

## A Unified Wardowski Contraction Approach to Multivalued Fixed Points in Cone Metric Spaces

Hawa Ibnouf Osman Ibnouf\*

*Department of Mathematics, College of Science, Qassim University, Saudi Arabia*

\*Corresponding author: h.ibnouf@qu.edu.sa

**Abstract.** In this paper, we investigate fixed point results for multivalued mappings defined on complete cone metric spaces (CCMS) by employing nonlinear Wardowski-type contraction conditions. Our approach extends the classical Banach contraction principle (BCP) and several of its generalizations by incorporating a control function belonging to the Wardowski class  $\mathfrak{F}$  and a Hausdorff cone metric (HCM) structure. The obtained results unify and generalize various fixed point theorems established in metric,  $b$ -metric, controlled metric, and multivalued settings. In particular, our theorems extend earlier results of Huang and Zhang, Krishnakumar and Marudai, and Wardowski to a broader nonlinear and multivalued cone metric framework. Illustrative examples are provided to demonstrate the applicability of the main results. These findings contribute to the ongoing development of nonlinear fixed point theory and provide a flexible tool for applications to integral and functional equations.

### 1. INTRODUCTION

The BCP [1] remains one of the most influential results in nonlinear analysis due to its simplicity and wide range of applications, particularly in the theory of integral and differential equations. Over the past decades, numerous generalizations of this principle have been proposed by relaxing either the underlying space or the contractive condition.

Among early extensions, Bakhtin [2] introduced almost metric spaces (MS), while further generalizations such as  $b$ -MSs were studied by Kamran et al. [3]. More recently, controlled and double controlled metric type spaces were developed to unify and extend several metric structures, as seen in the works of Mlaiki et al. [4] and Abdeljawad et al. [5]. These developments highlight the continuing interest in flexible distance structures.

A significant advancement in the theory of contractive mappings was made by Wardowski [14], who introduced a new class of nonlinear contractions governed by a function  $F$ . This concept has

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since been widely studied and extended, including multivalued versions [10, 11] and survey-level treatments [12]. Applications of  $F$ -contractions to integral equations and related problems can be found in [13].

On the other hand, cone metric spaces (CMS), introduced by Huang and Zhang [15], provide an ordered vector-valued generalization of MSs and have proven useful in studying nonlinear problems. Fixed point (FP) results for multivalued mappings (MVM) in CMSs were further explored by Krishnakumar and Marudai [16]. However, most existing results in CMSs rely on linear or rational contractive conditions.

Motivated by these observations, the aim of this paper is to develop FP theorems for MVMs in CMSs via Wardowski-type nonlinear contractions. Our results simultaneously generalize the BCP, Wardowski's  $F$ -contractions, and multivalued cone metric FP theorems. The obtained theorems extend and complement recent contributions in parametric, fuzzy, and digital metric frameworks [6–8].

## 2. PRELIMINARIES

Let  $\mathcal{E}$  be a real Banach space and let  $\mathcal{P} \subset \mathcal{E}$  be a cone ( $\mathcal{C}$ ) with interior  $\text{int}(\mathcal{P})$ . The partial order induced by  $\mathcal{P}$  is defined by  $u \leq v$  if and only if  $v - u \in \mathcal{P}$ . The zero element of  $\mathcal{E}$  is denoted by  $\theta$ .

**Definition 2.1.** Let  $\Psi$  be a nonempty (NE) set. A mapping  $S : \Psi \times \Psi \rightarrow \mathcal{E}$  is called a  $\mathcal{C}$  metric if, for all  $\delta, \sigma, \omega \in \Psi$ :

- (i)  $\theta \leq S(\delta, \sigma)$  and  $S(\delta, \sigma) = \theta$  if and only if  $\delta = \sigma$ ;
- (ii)  $S(\delta, \sigma) = S(\sigma, \delta)$ ;
- (iii)  $S(\delta, \omega) \leq S(\delta, \sigma) + S(\sigma, \omega)$ .

The pair  $(\Psi, S)$  is called a CMS.

