

A Novel N-Cubic Fuzzy DEMATEL Approach for Analyzing Causal Relationships in the Decision to Continue Studying in a Model School for the Elderly

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Abstract. This study presents a new analytical framework that combines the N-Cubic Fuzzy approach with the DEMATEL technique to examine how different factors interact to shape the decision to continue studying in a model school for the elderly in senior schools across Thailand. Unlike conventional fuzzy structures, the N-Cubic Fuzzy Set (NCFS) provides a flexible way to describe uncertainty, bringing together membership and non-membership ranges along with central tendencies and degrees of hesitation within a single model. This richer representation helps capture how human judgments vary under complex and ambiguous situations. To explore the main determinants of evaluation, a 27-item assessment was designed that covers four key aspects—personal, social, learning, and environmental. Each item was rated by a group of experts, and their linguistic evaluations were converted into NCFS values. The analysis applied a six-level influence scale, and the responses were combined through the N-Cubic Weighted Arithmetic Mean (NCFWAM). From this, a score matrix was obtained and adjusted using a tailored scoring function for NCFS, followed by a normalization step and the development of a total-relation matrix. Indices of prominence and relation were then computed to identify which factors exert the strongest causal effects. The findings indicate that the learning setting and social participation serve primarily as causal elements, whereas psychological well-being and daily functioning tend to be affected by them. Overall, the proposed NCFS-DEMATEL framework offers a practical means for analyzing multi-expert judgments in uncertain environments and provides insights that can inform policy design and enhance elderly care programs in real contexts. This study is aligned with Sustainable Development Goal 4 (Quality Education), highlighting lifelong learning and the importance of an appropriate learning environment for the elderly.

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1. INTRODUCTION

The fuzzy DEMATEL method builds on the traditional decision-making trial and evaluation laboratory (DEMATEL) framework by adding fuzzy set theory [1]. When the information is unclear or only available in words, this combination allows researchers to look into how the criteria affect each other. For decades, the method has become more popular in multi-criteria decision-making (MCDM) studies, especially when subjective judgments from domain experts are very important.

Several variations have been proposed to enhance its proficiency in managing linguistic ambiguity. For example, Abdullah and Goh [6] made a version based on Pythagorean fuzzy sets, and Gül [7] made the spherical fuzzy DEMATEL, which makes it even better at showing how vague expert opinions can be. Later, the intuitionistic fuzzy DEMATEL (IF-DEMATEL) was created to allow for uncertainty in language evaluation. Studies on bipolar and spherical bipolar fuzzy environments [8–12] have shown that they can be used to model both positive and negative trends in decision-making.

These changes have led to flexibility, but they still do not work well when there are multiple layers of uncertainty. In a lot of expert-driven systems, uncertainty is not just fuzzy; it is also interval-valued, bipolar, or hesitant, which are all hard to capture at the same time. A new idea called the N-Cubic Fuzzy Set (NCFS) was recently proposed to get around this problem [13]. The NCFS structure combines different types of uncertainty into one model, which gives us a better and more complete way to show how complex and structured vagueness works in real decision-making situations [15].

The N-Cubic fuzzy framework has not yet been used with causal analysis methods like DEMATEL, even though it is likely to be useful. This gap makes it hard to look at how different parts of a system depend on each other when a lot of uncertainty exists. The goal of this study is to fill that gap by introducing the N-Cubic Fuzzy DEMATEL, which combines the analytical basis of DEMATEL with the expressive modeling power of the N-Cubic fuzzy representation. A new score function, a normalization method, and an algorithm for figuring out total influence and causal prominence among criteria have all been created to defuzzify N-Cubic fuzzy information.

Here is a summary of what this paper adds to the field. (1) The study is the first to combine N-Cubic Fuzzy Sets with the DEMATEL framework, which gives us a mathematically sound way to look at causal relationships when there is a lot of uncertainty. (2) A new scoring and normalization system is created to make it easier to combine numbers in the N-Cubic environment. (3) A detailed method for calculating the total-relation matrix and finding causal and effect groups is created. (4) Finally, a numerical case study is used to check that the proposed method works and is possible in a real-world decision-making situation.

The rest of this paper is organized as follows: In Section 2, we go over the basic ideas behind fuzzy DEMATEL and N-Cubic fuzzy theory. In detail, Section 3 deals with the proposed N-Cubic Fuzzy DEMATEL method. Section 4 shows a numerical case study, and Section 5 provides with comments and suggestions for future work.

2. PRELIMINARIES

In this section, we provide essential background on fuzzy set extensions that lead to the formulation of the N-Cubic Fuzzy Set (NCFS), which serves as the foundation for the proposed model.

