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The q-Rung Simplified Neutrosophic Soft Set: Its Similarity Measure

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Abstract. In this era, the literature has paid increasing attention to uncertainty measures, particularly among researchers and experts who work with imprecise, incomplete, or vague data, which is common in real-world contexts affected by ongoing environmental and economic fluctuations. Numerous mathematical tools have been proposed to address uncertainty, including the q-rung orthopair neutrosophic set (q-RONS). This model significantly extends the capabilities of earlier frameworks, such as the Intuitionistic Fuzzy Set, Pythagorean Set, and Fermatian Set, by overcoming limitations caused by their restriction to powers less than or equal to 3. In q-RONS, the sum of the q-th powers of the truth-membership (T) and falsity-membership degrees (F) is less than or equal to 1, allowing for a broader representation of uncertain information. Despite the enhanced flexibility and effectiveness of q-RONS in handling uncertain data, existing approaches fall short in efficiently dealing with nominal-valued parameters and standard evidence required for decision-making applications. In this paper, we propose a novel hybrid structure called the q-rung simplified neutrosophic soft set (q-RSNSS), which generalizes previous models. We introduce the formal definition of our proposed model, along with its foundational concepts and mathematical properties, including operations such as union, intersection, and complement, supported by rigorous proofs. We also define fundamental algebraic operations between two neutrosophic numbers. Furthermore, we develop an algorithmic approach to multi-criteria decision-making (MCDM) under a similarity measures environment.

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1. Introduction

Multi-Criteria Decision Making (MCDM) is one of the worthy branches of operations research, with wide-ranging applications across engineering, social sciences, economics, and healthcare. It involves evaluating or ranking a finite number of alternatives based on multiple - often conflicting - criteria. Over recent years, MCDM and its fuzzy extensions have become the focus of numerous theoretical and applied studies, particularly in response to the growing complexity of real-world decision-making scenarios. Due to its practical and applied importance, it has become the cornerstone of decision-making science, as it works to provide appropriate alternatives that are consistent with the user's mentality. Decision-making with these technologies is often accompanied by a lot of uncertainty that arises from the complexity of daily life and the complexity of the alternatives used. Recently, there has been a growing interest in discovering mathematical techniques that work in conjunction with decision-making techniques to overcome uncertainty. To meet this demand, Zadeh [1] introduced an innovative mathematical concept called a fuzzy set (FS) to overcome the doubts and uncertainty associated with decision-making data. Unlike classical (crisp) sets, where elements either belong (the values equal 1) or do not belong (the values equal 0), fuzzy sets assign a membership grade ranging between 0 and 1. To better capture uncertainty when defining precise membership values is difficult, Atanassov [2] introduced the concept of IFS a generalization of fuzzy sets. In IFS, each element is characterized by independent membership and non-membership degrees, with a hesitation margin representing the uncertainty. Here, it is worth noting that if the false part equals zero, then the concept of IFS is converted into an FS. However, this concept lacks the poverty of specifying the parameters that characterize the values of alternatives. To meet this requirement, Molodtsov [3] presented the SS theory as a parameter-rich tool that characterises alternatives by their parameters. This tool was characterized by its high simplicity and clarity, which is why it attracted the attention of many researchers, one of whom was Smarandache [4] when he presented NSs as a generalization of FS and IFS. The NSS is a powerful tool for dealing with alternatives characterized by parametric parameters by giving three fuzzy states with values ranging from 0 to 1, where these states represent the principles of truth, falsehood, and neutrality in a mathematical way.

As an extension of the IFS, Yager [5] launched the PyFS with one property: that the value of the squares of both the truth and non-truth functions does not exceed one. This model is used brilliantly to address widespread uncertainty in applications. Ma and Xu [6] investigated several PyFS symmetric operators to address complex issues. Based on several metrics, including the distance function and score, Huang et al. [7] put up an MADM model. Nevertheless, in scenarios with strong conflicting preferences such as consumer choices influenced by economic contexts PyFS and IFS are shortened when both membership and non-membership degrees are high simultaneously. To overcome this restriction, Yager [8] put forward the q-ROFS, offering more flexibility, where the value q-th powers of the membership and non-membership degrees, where the sum to at almost all one, offering more flexibility as q increases. The information range of q-ROFS is wider than that of IFS and PyFS due to the multiple degrees of q. Therefore, q-ROFS is one of the most well-organised and implemented tools for curing lofty-level vagueness in daily life. Further mathematical developments by Peng [9] , Shu [10] , Ai [11] , Gao [12]. Al-Quran et al. [13] ,a novel hybrid model termed the q-rung simplified neutrosophic set (q-RSN), which generalizes existing intuitionistic, Pythagorean, and Fermatean neutrosophic frameworks. In this model, truth and falsity membership degrees are dependent and satisfy a q-rung orthopair condition, while indeterminacy remains independent. This led to the generalized framework of q-RSNs, which unifies previous models - like simplified neutrosophic sets(NS) ($q = 1$) [14], PyNS ($q = 2$) [15] , FNS ($q = 3$) [16] and q-RONS [17]- into a single adaptable structure. Hence, q-RSNs provide a comprehensive and customizable approach for expressing imprecise, indeterminate, and inconsistent information in increasingly complex decision-making environments.

The measurement of uncertainty is a crucial topic in uncertainty-related theories. This includes similarity measures, distance measures, and entropy, which have found widespread applications in image processing, clustering, pattern recognition, and case-based reasoning [18–21]. Majumdar and Samanta [22] initiated the exploration of uncertainty measures for soft sets and fuzzy soft sets, proposing initial similarity measures. Later, Kharal [23] introduced operations based on similarity and distance, applying them to corporate financial diagnosis. Jiang et al. [24] extended these ideas to intuitionistic fuzzy soft sets and interval-valued fuzzy soft sets, while Wang and Qu [25] proposed related measures for vague soft sets. Despite this progress, foundational issues remain. For example, fuzzy soft sets rely on different parameter sets, making existing similarity and distance measures only partially applicable. Additionally, Min [26] proposed a notion of similarity based on mappings between parameter sets, though it doesn't fully capture true similarity. Both entropy and similarity measures are vital for evaluating the degree of uncertainty and resemblance between sets. De Luca and Termini [27] formulated axioms for fuzzy entropy. Pappis and his colleagues [28, 29] approached similarity axiomatically and extensively explored it, more so than entropy. Liu et al. [30] proposed a new category of similarity measures and entropies is presented based on fuzzy equivalence. These measures have been extended to multiple set types, such as interval-valued fuzzy sets [31], fuzzy soft sets [32–35], and intuitionistic fuzzy soft sets [36], proving essential in various fields like decision-making, image processing, and pattern recognition.

The main contributions of this work are distributed as follows:

(1) First, we establish our proposed concept of q-RSNSS, which utilizes the features present in q-RSN and SS.
(2) Second, we move towards incorporating more fundamental features (basic operations) of this concept, such as union, intersection, and complement, which work efficiently in dealing with fuzzy data. We also work on verifying some mathematical properties related to set theory, such as De Morgan's property.
(3) Third, we work on building highly efficient similarity tools for dealing with q-RSNSS data.

(4) Fourth, we apply the proposed similarity tools to a common everyday application related to widely known decision theory.

1.1. **Article Sections.** This article is organized into five main parts, distributed as follows: Section 2 revisits some previous concepts closely related to our proposed concept. In Section 3, we provide the formal definition of our proposed concept as an extension of the concepts mentioned in Section 2, in addition to the basic processes defined within it. In Section 4, we continue to develop new results for our proposed concept, along with some numerical examples to illustrate the workings. In Section 5, we present the basic definitions of some two-dimensional similarity techniques that we used in this section to address a decision-making problem.

2. Primary Document

In this section we see many basic concepts initiated in previous studies.