Let  $CB(\Psi)$  denote the family of all NE, closed and bounded subsets of  $\Psi$ . For  $\mathcal{A}, \mathcal{B} \in CB(\Psi)$ , the HCM  $\mathcal{H} : CB(\Psi) \times CB(\Psi) \rightarrow \mathcal{E}$  is defined by

$$\mathcal{H}(\mathcal{A}, \mathcal{B}) = \sup \left\{ \sup_{\delta \in \mathcal{A}} S(\delta, \mathcal{B}), \sup_{\sigma \in \mathcal{B}} S(\mathcal{A}, \sigma) \right\}.$$

**Definition 2.2.** Let  $\mathfrak{F}$  be the family of functions  $\mathcal{F} : \text{int}(\mathcal{P}) \rightarrow \mathbb{R}$  satisfying:

- (W1)  $\mathcal{F}$  is strictly increasing;
- (W2)  $\omega_n \rightarrow \theta$  if and only if  $\mathcal{F}(\omega_n) \rightarrow -\infty$ ;
- (W3) There exists  $k \in (0, 1)$  such that  $\lim_{\omega \rightarrow \theta} \|\omega\|^k \mathcal{F}(\omega) = 0$ .

Such functions are called Wardowski control functions [14].

## 3. WARDOWSKI-TYPE FIXED POINT RESULTS FOR MULTIVALUED OPERATORS IN $\mathcal{C}$ METRIC SPACES

Let  $\mathcal{E}$  be a real Banach space and let  $\mathcal{P} \subset \mathcal{E}$  be a normal  $\mathcal{C}$  with NE interior  $\text{int}(\mathcal{P})$ . The partial order induced by  $\mathcal{P}$  is denoted by  $\leq$ , and  $\theta$  denotes the zero element of  $\mathcal{E}$ .

Let  $\Psi$  be a NE set and let  $S : \Psi \times \Psi \rightarrow \mathcal{E}$  be a  $\mathcal{C}$  metric. The pair  $(\Psi, S)$  is assumed to be complete.

Denote by  $CB(\Psi)$  the family of all NE, closed and bounded subsets of  $\Psi$ . For  $\mathcal{A}, \mathcal{B} \in CB(\Psi)$ , define the HCM  $\mathcal{H} : CB(\Psi) \times CB(\Psi) \rightarrow \mathcal{E}$  by

$$\mathcal{H}(\mathcal{A}, \mathcal{B}) = \sup \left\{ \sup_{\delta \in \mathcal{A}} S(\delta, \mathcal{B}), \sup_{\sigma \in \mathcal{B}} S(\mathcal{A}, \sigma) \right\}.$$

Let  $\mathfrak{F}$  denote the class of functions  $\mathcal{F} : \text{int}(\mathcal{P}) \rightarrow \mathbb{R}$  satisfying the following conditions:

(W1)  $\mathcal{F}$  is strictly increasing with respect to  $\leq$ ;

(W2) For any sequence  $\{\omega_n\} \subset \text{int}(\mathcal{P})$ ,

$$\omega_n \rightarrow \theta \iff \mathcal{F}(\omega_n) \rightarrow -\infty;$$

(W3) There exists  $k \in (0, 1)$  such that

$$\lim_{\omega \rightarrow \theta} \|\omega\|^k \mathcal{F}(\omega) = 0.$$

### 3.1. Main Results.

**Theorem 3.1.** Let  $(\Psi, S)$  be a CCMS and let  $\Pi : \Psi \rightarrow CB(\Psi)$  be a MVO. Assume that there exist  $\mathcal{F} \in \mathfrak{F}$  and  $\tau \in \text{int}(\mathcal{P})$  such that for all  $\delta, \sigma \in \Psi$  with  $\mathcal{H}(\Pi\delta, \Pi\sigma) > \theta$ ,

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta, \Pi\sigma)) \leq \mathcal{F}(S(\delta, \Pi\delta) + S(\sigma, \Pi\sigma)).$$

Then  $\Pi$  admits at least one FP in  $\Psi$ .

*Proof.* Let  $\delta_0 \in \Psi$  be arbitrary. Since  $\Pi\delta_0 \in CB(\Psi)$ , choose  $\delta_1 \in \Pi\delta_0$ . If  $\delta_1 = \delta_0$ , then  $\delta_0$  is a FP of  $\Pi$  and the proof is complete. Otherwise,  $\delta_1 \neq \delta_0$ .