2.1. Intuitionistic Fuzzy Sets (IFS). The concept of intuitionistic fuzzy sets (IFS), introduced by Atanassov [2], extends Zadeh's fuzzy set theory [1] by incorporating both membership and non-membership degrees. An IFS A in a universe of discourse X is defined as:

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$$

where $\mu_A(x) \in [0, 1]$ denotes the degree of membership, and $\nu_A(x) \in [0, 1]$ denotes the degree of non-membership of x in A , such that:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1, \quad \forall x \in X.$$

The hesitation margin (or indeterminacy degree) is given by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

2.2. Cubic Fuzzy Sets (CFS). A cubic fuzzy set (CFS) combines an interval-valued fuzzy set and an IFS to better capture uncertainty. It was introduced to handle cases where both a range of membership and a crisp intuitionistic value are meaningful [3]. A CFS B on X is defined as

$$B = \left\{ \left(x, [\mu_B^L(x), \mu_B^U(x)], \nu_B(x) \right) \mid x \in X \right\}$$

where $[\mu_B^L(x), \mu_B^U(x)] \subseteq [0, 1]$ is the interval-valued membership degree, and $\nu_B(x) \in [0, 1]$ is the non-membership degree.

2.3. N-Cubic Fuzzy Sets (NCFS). To model more complex and layered forms of uncertainty, the N-Cubic Fuzzy Set (NCFS) is proposed. It generalizes previous models by incorporating, lower and upper bounds of membership, corresponding uncertainty (confidence) values, and non-membership information, also with uncertainty.

Definition 2.1 (N-Cubic Fuzzy Set). Let X be a universal set. An N-Cubic fuzzy set z defined on X is given by

$$z = \left\{ \left(u, \left[(e_z^-(u), \omega_z^-(u)), (e_z^+(u), \omega_z^+(u)) \right], (c_z(u), \omega_z^c(u)) \right) \mid u \in X \right\}$$

where

- $e_z^-(u)$ and $e_z^+(u)$ represent the lower and upper bounds of the membership degree,
- $\omega_z^-(u)$ and $\omega_z^+(u)$ denote the confidence (or reliability) associated with each bound,
- $c_z(u)$ is the non-membership degree, and $\omega_z^c(u)$ is its corresponding uncertainty.

All components satisfy

$$0 \leq e_z^-(u) \leq e_z^+(u) \leq 1, \quad 0 \leq c_z(u), \omega_z^-(u), \omega_z^+(u), \omega_z^c(u) \leq 1.$$

This structure allows the NCFS to flexibly capture asymmetric uncertainty, confidence levels, and dual-valued interpretations of each element in X .

3. PRELIMINARIES AND DEFINITIONS

Definition 3.1. Let $X = \{x_1, x_2, \dots, x_n\}$ be a set of criteria or factors. An **N-Cubic Fuzzy Number (NCFN)** \tilde{z}_{ij} is defined as

$$\tilde{z}_{ij} = \left\langle \left[(e_{ij}^-, \omega_{ij}^-), (e_{ij}^+, \omega_{ij}^+) \right], (c_{ij}, \omega_{ij}^c) \right\rangle$$

where

- $(e_{ij}^-, \omega_{ij}^-)$ and $(e_{ij}^+, \omega_{ij}^+)$ are lower and upper bounds of membership degrees with their confidence levels.
- (c_{ij}, ω_{ij}^c) denotes the non-membership degree and its uncertainty.

Definition 3.2. The **N-Cubic Fuzzy Direct-Relation Matrix (NCFDRM)** is denoted by $\tilde{Z} = [\tilde{z}_{ij}]_{n \times n}$, where \tilde{z}_{ij} expresses the degree of influence of criterion x_i on x_j .

Definition 3.3. A **Score Function** $S(\tilde{z}_{ij})$ to convert an NCFN to a crisp value is defined as

$$S(\tilde{z}_{ij}) = \frac{1}{6} \left(e_{ij}^- + e_{ij}^+ + c_{ij} + \frac{\omega_{ij}^- + \omega_{ij}^+ + \omega_{ij}^c}{2\pi} \right)$$

Definition 3.4 (Operations on N-Cubic Fuzzy Sets (Proposed)). Let $A = \left\langle \left[(e_A^-, \omega_A^-), (e_A^+, \omega_A^+) \right], (c_A, \omega_A^c) \right\rangle$ and $B = \left\langle \left[(e_B^-, \omega_B^-), (e_B^+, \omega_B^+) \right], (c_B, \omega_B^c) \right\rangle$ be two N-Cubic fuzzy numbers.