Definition 2.1. *Let $\mathring{U}$ be a universe of discourse. Then, $\mathring{\psi} = \left\{\left(\ddot{x}, \langle \mathcal{T}_{\mathring{\psi}}(\ddot{x}), \mathcal{I}_{\mathring{\psi}}(\ddot{x}), \mathcal{F}_{\mathring{\psi}}(\ddot{x})\rangle\right) : \ddot{x} \in \mathring{U}\right\}$, is called*

(1) *a simplified neutrosophic set (NS) ([14]) if $0 \le \mathcal{T}_{\mathring{\psi}} + \mathcal{I}_{\mathring{\psi}} + \mathcal{F}_{\mathring{\psi}} \le 3$.*
(2) *A Pythagorean neutrosophic set (PyNS) ([15]) if $0 \le (\mathcal{T}_{\mathring{\psi}})^2 + (\mathcal{F}_{\mathring{\psi}})^2 \le 1$ or $0 \le (\mathcal{T}_{\mathring{\psi}})^2 + (\mathcal{I}_{\mathring{\psi}})^2 + (\mathsf{F}_{\mathring{\psi}})^2 \le 2$*
(3) *a Fermatean neutrosophic set (FNS) ([16]) if $0 \le (\mathcal{T}_{\mathring{\psi}})^3 + (\mathcal{F}_{\mathring{\psi}})^3 \le 1$ or $0 \le (\mathcal{T}_{\mathring{\psi}})^3 + (\mathcal{I}_{\mathring{\psi}})^3 + (\mathcal{F}_{\mathring{\psi}})^3 \le 2$*
(4) *a q-rung orthopair neutrosophic set $(q - RONS)$ ([17]) if $0 \le (\mathcal{T}_{\mathring{\psi}})^q + (\mathcal{F}_{\mathring{\psi}})^q \le 1$ or $0 \le (\mathcal{T}_{\mathring{\psi}})^q + (\mathcal{I}_{\mathring{\psi}})^q + (\mathcal{F}_{\mathring{\psi}})^q \le 2$ for $q \ge 1$.*

Definition 2.2. *[3] Let $\mathring{U} = \{\ddot{x}_1, \ddot{x}_2, ..., \ddot{x}_n\}$ and $\mathring{E} = \{\mathring{e}_1, \mathring{e}_2, ..., \mathring{e}_n\}$ be a reference set and attribute set, respectively. Then a soft set (SS) over $\mathring{U}$ given as structures as follows:*

$$\mathring{A} = \left\{(\mathring{e}, \langle \mathsf{A}(\mathring{e})\rangle) : \mathring{e} \in \mathring{E}\right\},$$

where the function $\mathring{A}$ given by following mapping:

$$\mathring{A} : \mathring{E} \to P^{\mathring{U}}$$

Here $P^{\mathring{U}}$ refer to collection of subsets of reference set $\mathring{U}$.

Definition 2.3. *[13] Let $\mathring{U}$ denote the universal set. A $\mathfrak{q}$-RSN set $\mathring{K}$ in $\mathring{U}$ is a set of ordered sequences*

$$\mathring{K} = \left\{\left(\ddot{x}, \langle \mathcal{T}_{\mathring{K}}(\ddot{x}), \mathcal{I}_{\mathring{K}}(\ddot{x}), \mathcal{F}_{\mathring{K}}(\ddot{x})\rangle\right) : \ddot{x} \in \mathring{U}\right\}.$$

Here, $\mathcal{T}_{\mathring{K}} : \mathring{U} \to [0,1]$, $\mathcal{I}_{\mathring{K}} : \mathring{U} \to [0,1]$, and $\mathcal{F}_{\mathring{K}} : \mathring{U} \to [0,1]$ are identified as the membership values corresponding to the concepts of truth, indeterminacy, and falsity, respectively, where $0 \le (\mathcal{T}_{\mathring{K}}(\ddot{x}))^{\mathfrak{q}} + (\mathcal{F}_{\mathring{K}}(\ddot{x}))^{\mathfrak{q}} \le 1$, $\mathfrak{q} \ge 1$ for every $\ddot{x} \in \mathring{U}$. The components $\mathcal{T}_{\mathring{K}}(\ddot{x})$ and $\mathcal{F}_{\mathring{K}}(\ddot{x})$ are dependent and the component $\mathcal{I}_{\mathring{K}}(\ddot{x})$ is independent. Consequently, this tuple adheres to restriction: $0 \le (\mathcal{T}_{\mathring{K}}(\ddot{x}))^{\mathfrak{q}} + \mathcal{I}_{\mathring{K}}(\ddot{x}) + (\mathcal{F}_{\mathring{K}}(\ddot{x}))^{\mathfrak{q}} \le 2$ and naturally the restriction $0 \le (\mathcal{T}_{\mathring{K}}(\ddot{x}))^{\mathfrak{q}} + (\mathcal{I}_{\mathring{K}}(\ddot{x}))^{\mathfrak{q}} + (\mathcal{F}_{\mathring{K}}(\ddot{x}))^{\mathfrak{q}} \le 2$.

Definition 2.4. *[13] Let* $\mathring{K} = \left\{\left(\ddot{x}, \langle \mathcal{T}_{\mathring{K}}(\ddot{x}), \mathcal{I}_{\mathring{K}}(\ddot{x}), \mathcal{F}_{\mathring{K}}(\ddot{x})\rangle\right) : \ddot{x} \in \mathring{U}\right\}$ *be a* q*-RSN set over* $\mathring{U}$*.Then,*

(1) $\mathring{K}$ *is referred to as an absolute* q*-RSN set, denoted by* $\mathring{K}_{||}$*, if* $\mathcal{T}_{\mathring{K}}(\ddot{x}) = 1$*, and* $\mathcal{I}_{\mathring{K}}(\ddot{x}) = \mathcal{F}_{\mathring{K}}(\ddot{x}) = 0$*, i.e.,* $\mathring{K}_{||} = \langle 1, 0, 0\rangle$*.*

(2) $\mathring{K}$ *is referred to as a null* q*-RSN set, denoted by* $\mathring{K}$*, if* $\mathcal{T}_{\mathring{K}}(\ddot{x}) = 0$*, and* $\mathcal{I}_{\mathring{K}}(\ddot{x}) = \mathcal{F}_{\mathring{K}}(\ddot{x}) = 1$*, i.e.,* $\mathring{K} = \langle 0, 1, 1\rangle$*.*

3. q-Rung Simplified Neutrosophic Soft Set

In this section, we will provide the main definition of our proposed concept q-RSNSS via integrating both SS and q-RSNS into one model. Based on this mathematical model, we initiated the fundamental set theory operations that can be use in given more reuislts.

Definition 3.1. *Let* $\mathring{U} = \{\mathring{\ddot{x}}_1, \mathring{\ddot{x}}_2, ..., \mathring{\ddot{x}}_n\}$ *be a limited underlying set accompanied by* $\mathring{\mathtt{E}} = \{\mathring{e}_1, \mathring{e}_2, ..., \mathring{e}_n\}$ *as the assemblage of attributes. Let* $\mathring{\mathtt{A}} \neq \emptyset \subseteq \mathring{\mathtt{E}}$ *and* $q - RSN^{\mathring{U}}$ *represent the aggregate of all* $q - RSN$ *subsets over* $\mathring{U}$*. A pair* $(\mathring{\varphi}, \mathring{\mathtt{A}})$ *or* $\mathring{\varphi}_{\mathring{\mathtt{A}}}$ *is called a* $q - RSNSS$ *over* $\mathring{U}$*, where*

$$\mathring{\varphi} : \mathring{\mathtt{A}} \to q - RSN^{\mathring{U}}$$

is mapping, and is given as

$$\begin{aligned}(\mathring{\varphi}, \mathring{\mathtt{A}}) &= \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})\rangle\right\}\right) : \mathring{e} \in \mathring{\mathtt{A}}, \mathring{\ddot{x}} \in \mathring{U}\right\} \\ &= \left\{\left(\mathring{e}, \left\{\frac{\mathring{\ddot{x}}}{\langle \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})\rangle}\right\}\right) : \mathring{e} \in \mathring{\mathtt{A}}, \mathring{\ddot{x}} \in \mathring{U}\right\} \\ &= \left\{\left(\mathring{e}, \left\{\frac{\langle \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})\rangle}{\mathring{\ddot{x}}}\right\}\right) : \mathring{e} \in \mathring{\mathtt{A}}, \mathring{\ddot{x}} \in \mathring{U}\right\}\end{aligned}$$

where $\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}} : \mathring{U} \to [0, 1]$*,* $\mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}} : \mathring{U} \to [0, 1]$*, and* $\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}} : \mathring{U} \to [0, 1]$ *are three mapping along with the property*

$$0 \leq [\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})]^{\mathfrak{q}} + [\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})]^{\mathfrak{q}} \leq 1 \qquad (\mathfrak{q} \geq 1).$$