Proceed inductively. Having chosen  $\delta_n \in \Psi$ , select  $\delta_{n+1} \in \Pi\delta_n$  such that

$$S(\delta_n, \delta_{n+1}) \leq S(\delta_n, \Pi\delta_n) + \varepsilon_n,$$

where  $\{\varepsilon_n\}$  is a sequence in  $\text{int}(\mathcal{P})$  satisfying  $\varepsilon_n \rightarrow \theta$ .

For  $n \geq 1$ , since  $\delta_{n+1} \in \Pi\delta_n$  and  $\delta_{n+2} \in \Pi\delta_{n+1}$ , we have

$$S(\delta_{n+1}, \delta_{n+2}) \leq \mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1}).$$

If  $\mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1}) = \theta$ , then  $\Pi\delta_n = \Pi\delta_{n+1}$  and hence  $\delta_{n+1} \in \Pi\delta_{n+1}$ , which shows that  $\delta_{n+1}$  is a FP. Thus, we assume  $\mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1}) > \theta$ .

By the contractive condition, we obtain

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1})) \leq \mathcal{F}(S(\delta_n, \Pi\delta_n) + S(\delta_{n+1}, \Pi\delta_{n+1})).$$

Using the monotonicity of  $\mathcal{F}$  and the construction of  $\{\delta_n\}$ , it follows that

$$\tau + \mathcal{F}(S(\delta_{n+1}, \delta_{n+2})) \leq \mathcal{F}(S(\delta_n, \delta_{n+1}) + S(\delta_{n+1}, \delta_{n+2})).$$

Since  $\mathcal{F}$  satisfies condition (W2), the above inequality implies that

$$S(\delta_n, \delta_{n+1}) \rightarrow \theta \text{ as } n \rightarrow \infty.$$

Next, we show that  $\{\delta_n\}$  is a Cauchy sequence. For  $m > n$ , by the  $C$  metric triangle inequality, we have

$$S(\delta_n, \delta_m) \leq \sum_{i=n}^{m-1} S(\delta_i, \delta_{i+1}).$$

Using condition (W3) and the convergence of  $\sum_{i=n}^{\infty} S(\delta_i, \delta_{i+1})$  in  $\mathcal{P}$ , we deduce that

$$S(\delta_n, \delta_m) \rightarrow \theta \quad \text{as } n, m \rightarrow \infty.$$

Thus,  $\{\delta_n\}$  is a Cauchy sequence in  $(\Psi, S)$ .

Since  $(\Psi, S)$  is complete, there exists  $\delta^* \in \Psi$  such that

$$\delta_n \rightarrow \delta^* \quad \text{as } n \rightarrow \infty.$$

Finally, we show that  $\delta^*$  is a FP of  $\Pi$ . Assume, on the contrary, that  $\delta^* \notin \Pi\delta^*$ . Then  $\mathcal{H}(\Pi\delta_n, \Pi\delta^*) > \theta$  for sufficiently large  $n$ . Applying the contractive condition with  $(\delta, \sigma) = (\delta_n, \delta^*)$  and letting  $n \rightarrow \infty$ , we obtain a contradiction with condition (W2). Hence,  $\delta^* \in \Pi\delta^*$ .

Therefore,  $\Pi$  admits at least one FP in  $\Psi$ . □

**Example 3.1.** Let  $\Psi = [0, 1]$  and let  $\mathcal{E} = \mathbb{R}^2$ . Define the  $C$

$$\mathcal{P} = \{(u, v) \in \mathbb{R}^2 : u \geq 0, v \geq 0\},$$

which is a normal  $C$  with NE interior. The zero element is  $\theta = (0, 0)$ .

Define the  $C$  metric  $S : \Psi \times \Psi \rightarrow \mathcal{E}$  by

$$S(\delta, \sigma) = (|\delta - \sigma|, 2|\delta - \sigma|).$$

It is straightforward to verify that  $(\Psi, S)$  is a CCMS.

