique solution to the optimization problem

$$\min_{(x_1,x_2,\dots,x_n)\in C}\sum_{i=1}^{n}(x_i - 0.5 + 0.5\theta(u_i))^2. \tag{4.2}$$

The challenge in this study lies in converting the supply security problem into a convex optimization problem.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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