Here $\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})$ *and* $\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})$ *are identified as the membership values corresponding to the concepts of truth, indeterminacy, and falsity, respectively.The components* $\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})$ *and* $\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})$ *are dependent and the component* $\mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})$ *is independent. Consequently, this tuple adheres to restriction:* $0 \leq (\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}))^{\mathfrak{q}} + \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}) + (\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}))^{\mathfrak{q}} \leq 2$ *and naturally the restriction* $0 \leq (\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}))^{\mathfrak{q}} + (\mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}))^{\mathfrak{q}} + (\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}))^{\mathfrak{q}} \leq 2$*.*

For the $q - RSNSS$ $\mathring{\varphi}_{\mathring{\mathtt{A}}} = \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})\rangle\right\}\right) : \mathring{e} \in \mathring{\mathtt{A}}, \mathring{\ddot{x}} \in \mathring{U}\right\}$*, which is three components* $\left\langle \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\ddot{x}})\right\rangle$ *are said to q-Rung simplified neutrosophic numbers* $(q - RSNNs)$ *and each* $q - RSNN$ *can be denoted by* $\left\langle \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}, \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}, \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}\right\rangle$*.*

Example 3.1. *Consider* $\mathring{U} = \{\mathring{\ddot{x}}_1, \mathring{\ddot{x}}_2\}$ *(e.g., two mobile phones) and* $\mathring{\mathtt{E}} = \{\mathring{e}_1 = cheap, \mathring{e}_2 = durable\}$*, also assume that* $\mathring{\mathtt{A}} = \{\mathring{e}_1 = cheap\}$ *and* $q = 2$*.*

We define a $q - RSNSS$ $\mathring{\varphi}_{\mathring{\mathtt{A}}}$ *over* $\mathring{U}$ *as:*

$$\mathring{\varphi}_{\mathring{A}} = \left\{ \left(\mathring{e}_1 = cheap, \begin{cases} \mathring{\ddot{x}}_1 \mapsto (0.3, 0.2, 0.6) \\ \mathring{\ddot{x}}_2 \mapsto (0.4, 0.1, 0.7) \end{cases} \right) \right\}$$

Now, Verify the q-Rung condition *($q = 2$):*

- *For $\mathring{\ddot{x}}_1$: $0.3^2 + 0.6^2 = 0.09 + 0.36 = 0.45 \leq 1 \quad \Rightarrow$ Valid*
- *For $\mathring{\ddot{x}}_2$: $0.4^2 + 0.7^2 = 0.16 + 0.49 = 0.65 \leq 1 \quad \Rightarrow$ Valid*

Hence, this is a valid $q-RSNSS$.

Remark 3.1. *Let $ⓣ_{ij} = \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{e}_i)(\mathring{\ddot{x}}_i)$, $ⓘ_{ij} = \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{e}_i)(\mathring{\ddot{x}}_i)$ and $ⓕ_{ij} = \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{e}_i)(\mathring{\ddot{x}}_i)$ be $q-RSNS$ set $\mathring{\varphi}_{\mathring{A}}$, where $i = 1, 2, 3, ..., m$, $j = 1, 2, 3, ..., n$. Then,the tabular form of $q-RSNSS$ $\mathring{\varphi}_{\mathring{A}}$ is given in Table 1.*

$\mathring{\varphi}_{\mathring{A}}$	$\mathring{e}_1$	$\mathring{e}_2$	$\cdots$	$\mathring{e}_n$
$\mathring{\ddot{x}}_1$	$(ⓣ_{11}, ⓘ_{11}, ⓕ_{11})$	$(ⓣ_{12}, ⓘ_{12}, ⓕ_{12})$	$\cdots$	$(ⓣ_{1n}, ⓘ_{1n}, ⓕ_{1n})$
$\mathring{\ddot{x}}_2$	$((ⓣ_{21}, ⓘ_{21}, ⓕ_{21})$	$(ⓣ_{22}, ⓘ_{22}, ⓕ_{22}$	$\cdots$	$(ⓣ_{2n}, ⓘ_{2n}, ⓕ_{2n})$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathring{\ddot{x}}_m$	$((ⓣ_{m1}, ⓘ_{m1}, ⓕ_{m1})$	$((ⓣ_{m2}, ⓘ_{m2}, ⓕ_{m2})$	$\cdots$	$((ⓣ_{mn}, ⓘ_{mn}, ⓕ_{mn})$

Definition 3.2. *Let*

$$(\mathring{\varphi}, \mathring{A}) = \left\{ \left(\mathring{e}, \left\{ \mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle \right\} \right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U} \right\}$$

be a q-RSNSS on $\mathring{U} = \{\mathring{\ddot{x}}_1, \mathring{\ddot{x}}_2, ..., \mathring{\ddot{x}}_n\}$, then $(\mathring{\varphi}, \mathring{A})$ is called q-RSNS-Null Set where $\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = 0$, $\mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = 0$ and $\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = 1$.

Definition 3.3. *Let*

$$(\mathring{\varphi}, \mathring{A}) = \left\{ \left(\mathring{e}, \left\{ \mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle \right\} \right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U} \right\}$$

be a q-RSNSS on $\mathring{U} = \{\mathring{\ddot{x}}_1, \mathring{\ddot{x}}_2, ..., \mathring{\ddot{x}}_n\}$, then $(\mathring{\varphi}, \mathring{A})$ is called q-RSNS-absolute Set where $\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = 1$, $\mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = 1$ and $\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = 0$.

Definition 3.4. *Let*

$$(\mathring{\varphi}_1, \mathring{A}) = \left\{ \left(\mathring{e}, \left\{ \mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle \right\} \right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U} \right\}$$

and

$$(\mathring{\varphi}_2, \mathring{B}) = \left\{ \left(\mathring{e}, \left\{ \mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\ddot{x}), \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\ddot{x}), \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}}) \rangle \right\} \right) : \mathring{e} \in \mathring{B}, \mathring{\ddot{x}} \in \mathring{U} \right\}$$

be two q-RSNSSs on $\mathring{U} = \{\mathring{\ddot{x}}_1, \mathring{\ddot{x}}_2, ..., \mathring{\ddot{x}}_n\}$, then $(\mathring{\varphi}_1, \mathring{A}) \leq (\mathring{\varphi}_2, \mathring{B})$ if $\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \geq \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\ddot{x})$, $\mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \leq \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}})$ and $\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \leq \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}})$.

Definition 3.5. *Let*

$$(\mathring{\varphi}_1, \mathring{A}) = \left\{ \left(\mathring{e}, \left\{ \mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle \right\} \right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U} \right\}$$

and

$$(\mathring{\varphi}_2, \mathring{B}) = \left\{ \left(\mathring{e}, \left\{ \mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\ddot{x}), \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\ddot{x}), \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}}) \rangle \right\} \right) : \mathring{e} \in \mathring{B}, \mathring{\ddot{x}} \in \mathring{U} \right\}$$

be two q-RSNSSs on $\mathring{U} = \{\mathring{\ddot{x}}_1, \mathring{\ddot{x}}_2, ..., \mathring{\ddot{x}}_n\}$, *then* $(\mathring{\varphi}_1, \mathring{A}) = (\mathring{\varphi}_2, \mathring{B})$ *if* $\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}})$, $\mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}})$ *and* $\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) = \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}})$.