Let  $CB(\Psi)$  denote the family of all NE closed and bounded subsets of  $\Psi$ . Define the MVO

$$\Pi(\delta) = \left[0, \frac{\delta}{2}\right], \quad \delta \in \Psi.$$

For  $\delta, \sigma \in \Psi$ , the HCM is given by

$$\mathcal{H}(\Pi\delta, \Pi\sigma) = \left(\left|\frac{\delta - \sigma}{2}\right|, |\delta - \sigma|\right).$$

Define the Wardowski function  $\mathcal{F} : \text{int}(\mathcal{P}) \rightarrow \mathbb{R}$  by

$$\mathcal{F}(u, v) = \ln(u + v),$$

which belongs to the class  $\mathfrak{F}$ .

Choose  $\tau = (0.1, 0.1) \in \text{int}(\mathcal{P})$ . Then, for  $\delta, \sigma \in \Psi$  with  $\delta \neq \sigma$ , we have

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta, \Pi\sigma)) = (0.1, 0.1) + \ln\left(\frac{3}{2}|\delta - \sigma|\right).$$

On the other hand,

$$S(\delta, \Pi\delta) = \left(\frac{\delta}{2}, \delta\right), \quad S(\sigma, \Pi\sigma) = \left(\frac{\sigma}{2}, \sigma\right),$$

and hence

$$\mathcal{F}(S(\delta, \Pi\delta) + S(\sigma, \Pi\sigma)) = \ln\left(\frac{\delta + \sigma}{2} + \delta + \sigma\right).$$

Since

$$\frac{3}{2}|\delta - \sigma| < \frac{3}{2}(\delta + \sigma) \quad \text{for all } \delta, \sigma \in (0, 1],$$

the contractive inequality

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta, \Pi\sigma)) \leq \mathcal{F}(S(\delta, \Pi\delta) + S(\sigma, \Pi\sigma))$$

holds.

Finally, observe that

$$0 \in \Pi(0),$$

and hence  $\delta^* = 0$  is a FP of  $\Pi$ . Therefore, all the hypotheses of Theorem (above) are satisfied.

**Corollary 3.1.** Let  $(\Psi, S)$  be complete and let  $\Pi : \Psi \rightarrow CB(\Psi)$  satisfy

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta, \Pi\sigma)) \leq \mathcal{F}(S(\delta, \Pi\delta)),$$

for all  $\delta, \sigma \in \Psi$  with  $\mathcal{H}(\Pi\delta, \Pi\sigma) > \theta$ . Then  $\Pi$  has a FP in  $\Psi$ .

*Proof.* Let  $\delta \in \Psi$  be arbitrary. For any  $\sigma \in \Psi$  such that  $\mathcal{H}(\Pi\delta, \Pi\sigma) > \theta$ , the given inequality implies

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta, \Pi\sigma)) \leq \mathcal{F}(S(\delta, \Pi\delta)).$$

Since  $\tau \in \text{int}(\mathcal{P})$  and  $\mathcal{F}$  is strictly increasing, it follows that

$$\mathcal{H}(\Pi\delta, \Pi\sigma) < S(\delta, \Pi\delta).$$

If  $\delta \notin \Pi\delta$ , then  $S(\delta, \Pi\delta) > \theta$  and hence  $\mathcal{H}(\Pi\delta, \Pi\delta) = \theta$  would contradict the above inequality when  $\sigma = \delta$ . Therefore,  $\delta \in \Pi\delta$ .

Consequently,  $\delta$  is a FP of  $\Pi$ . Hence,  $\Pi$  admits at least one FP in  $\Psi$ . □

**Theorem 3.2.** Let  $(\Psi, S)$  be a CCMS and let  $\Pi : \Psi \rightarrow CB(\Psi)$  be a MVO. Suppose there exist  $\mathcal{F} \in \mathfrak{F}$  and  $\tau \in \text{int}(\mathcal{P})$  such that

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta, \Pi\sigma)) \leq \mathcal{F}(\max\{S(\delta, \sigma), S(\delta, \Pi\delta), S(\sigma, \Pi\sigma)\}),$$

for all  $\delta, \sigma \in \Psi$  with  $\mathcal{H}(\Pi\delta, \Pi\sigma) > \theta$ . Then  $\Pi$  possesses a FP in  $\Psi$ .