4. Basic Operations on $q - RSNSSs$

In this section, we introduce some basic theoretic operations on $q - RSNSSs$ such as the complement, union and intersection. We also give some properties on $q - RSNSSs$.

Let $(\mathring{\varphi}_1, \mathring{A}) = \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\}$ and $(\mathring{\varphi}_2, \mathring{B}) = \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{B}, \mathring{\ddot{x}} \in \mathring{U}\right\}$ be two $q - RSNSSs$ over the same universe $\mathring{U}$.

The fundamental operations are defined as follows:

4.1. **Complement of q-RSNSS.**

Definition 4.1. *The complement of a* $q - RSNSSs$ $(\mathring{\varphi}, \mathring{A})$, *denoted by* $(\mathring{\varphi}, \mathring{A})^c$, *is defined as*

$$(\mathring{\varphi}, \mathring{A})^c = \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\}$$

where for each $\mathring{\ddot{x}} \in \mathring{U}$,

$$\mathcal{T}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}) = \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \quad \mathcal{I}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}) = 1 - \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \quad \mathcal{F}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}) = \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}).$$

Example 4.1. *Consider the approximation given in Example 1, If* $\langle \mathring{\ddot{x}}_1, 0.3, 0.2, 0.6\rangle \in \mathring{\varphi}(\mathring{e}_1)$, *then* $\langle \mathring{\ddot{x}}_1, 0.6, 0.8, 0.3\rangle \in \mathring{\varphi}^c(\mathring{e}_1)$.

Theorem 4.1. *If* $(\mathring{\varphi}, \mathring{A}) \in q - RSNSSs$, *then* $\left((\mathring{\varphi}, \mathring{A})^c\right)^c = (\mathring{\varphi}, \mathring{A})$.

Proof. This proof depends on the definition 10.

Firstly take $(\mathring{\varphi}_1, \mathring{A}) = \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\}$

and take
$$\begin{aligned}(\mathring{\varphi}, \mathring{A})^c &= \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\} \\ &= \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \left[1 - \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}})\right], \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}})\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\}\end{aligned}$$

Now take $\left((\mathring{\varphi}, \mathring{A})^c\right)^c$

$$\begin{aligned}\left((\mathring{\varphi}, \mathring{A})^c\right)^c &= \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{F}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}), \left[1 - \mathcal{I}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}})\right], \mathcal{T}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\} \\ &= \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \left[1 - (1 - \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}))\right], \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\} \\ &= \left\{\left(\mathring{e}, \left\{\mathring{\ddot{x}}, \langle \mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\ddot{x}}) \rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\ddot{x}} \in \mathring{U}\right\} \\ &= (\mathring{\varphi}, \mathring{A})\end{aligned}$$

□

4.2. **Union of q-RSNSSs.**

Definition 4.2. *The union of* $(\mathring{\varphi}_1, \mathring{\mathtt{A}})$ *and* $(\mathring{\varphi}_2, \mathring{\mathtt{B}})$*, denoted by* $(\mathring{\varphi}, \mathring{\mathtt{C}}) = (\mathring{\varphi}_1, \mathring{\mathtt{A}}) \cup (\mathring{\varphi}_2, \mathring{\mathtt{B}})$ *, where* $\mathring{\mathtt{C}} = \mathring{\mathtt{A}} \cup \mathring{\mathtt{B}}$*, is given by*

$$\mathring{\varphi}(\mathring{e}) = \begin{cases} \mathring{\varphi}_1(\mathring{e}), & \text{if } \mathring{e} \in \mathring{\mathtt{A}} - \mathring{\mathtt{B}}, \\ \mathring{\varphi}_2(\mathring{e}), & \text{if } \mathring{e} \in \mathring{\mathtt{B}} - \mathring{\mathtt{A}}, \\ \mathring{\varphi}_1(\mathring{e}) \cup \mathring{\varphi}_2(\mathring{e}), & \text{if } \mathring{e} \in \mathring{\mathtt{A}} \cap \mathring{\mathtt{B}}. \end{cases}$$

For each $\mathring{e} \in \mathring{\mathtt{A}} \cap \mathring{\mathtt{B}}$ *and* $\mathring{\dot{x}} \in \mathring{U}$*, the union operation between* $\mathring{\varphi}_1(\mathring{e})$ *and* $\mathring{\varphi}_2(\mathring{e})$*) is defined component–wise as*

$$\begin{aligned} \mathcal{T}_{\mathring{\varphi}(\mathring{e})}(\mathring{\dot{x}}) &= \max\Big(\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\dot{x}}), \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{B}}}}(\mathring{\dot{x}})\Big), \\ \mathcal{I}_{\mathring{\varphi}(\mathring{e})}(\mathring{\dot{x}}) &= \min\Big(\mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\dot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{B}}}}(\mathring{\dot{x}})\Big), \\ \mathcal{F}_{\mathring{\varphi}(\mathring{e})}(\mathring{\dot{x}}) &= \min\Big(\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\dot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{B}}}}(\mathring{\dot{x}})\Big). \end{aligned}$$

Example 4.2. *Let* $\mathring{U} = \{\mathring{\dot{x}}_1, \mathring{\dot{x}}_2\}$ *represent a universe of discourse and let the parameter sets be* $\mathring{\mathtt{A}} = \{\mathring{e}_1, \mathring{e}_3\}$ *and* $\mathring{\mathtt{B}} = \{\mathring{e}_1, \mathring{e}_2, , \mathring{e}_3\}$ *. Two* $4-$*RSNSSs , denoted as* $(\mathring{\varphi}_1, \mathring{\mathtt{A}})$ *and* $(\mathring{\varphi}_2, \mathring{\mathtt{B}})$*, are provided as follows:*

$\mathring{\varphi}_{\mathring{\mathtt{A}}}$	$\mathring{e}_1$	$\mathring{e}_3$
$\mathring{\dot{x}}_1$	$(0.6, 0.2, 0.3)$	$(0.4, 0.3, 0.1)$
$\mathring{\dot{x}}_2$	$(0.7, 0.1, 0.2)$	$(0.5, 0.2, 0.2)$

$\mathring{\varphi}_{\mathring{\mathtt{B}}}$	$\mathring{e}_1$	$\mathring{e}_2$	$\mathring{e}_3$
$\mathring{\dot{x}}_1$	$(0.5, 0.8, 0.2)$	$(0.7, 0.1, 0.2)$	$(0.5, 0.3, 0.4)$
$\mathring{\dot{x}}_2$	$(0.5, 0.4, 0.3)$	$(0.6, 0.2, 0.3)$	$(0.6, 0.1, 0.4)$

The union $(\mathring{\varphi}_1, \mathring{\mathtt{A}}) \cup (\mathring{\varphi}_2, \mathring{\mathtt{B}})$ *is calculated as follows:*

$\mathring{\varphi}_{\mathring{\mathtt{A}} \cup \mathring{\mathtt{B}}}$	$\mathring{e}_1$	$\mathring{e}_2$	$\mathring{e}_2$
$\mathring{\dot{x}}_1$	$(0.6, 0.2, 0.2)$	$(0.7, 0.1, 0.2)$	$(0.5, 0.3, 0.1)$
$\mathring{\dot{x}}_2$	$(0.7, 0.1, 0.2)$	$(0.6, 0.2, 0.3)$	$(0.6, 0.1, 0.2)$

4.3. **Intersection of q-RSNSSs.**

Definition 4.3. *The Intersection of* $(\mathring{\varphi}_1, \mathring{\mathtt{A}})$ *and* $(\mathring{\varphi}_2, \mathring{\mathtt{B}})$*, denoted by* $(\mathring{\varphi}, \mathring{\mathtt{C}}) = (\mathring{\varphi}_1, \mathring{\mathtt{A}}) \cap (\mathring{\varphi}_2, \mathring{\mathtt{B}})$ *, where* $\mathring{\mathtt{C}} = \mathring{\mathtt{A}} \cap \mathring{\mathtt{B}}$*, is defined as*

$$\begin{aligned} \mathcal{T}_{\mathring{\varphi}(\mathring{e})}(\mathring{\dot{x}}) &= \min\Big(\mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\dot{x}}), \mathcal{T}_{\mathring{\varphi}_{\mathring{\mathtt{B}}}}(\mathring{\dot{x}})\Big), \\ \mathcal{I}_{\mathring{\varphi}(\mathring{e})}(\mathring{\dot{x}}) &= \max\Big(\mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\dot{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{\mathtt{B}}}}(\mathring{\dot{x}})\Big), \\ \mathcal{F}_{\mathring{\varphi}(\mathring{e})}(\mathring{\dot{x}}) &= \max\Big(\mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{A}}}}(\mathring{\dot{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{\mathtt{B}}}}(\mathring{\dot{x}})\Big). \end{aligned}$$

Example 4.3. *Using the same* $(\mathring{\varphi}_1, \mathring{\mathtt{A}})$ *and* $(\mathring{\varphi}_2, \mathring{\mathtt{B}})$ *from Example (3),* $(\mathring{\varphi}_1, \mathring{\mathtt{A}}) \cap (\mathring{\varphi}_2, \mathring{\mathtt{B}})$ *intersects as follows:*

$\mathring{\varphi}_{\mathring{A}\cap\mathring{B}}$	$\mathring{e}_1$	$\mathring{e}_2$	$\mathring{e}_2$
$\mathring{\mathring{x}}_1$	$(0.5, 0.8, 0.3)$	$(0.7, 0.1, 0.2)$	$(0.4, 0.3, 0.4)$
$\mathring{\mathring{x}}_2$	$(0.5, 0.4, 0.3)$	$(0.6, 0.2, 0.3)$	$(0.5, 0.2, 0.4)$

Proposition 4.1. *If $(\mathring{\varphi}_1, \mathring{A})$ and $(\mathring{\varphi}_2, \mathring{B})$ be two $q-RSNSSs$ on $\mathring{U}$, then*

(1) $((\mathring{\varphi}_1, \mathring{A}) \cap (\mathring{\varphi}_2, \mathring{B}))^c = (\mathring{\varphi}_1, \mathring{A})^c \cup (\mathring{\varphi}_2, \mathring{B})^c$
(2) $((\mathring{\varphi}_1, \mathring{A}) \cup (\mathring{\varphi}_2, \mathring{B}))^c = (\mathring{\varphi}_1, \mathring{A})^c \cap (\mathring{\varphi}_2, \mathring{B})^c$

Proof. (1) For the $q-RSNSSs$ $(\mathring{\varphi}_1, \mathring{A})$ and $(\mathring{\varphi}_2, \mathring{B})$, the Intersection $(\mathring{\varphi}_1, \mathring{A}) \cap (\mathring{\varphi}_2, \mathring{B}) = (\mathring{\varphi}, \mathring{C})$, where $\mathring{C} = \mathring{A} \cap \mathring{B}$, is defined component-wise by:

$$(\mathring{\varphi}_1, \mathring{A}) \cap (\mathring{\varphi}_2, \mathring{B}) = \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \mathcal{T}_{\mathring{\varphi}(\mathring{e})}(\mathring{\mathring{x}}), \mathcal{I}_{\mathring{\varphi}(\mathring{e})}(\mathring{\mathring{x}}), \mathcal{F}_{\mathring{\varphi}(\mathring{e})}(\mathring{\mathring{x}})\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$
$$= \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \min\left(\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), \max\left(\mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), \max\left(\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right)\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$

Firstly, for any $q-RSNSSs$, the complement is defined as follows: $((\mathring{\varphi}_1, \mathring{A}) \cap (\mathring{\varphi}_2, \mathring{B}))^c$

$$= \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \mathcal{T}_{\mathring{\varphi}^c(\mathring{e})}(\mathring{\mathring{x}}), \mathcal{I}_{\mathring{\varphi}^c(\mathring{e})}(\mathring{\mathring{x}}), \mathcal{F}_{\mathring{\varphi}^c(\mathring{e})}(\mathring{\mathring{x}})\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$
$$= \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \mathcal{F}_{\mathring{\varphi}(\mathring{e})}(\mathring{\mathring{x}}), 1 - \mathcal{I}_{\mathring{\varphi}(\mathring{e})}(\mathring{\mathring{x}}), \mathcal{T}_{\mathring{\varphi}(\mathring{e})}(\mathring{\mathring{x}})\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$
$$= \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \max\left(\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), 1 - \max\left(\mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), \min\left(\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right)\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$
$$= \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \max\left(\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), \min\left(1 - \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), 1 - \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), \min\left(\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right)\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$

Now, according to the definition of union, we have

$$(\mathring{\varphi}_1, \mathring{A})^c \cup (\mathring{\varphi}_2, \mathring{B})^c$$
$$= \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \max\left(\mathcal{T}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{T}_{\mathring{\varphi}^c_{\mathring{B}}}(\mathring{\mathring{x}})\right), \min\left(\mathcal{I}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{I}_{\mathring{\varphi}^c_{\mathring{B}}}(\mathring{\mathring{x}})\right), \min\left(\mathcal{F}_{\mathring{\varphi}^c_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{F}_{\mathring{\varphi}^c_{\mathring{B}}}(\mathring{\mathring{x}})\right)\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$
$$= \left\{\left(\mathring{e}, \left\{\mathring{\mathring{x}}, \langle \max\left(\mathcal{F}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{F}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), \min\left(1 - \mathcal{I}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), 1 - \mathcal{I}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right), \min\left(\mathcal{T}_{\mathring{\varphi}_{\mathring{A}}}(\mathring{\mathring{x}}), \mathcal{T}_{\mathring{\varphi}_{\mathring{B}}}(\mathring{\mathring{x}})\right)\rangle\right\}\right) : \mathring{e} \in \mathring{A}, \mathring{\mathring{x}} \in \mathring{U}\right\}$$

Therefore, $((\mathring{\varphi}_1, \mathring{A}) \cap (\mathring{\varphi}_2, \mathring{B}))^c$ and $(\mathring{\varphi}_1, \mathring{A})^c \cup (\mathring{\varphi}_2, \mathring{B})^c$ are the same operators, for all $\mathring{e} \in \mathring{A} \cap \mathring{B}$ and $\mathring{\mathring{x}} \in \mathring{U}$ which implies that $((\mathring{\varphi}_1, \mathring{A}) \cap (\mathring{\varphi}_2, \mathring{B}))^c = (\mathring{\varphi}_1, \mathring{A})^c \cup (\mathring{\varphi}_2, \mathring{B})^c$ and this completes the proof.

(2) We can proof in a similar fashion to (1).

□

4.4. AND and OR operations of of q-RSNSSs.

Definition 4.4. *Let $(\mathring{\varphi}_1, \mathring{A})$ and $(\mathring{\varphi}_2, \mathring{B})$ be two $q-RSNSSs$, then $(\mathring{\varphi}_1, \mathring{A})$ AND $(\mathring{\varphi}_2, \mathring{B})$ is the $q-RSNSS$ denoted by $(\mathring{\varphi}_1, \mathring{A}) \wedge (\mathring{\varphi}_2, \mathring{B})$ and defined by $(\mathring{\varphi}_1, \mathring{A}) \wedge (\mathring{\varphi}_2, \mathring{B}) = (\mathring{\varphi}_\wedge, \mathring{A} \times \mathring{B})$, where $\mathring{\varphi}_\wedge(\mathring{a}, \mathring{b}) = \mathring{\varphi}_1(\mathring{a}) \cap \mathring{\varphi}_2(\mathring{b})$ for all $(\mathring{a}, \mathring{b}) \in \mathring{A} \times \mathring{B}$ is the operation of intersection of two $q-RSNSs$ on $\mathring{U}$.That is, the membership functions of $\mathring{\varphi}_\wedge(\mathring{a}, \mathring{b})$ are given by:*

$$\mathcal{T}_{\mathring{\varphi}_\wedge(\mathring{a},\mathring{b})}(\mathring{\mathring{x}}) = \min\left(\mathcal{T}_{\mathring{\varphi}_1(\mathring{a})}(\mathring{\mathring{x}}), \mathcal{T}_{\mathring{\varphi}_2(\mathring{a})}(\mathring{\mathring{x}})\right),$$
$$\mathcal{I}_{\mathring{\varphi}_\wedge(\mathring{a},\mathring{b})}(\mathring{\mathring{x}}) = \max\left(\mathcal{I}_{\mathring{\varphi}_1(\mathring{a})}(\mathring{\mathring{x}}), \mathcal{I}_{\mathring{\varphi}_2(\mathring{b})}(\mathring{\mathring{x}})\right),$$

$$\mathcal{F}_{\mathring{\varphi}_{\wedge}(\mathring{a},\mathring{b})}(\mathring{\ddot{x}}) = \max\Big(\mathcal{F}_{\mathring{\varphi}_1(\mathring{a})}(\mathring{\ddot{x}}), \mathcal{F}_{\mathring{\varphi}_1(\mathring{a})}(\mathring{\ddot{x}})\Big).$$