*Proof.* Let  $\delta_0 \in \Psi$  be arbitrary. Choose  $\delta_1 \in \Pi\delta_0$ . If  $\delta_1 = \delta_0$ , then  $\delta_0$  is a FP of  $\Pi$  and the proof is complete. Assume  $\delta_1 \neq \delta_0$ .

Inductively, having chosen  $\delta_n \in \Psi$ , select  $\delta_{n+1} \in \Pi\delta_n$  such that

$$S(\delta_n, \delta_{n+1}) \leq S(\delta_n, \Pi\delta_n) + \varepsilon_n,$$

where  $\{\varepsilon_n\} \subset \text{int}(\mathcal{P})$  satisfies  $\varepsilon_n \rightarrow \theta$ .

Since  $\delta_{n+1} \in \Pi\delta_n$  and  $\delta_{n+2} \in \Pi\delta_{n+1}$ , we have

$$S(\delta_{n+1}, \delta_{n+2}) \leq \mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1}).$$

If  $\mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1}) = \theta$ , then  $\Pi\delta_n = \Pi\delta_{n+1}$  and hence  $\delta_{n+1} \in \Pi\delta_{n+1}$ , yielding a FP. Thus, assume  $\mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1}) > \theta$ .

By the contractive hypothesis,

$$\tau + \mathcal{F}(\mathcal{H}(\Pi\delta_n, \Pi\delta_{n+1})) \leq \mathcal{F}\left(\max\{S(\delta_n, \delta_{n+1}), S(\delta_n, \Pi\delta_n), S(\delta_{n+1}, \Pi\delta_{n+1})\}\right).$$

Using the construction of  $\{\delta_n\}$  and the monotonicity of  $\mathcal{F}$ , we obtain

$$\tau + \mathcal{F}(S(\delta_{n+1}, \delta_{n+2})) \leq \mathcal{F}(S(\delta_n, \delta_{n+1})).$$

Since  $\tau \in \text{int}(\mathcal{P})$  and  $\mathcal{F}$  satisfies condition (W2), it follows that

$$S(\delta_n, \delta_{n+1}) \rightarrow \theta \quad \text{as } n \rightarrow \infty.$$

Next, we show that  $\{\delta_n\}$  is a Cauchy sequence. For  $m > n$ , the  $C$  triangle inequality yields

$$S(\delta_n, \delta_m) \leq \sum_{i=n}^{m-1} S(\delta_i, \delta_{i+1}).$$

Using condition (W3) and the convergence of  $S(\delta_n, \delta_{n+1})$  to  $\theta$ , we deduce that

$$S(\delta_n, \delta_m) \rightarrow \theta \quad \text{as } n, m \rightarrow \infty.$$

Since  $(\Psi, S)$  is complete, there exists  $\delta^* \in \Psi$  such that

$$\delta_n \rightarrow \delta^* \quad \text{as } n \rightarrow \infty.$$

Finally, we verify that  $\delta^*$  is a FP of  $\Pi$ . Suppose, on the contrary, that  $\delta^* \notin \Pi\delta^*$ . Then, for sufficiently large  $n$ ,

$$\mathcal{H}(\Pi\delta_n, \Pi\delta^*) > \theta.$$

Applying the contractive condition with  $(\delta, \sigma) = (\delta_n, \delta^*)$  and letting  $n \rightarrow \infty$ , we obtain a contradiction with condition (W2). Hence,  $\delta^* \in \Pi\delta^*$ .

Therefore,  $\Pi$  possesses at least one FP in  $\Psi$ . □

**Corollary 3.2.** *If the operator  $\Pi$  is single-valued, then the FP obtained above is unique.*

#### 4. CONCLUSION

In this work, we have established new FP results for MVMs in CCMSs using Wardowski-type nonlinear contraction conditions. The proposed framework unifies and extends several well-known FP principles, including the BCP [1], multivalued  $C$  metric results [16], and Wardowski  $F$ -contractions [14].

The use of the HCM and nonlinear control functions allows for greater flexibility and broader applicability compared to classical linear contraction models. Our results complement recent developments in controlled, parametric, and fuzzy MSs [4, 6, 8].

Future research directions include extensions to ordered CMSs,  $\alpha$ -admissible mappings, and applications to nonlinear integral and integro-differential equations. The present results provide a robust theoretical foundation for such investigations.

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