Definition 4.5. *Let $(\mathring{\varphi}_1, \mathring{A})$ and $(\mathring{\varphi}_2, \mathring{B})$ be two $q-RSNSSs$, then $(\mathring{\varphi}_1, \mathring{A})$ OR $(\mathring{\varphi}_2, \mathring{B})$ is the $q-RSNSS$ denoted by $(\mathring{\varphi}_1, \mathring{A}) \vee (\mathring{\varphi}_2, \mathring{B})$ and defined by $(\mathring{\varphi}_1, \mathring{A}) \vee (\mathring{\varphi}_2, \mathring{B}) = (\mathring{\varphi}_{\vee}, \mathring{A} \times \mathring{B})$, where $\mathring{\varphi}_{\vee}(\mathring{a}, \mathring{b}) = \mathring{\varphi}_1(\mathring{a}) \cup \mathring{\varphi}_2(\mathring{b})$ for all $(\mathring{a}, \mathring{b}) \in \mathring{A} \times \mathring{B}$ is the operation of intersection of two $q-RSNSs$ on $\mathring{U}$.That is, the membership functions of $\mathring{\varphi}_{\vee}(\mathring{a}, \mathring{b})$ are given by:*

$$\mathcal{T}_{\mathring{\varphi}_{\vee}(\mathring{a},\mathring{b})}(\mathring{\ddot{x}}) = \max\Big(\mathcal{T}_{\mathring{\varphi}_1(\mathring{a})}(\mathring{\ddot{x}}), \mathring{\varphi}_2(\mathring{b})(\mathring{\ddot{x}})\Big),$$
$$\mathcal{I}_{\mathring{\varphi}_{\vee}(\mathring{a},\mathring{b})}(\mathring{\ddot{x}}) = \min\Big(\mathcal{I}_{\mathring{\varphi}_1(\mathring{a})}(\mathring{\ddot{x}}), \mathring{\varphi}_2(\mathring{b})(\mathring{\ddot{x}})\Big),$$
$$\mathcal{F}_{\mathring{\varphi}_{\vee}(\mathring{a},\mathring{b})}(\mathring{\ddot{x}}) = \min\Big(\mathcal{F}_{\mathring{\varphi}_1(\mathring{a})}(\mathring{\ddot{x}}), \mathring{\varphi}_2(\mathring{b})(\mathring{\ddot{x}})\Big).$$

Example 4.4. *Using the same $(\mathring{\varphi}_1, \mathring{A})$ and $(\mathring{\varphi}_2, \mathring{B})$ from Example (3).*

Then $(\mathring{\varphi}_1, \mathring{A}) \wedge (\mathring{\varphi}_2, \mathring{B})$ is calculated as follows:

$\mathring{\varphi}_{\wedge}$	$(\mathring{e}_1, \mathring{e}_1)$	$(\mathring{e}_1, \mathring{e}_2)$	$(\mathring{e}_1, \mathring{e}_3)$	$(\mathring{e}_3, \mathring{e}_1)$	$(\mathring{e}_3, \mathring{e}_2)$	$(\mathring{e}_3, \mathring{e}_3)$
$\mathring{\ddot{x}}_1$	(0.5, 0.8, 0.3)	(0.6, 0.2, 0.3)	(0.5, 0.3, 0.4)	(0.4, 0.8, 0.2)	(0.4, 0.3, 0.2)	(0.4, 0.3, 0.4)
$\mathring{\ddot{x}}_2$	(0.5, 0.4, 0.3)	(0.6, 0.2, 0.3)	(0.6, 0.1, 0.4)	(0.5, 0.4, 0.3)	(0.5, 0.2, 0.3)	(0.5, 0.2, 0.4)

Then $(\mathring{\varphi}_1, \mathring{A}) \vee (\mathring{\varphi}_2, \mathring{B})$ is calculated as follows:

$\mathring{\varphi}_{\vee}$	$(\mathring{e}_1, \mathring{e}_1)$	$(\mathring{e}_1, \mathring{e}_2)$	$(\mathring{e}_1, \mathring{e}_3)$	$(\mathring{e}_3, \mathring{e}_1)$	$(\mathring{e}_3, \mathring{e}_2)$	$(\mathring{e}_3, \mathring{e}_3)$
$\mathring{\ddot{x}}_1$	(0.6, 0.2, 0.2)	(0.7, 0.1, 0.2)	(0.6, 0.2, 0.3)	(0.5, 0.3, 0.1)	(0.7, 0.1, 0.1)	(0.5, 0.3, 0.1)
$\mathring{\ddot{x}}_2$	(0.7, 0.1, 0.2)	(0.7, 0.1, 0.2)	(0.7, 0.1, 0.2)	(0.5, 0.2, 0.2)	(0.6, 0.2, 0.2)	(0.6, 0.1, 0.2)

Proposition 4.2. *If $(\mathring{\varphi}_1, \mathring{A})$ and $(\mathring{\varphi}_2, \mathring{B})$ be two $q-RSNSSs$ on $\mathring{U}$, then*

(1) $((\mathring{\varphi}_1, \mathring{A}) \wedge (\mathring{\varphi}_2, \mathring{B}))^c = (\mathring{\varphi}_1, \mathring{A})^c \vee (\mathring{\varphi}_2, \mathring{B})^c$
(2) $((\mathring{\varphi}_1, \mathring{A}) \vee (\mathring{\varphi}_2, \mathring{B}))^c = (\mathring{\varphi}_1, \mathring{A})^c \wedge (\mathring{\varphi}_2, \mathring{B})^c$

Proof. (1) Suppose that $(\mathring{\varphi}_1, \mathring{A})$ and $(\mathring{\varphi}_2, \mathring{B})$ are two $q-RSNSSs$ over a soft universe $\mathring{U}$ defined as $(\mathring{\varphi}, \mathring{A}) = \mathring{\varphi}_1(\mathring{e})$ for all $\mathring{e} \in \mathring{A} \subseteq \mathring{E}$ and $(\mathring{\varphi}, \mathring{B}) = \mathring{\varphi}_2(\mathring{e})$ for all $\mathring{e} \in \mathring{B} \subseteq \mathring{E}$. By Definitions 13 and 14 it follows that:

$$\begin{aligned}
((\mathring{\varphi}_1, \mathring{A}) \wedge (\mathring{\varphi}_2, \mathring{B}))^c &= [\mathring{\varphi}_1(\mathring{e}) \wedge \mathring{\varphi}_2(\mathring{e})]^c \\
&= [\mathring{\varphi}_1(\mathring{e}) \cap \mathring{\varphi}_2(\mathring{e})]^c \\
&= c[\mathring{\varphi}_1(\mathring{e}) \cap \mathring{\varphi}_2(\mathring{e})] \\
&= c[\mathring{\varphi}_1(\mathring{e})] \cup c[\mathring{\varphi}_2(\mathring{e})] \\
&= [\mathring{\varphi}_1(\mathring{e})]^c \vee [\mathring{\varphi}_2(\mathring{e})]^c \\
&= (\mathring{\varphi}_1, \mathring{A})^c \vee (\mathring{\varphi}_2, \mathring{B})^c.
\end{aligned}$$

(2) We can proof in a similar fashion to (1).

□

5. A Novel Similarity Measure of q-Rung Simplified Neutrosophic Soft Sets

Definition 5.1. *Assume that* $(\mathring{\varphi}, \mathring{A})$ *and* $(\mathring{\varphi}, \mathring{B})$ *be two q-RSNSSs define on* $\mathring{U} = \{\mathring{x}_1, \mathring{x}_2, ..., \mathring{x}_n\}$ *then* $\widehat{S}((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B}))$ *is called a similarity measure between two q-RSNSSs* $(\mathring{\varphi}, \mathring{A})$ *and* $(\mathring{\varphi}, \mathring{B})$ *where* $\widehat{S}$ *is a mapping given as following* $\widehat{S}$*: q-RSNSS*$(\mathring{U})\times$*q-RSNSS*$(\mathring{U})\rightarrow[0,1]$. *Here* $\widehat{S}$ *satisfied the following point:*

(1) $0 \leq \widehat{S}\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) \leq 1$.
(2) $\widehat{S}\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = 1$ *iff* $(\mathring{\varphi}, \mathring{A}) = (\mathring{\varphi}, \mathring{B})$.
(3) $\widehat{S}\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = \widehat{S}\left((\mathring{\varphi}, \mathring{B}), (\mathring{\varphi}, \mathring{A})\right)$.
(4) $\widehat{S}\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{C})\right) \leq \widehat{S}\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right)$ *and* $\widehat{S}\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{C})\right) \leq \widehat{S}\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right)$ *if* $\mathring{A} \subseteq \mathring{B} \subseteq \mathring{C}$.

Definition 5.2. *Let* $(\mathring{\varphi}, \mathring{A})$ *and* $(\mathring{\varphi}, \mathring{B})$ *be two q-RSNSSs define on* $\mathring{U} = \{\mathring{x}_1, \mathring{x}_2, ..., \mathring{x}_n\}$ *then the following points are define as is a similarity measure between q-RSNSSs.*

(1) $\widehat{S}_1\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = 1 - \frac{1}{3|U|}\sum_{\mathring{x}\in U}\Big(\left|\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right| + \left|\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right| + \left|\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\Big)$.

(2) $\widehat{S}_2\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = 1 - \frac{1}{3|U|}\sum_{\mathring{x}\in U}\Big(\left|\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right| - \left|\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right| - \left|\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\Big)$.

(3) $\widehat{S}_3\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = 1 - \frac{1}{3|U|}\sum_{\mathring{x}\in U}\Big(\left|\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right| \vee \left|\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right| \vee \left|\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x}) - \mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\Big)$.

(4) $\widehat{S}_4\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = \frac{1}{|U|}\sum_{\mathring{x}\in U}\left\{\frac{1-\Big(\left|\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\Big)}{1+\Big(\left|\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\Big)}\right\}$.

(5) $\widehat{S}_5\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = \sum_{\mathring{x}\in U}\left\{\frac{1-\Big(\left|\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|F^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-F^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\Big)}{1+\Big(\left|\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\vee\left|\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})-\mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right|\Big)}\right\}$.

(6) $\widehat{S}_6\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = \alpha * \frac{\sum_{\mathring{x}\in U}\left(\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\wedge\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}{\sum_{\mathring{x}\in U}\left(T^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\vee\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)} + \beta * \frac{\sum_{\mathring{x}\in U}\left(\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\wedge\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}{\sum_{\mathring{x}\in U}\left(\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\vee\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)} + \psi * \frac{\sum_{\mathring{x}\in U}\left(F^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\wedge\mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}{\sum_{\mathring{x}\in U}\left(\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\vee\mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}$.

where $\alpha + \beta + \psi \in [0,1]$ *and* $\alpha + \beta + \psi = 1$

(7) $\widehat{S}_7\left((\mathring{\varphi}, \mathring{A}), (\mathring{\varphi}, \mathring{B})\right) = \frac{\alpha}{|U|} * \frac{\sum_{\mathring{x}\in U}\left(\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\wedge\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}{\sum_{\mathring{x}\in U}\left(\mathcal{T}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\vee\mathcal{T}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)} + \frac{\beta}{|U|} * \frac{\sum_{x\in U}\left(\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\wedge\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}{\sum_{\mathring{x}\in U}\left(\mathcal{I}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\vee\mathcal{I}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)} + \frac{\psi}{|U|} * \frac{\sum_{\mathring{x}\in U}\left(\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\wedge\mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}{\sum_{\mathring{x}\in U}\left(\mathcal{F}^2_{\mathring{\varphi}_{\mathring{A}}}(\mathring{x})\vee\mathcal{F}^2_{\mathring{\varphi}_{\mathring{B}}}(\mathring{x})\right)}$.

where $\alpha + \beta + \psi \in [0,1]$ *and* $\alpha + \beta + \psi = 1$

5.1. Application of the similarity measures between q-RSNSSs in a pattern recognition problem.

In this subsection, we test the performance of the proposed similarity measures in dealing with MCDM problems related to pattern recognition. We test the techniques mentioned in the definition above to identify the best pattern to use. In this case, we are testing medical patterns to determine whether certain individuals are susceptible to seasonal influenza (illness) based on their symptoms.

Example 5.1. *Assume that* $\mathring{U} = \{\mathring{\dot{x}}_1, \mathring{\dot{x}}_2\}$ *represents the degree of* ***severity*** $\mathring{x}_1$ *and the degree of* ***mildness*** $\mathring{x}_2$ *and* $\mathring{\mathtt{E}} = \{\mathring{e}_1, \mathring{e}_2, \mathring{e}_3, \mathring{e}_4, \mathring{e}_5\}$ *represents set of certain visible symptoms where* $\mathring{e}_1$ *=headache,*$\mathring{e}_2$ *=nausea and vomiting,* $\mathring{e}_3$ *=fatigue,* $\mathring{e}_4$ *=weakness,*$\mathring{e}_5$ *=skin changes. Next, we start arranging the following algorithm to deal with this problem:*

Step 1. *Build a first model* $(\mathring{\varphi}, \mathring{\mathtt{E}})$ *for seasonal influenza (illness) depending on medical experts.*

$$(\mathring{\varphi}, \mathring{\mathtt{E}}) = \begin{Bmatrix} \mathring{e}_1, \frac{(\langle 0.3,0.4,0.4\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.3,0.7,0.1\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_2, \frac{(\langle 0.1,0.6,0.3\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.3,0.7,0.5\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_3, \frac{(\langle 0.6,0.4,0.8\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.5,0.4,0.8\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_4, \frac{(\langle 0.7,0.5,0.9\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.5,0.4,0.8\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_5, \frac{(\langle 0.7,0.8,0.1\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.4,0.6,0.3\rangle)}{\mathring{\dot{x}}_2} \end{Bmatrix}$$

Step 2. *Build a first the three models* $(\mathring{\varphi}, \mathtt{A}_1 \mathring{\subseteq} \mathtt{E})$, $(\mathring{\varphi}, \mathtt{A}_2 \mathring{\subseteq} \mathtt{E})$ *and* $(\mathring{\varphi}, \mathtt{A}_3 \mathring{\subseteq} \mathtt{E})$ *for patients* A_1, A_2, A_3 *respectively, depending on medical experts.*

$$(\mathring{\varphi}, \mathtt{A}_1 \mathring{\subseteq} \mathtt{E}) = \begin{Bmatrix} \mathring{e}_1, \frac{(\langle 0.5,0.2,0.6\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.7,0.4,0.3\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_2, \frac{(\langle 0.5,0.8,0.9\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.2,0.5,0.53\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_3, \frac{(\langle 0.3,0.1,0.1\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.4,0.3,0.6\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_4, \frac{(\langle 0.4,0.5,0.2\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.5,0.5,0.8\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_5, \frac{(\langle 0.1,0.1,0.7\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.5,0.3,0.9\rangle)}{\mathring{\dot{x}}_2} \end{Bmatrix}$$

$$(\mathring{\varphi}, \mathtt{A}_2 \mathring{\subseteq} \mathtt{E}) = \begin{Bmatrix} \mathring{e}_1, \frac{(\langle 0.2,0.1,0.9\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.5,0.3,0.4\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_2, \frac{(\langle 0.6,0.1,0.9\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.9,0.5,0.3\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_3, \frac{(\langle 0.3,0.1,0.1\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.7,0.4,0.6\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_4, \frac{(\langle 0.5,0.5,0.4\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.8,0.3,0.9\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_5, \frac{(\langle 0.4,0.8,0.9\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.7,0.9,0.3\rangle)}{\mathring{\dot{x}}_2} \end{Bmatrix}$$

and

$$(\mathring{\varphi}, \mathtt{A}_3 \mathring{\subseteq} \mathtt{E}) = \begin{Bmatrix} \mathring{e}_1, \frac{(\langle 0.5,0.2,0.6\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.7,0.4,0.3\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_2, \frac{(\langle 0.1,0.3,0.2\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.5,0.2,0.8\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_3, \frac{(\langle 0.3,0.8,0.5\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.9,0.5,0.7\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_4, \frac{(\langle 0.2,0.6,0.1\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.3,0.7,0.2\rangle)}{\mathring{\dot{x}}_2} \\ \mathring{e}_5, \frac{(\langle 0.2,0.6,0.7\rangle)}{\mathring{\dot{x}}_1}, \frac{(\langle 0.4,0.7,0.3\rangle)}{\mathring{\dot{x}}_2} \end{Bmatrix}$$

Step 3. *We apply the tools for the similarity procedures provided in Definition 11, where we obtain the results shown in Table 1. Accordingly, the results obtained in Table No. 1 show the variation in the severity of the injury from* $\mathtt{A}_1$ *to* $\mathtt{A}_3$.

Table 1. Comparative examination of existing models using key criteria.

SM Methods	A_1	A_2	A_3	*Ranking*
$\widehat{S}_1\left((\mathring{\varphi},\mathring{\mathtt{E}}),(\mathring{\varphi},\mathring{\mathtt{A}}_{\mathtt{i}})\right)$	*0.5285*	*0.4632*	*0.5061*	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$
$\widehat{S}_2\left((\mathring{\varphi},\mathring{\mathtt{E}}),(\mathring{\varphi},\mathring{\mathtt{A}}_{\mathtt{i}})\right)$	*0.0653*	*0.0372*	*0.0642*	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$
$\widehat{S}_3\left((\mathring{\varphi},\mathring{\mathtt{E}}),(\mathring{\varphi},\mathring{\mathtt{A}}_{\mathtt{i}})\right)$	*0.7742*	*0.4491*	*0.6739*	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$
$\widehat{S}_4\left((\mathring{\varphi},\mathring{\mathtt{E}}),(\mathring{\varphi},\mathring{\mathtt{A}}_{\mathtt{i}})\right)$	*0.6206*	*0.5722*	*0.5978*	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$
$\widehat{S}_5\left((\mathring{\varphi},\mathring{\mathtt{E}}),(\mathring{\varphi},\mathring{\mathtt{A}}_{\mathtt{i}})\right)$	*0.6378*	*0.5423*	*0.6003*	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$
$\widehat{S}_6\left((\mathring{\varphi},\mathring{\mathtt{E}}),(\mathring{\varphi},\mathring{\mathtt{A}}_{\mathtt{i}})\right)$	*0.3852*	*0.2291*	*0.3093*	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$
$\widehat{S}_7\left((\mathring{\varphi},\mathring{\mathtt{E}}),(\mathring{\varphi},\mathring{\mathtt{A}}_{\mathtt{i}})\right)$	*0.4582*	*0.3421*	*0.4096*	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$

Table 1 presents a comparative analysis of several existing similarity measures implemented to the considered neutrosophic models under the given criteria. Each row in the table corresponds to a particular similarity measure $\widehat{S}_i\,(.)$, while the columns $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ represent alternative diagnostic instances derived from the symptom set.

The numerical values listed in the table indicate the degree of similarity between the neutrosophic representation of the symptom set and each diagnostic alternative. A larger similarity value implies a stronger correspondence between the observed symptoms and the evaluated alternative. The very last column summarizes the ranking of alternatives, which is obtained by arranging the similarity values in descending order.

It is observed that although different similarity measures give slightly different numerical values, the ranking results remain consistent across most measures. Specifically,alternative $\mathcal{A}_1$ achieves the highest degree of similarity in most cases, followed by $\mathcal{A}_3$, even as $\mathcal{A}_2$ records the lowest degree of similarity. This consistency highlights the robustness and stability of the proposed decision-making framework.

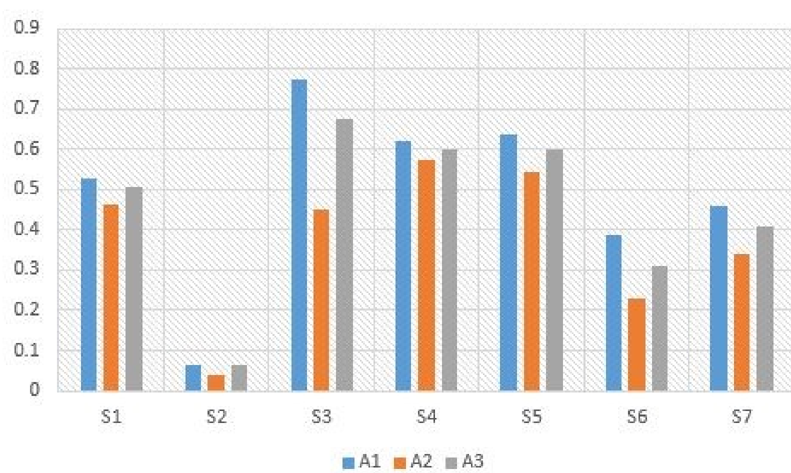

Figure 1. Statistical chart showing the difference between the outputs of the similarity tools

Figure 1 shows a statistical chart depicting illustrates the comparison of the severity of the infection among the three patients, where our proposed concept and the proposed similarity tools were employed in this work to measure the strength of the symptoms associated with the three patients. The graphical results clearly support the numerical outcomes reported in Table 1. Specifically, alternative $\mathscr{A}_1$ consistently exhibits higher similarity values across different similarity measures, confirming its dominance as the most suitable diagnostic decision. Moreover, the limited variation among the bars demonstrates the effectiveness of the proposed approach in reducing uncertainty.

6. Conclusions

In this work we combined the advantages of the previous concepts into one concept called the q-rung neutrosophic soft set (q-RNSS). Based on this model we introduced the formal definition of our proposed model, along with its foundational concepts and mathematical properties, including operations such as union, intersection, and complement, supported by rigorous proofs. We also define fundamental algebraic operations between two neutrosophic numbers. In addition, we have presented several similarity techniques to find the similarity ratio between two objects from q-RNSS environment. Furthermore, we develop an algorithmic approach to multi-criteria decision-making (MCDM) under a similarity measures environment.Finely, we hope that these tools will increase the flexibility of decision-makers in formulating statements of everyday problems. In addition, we recommend that researchers adopt this model in future work by adapting this model to many real-life applications.

Abbreviations

FS	Fuzzy Set
IFS	Intuitionistic fuzzy sets
NS	Neutrosophic Set
SS	Soft Set
NSS	Neutrosophic soft sets
PyFS	Pythagorean fuzzy set
q-ROFS	q-Rung orthopair fuzzy set
q-RSNs	q-Rung simplified neutrosophic sets
PyNS	Pythagorean neutrosophic sets
FNS	Fermatean neutrosophic set
q-RONS	q-rung orthopair neutrosophic set
$\mathcal{T}$	truth membership degree
$\mathcal{I}$	indeterminacy membership degree
$\mathcal{F}$	falsity membership degree
DM	Decision-Making
U	Universal of discourse.

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