Then, the union, intersection, and complement of A and B are defined as

- **Union** ($A \cup B$)

$$\begin{aligned} e_{A \cup B}^- &= \min\{e_A^-, e_B^-\}, & e_{A \cup B}^+ &= \max\{e_A^+, e_B^+\}, \\ \omega_{A \cup B}^- &= \max\{\omega_A^-, \omega_B^-\}, & \omega_{A \cup B}^+ &= \max\{\omega_A^+, \omega_B^+\}, \\ c_{A \cup B} &= \min\{c_A, c_B\}, & \omega_{A \cup B}^c &= \max\{\omega_A^c, \omega_B^c\}. \end{aligned}$$

- **Intersection** ($A \cap B$)

$$\begin{aligned} e_{A \cap B}^- &= \max\{e_A^-, e_B^-\}, & e_{A \cap B}^+ &= \min\{e_A^+, e_B^+\}, \\ \omega_{A \cap B}^- &= \min\{\omega_A^-, \omega_B^-\}, & \omega_{A \cap B}^+ &= \min\{\omega_A^+, \omega_B^+\}, \\ c_{A \cap B} &= \max\{c_A, c_B\}, & \omega_{A \cap B}^c &= \min\{\omega_A^c, \omega_B^c\}. \end{aligned}$$

- **Complement** (\bar{A})

$$\begin{aligned} e_{\bar{A}}^- &= 1 - e_A^+, & e_{\bar{A}}^+ &= 1 - e_A^-, \\ \omega_{\bar{A}}^- &= \omega_A^+, & \omega_{\bar{A}}^+ &= \omega_A^-, \\ c_{\bar{A}} &= 1 - c_A, & \omega_{\bar{A}}^c &= \omega_A^c. \end{aligned}$$

These operations are defined as element-wise for each $x \in X$.

Definition 3.5 (Proposed Operations on N-Cubic Fuzzy Sets). Let

$$A = \langle [(e_A^-, \omega_A^-), (e_A^+, \omega_A^+)], (c_A, \omega_A^c) \rangle, \quad B = \langle [(e_B^-, \omega_B^-), (e_B^+, \omega_B^+)], (c_B, \omega_B^c) \rangle$$

be two N-Cubic fuzzy numbers defined on a universe of discourse X . Then, the operations are defined as follows:

• **Union**

$$A \sqcup B = \langle [(\min\{e_A^-, e_B^-\}, \max\{\omega_A^-, \omega_B^-\}), (\max\{e_A^+, e_B^+\}, \max\{\omega_A^+, \omega_B^+\})], (\min\{c_A, c_B\}, \max\{\omega_A^c, \omega_B^c\}) \rangle$$

• **Intersection**

$$A \sqcap B = \langle [(\max\{e_A^-, e_B^-\}, \min\{\omega_A^-, \omega_B^-\}), (\min\{e_A^+, e_B^+\}, \min\{\omega_A^+, \omega_B^+\})], (\max\{c_A, c_B\}, \min\{\omega_A^c, \omega_B^c\}) \rangle$$

• **Complement**

$$\bar{A} = \langle [(1 - e_A^+, \omega_A^+), (1 - e_A^-, \omega_A^-)], (1 - c_A, \omega_A^c) \rangle$$

• **Addition**

$$A \oplus B = \left\langle \left[\left(\sqrt{e_A^{-2} + e_B^{-2} - e_A^{-2}e_B^{-2}}, \omega_A^- + \omega_B^- \right), \left(\sqrt{e_A^{+2} + e_B^{+2} - e_A^{+2}e_B^{+2}}, \omega_A^+ + \omega_B^+ \right) \right], \left(\sqrt{c_A^2 + c_B^2 - c_A^2c_B^2}, \omega_A^c + \omega_B^c \right) \right\rangle$$

• **Multiplication**

$$A \otimes B = \langle [(e_A^- e_B^-, \min\{\omega_A^-, \omega_B^-\}), (e_A^+ e_B^+, \min\{\omega_A^+, \omega_B^+\})], (c_A c_B, \min\{\omega_A^c, \omega_B^c\}) \rangle$$

• **Scalar Multiplication** For $\lambda > 0$,

$$\lambda A = \langle [(1 - (1 - e_A^{-2})^\lambda)^{1/2}, \lambda \cdot \omega_A^-], (1 - (1 - e_A^{+2})^\lambda)^{1/2}, \lambda \cdot \omega_A^+], ((c_A)^\lambda, \lambda \cdot \omega_A^c) \rangle$$

• **Power** For $\lambda > 0$,

$$A^\lambda = \langle [(e_A^-)^\lambda, (\omega_A^-)^\lambda], ((e_A^+)^\lambda, (\omega_A^+)^\lambda)], ((c_A)^\lambda, (\omega_A^c)^\lambda) \rangle$$

Theorem 3.1 (Algebraic Properties of N-Cubic Fuzzy Sets). Let A and B be two N-Cubic fuzzy numbers, and $\lambda, \lambda_1, \lambda_2 > 0$. Then, the following properties hold:

- $A \oplus B = B \oplus A$ (Commutativity of addition)
- $A \otimes B = B \otimes A$ (Commutativity of multiplication)
- $\lambda(A \oplus B) = \lambda A \oplus \lambda B$ (Distributivity over scalar)
- $\lambda_1 A \oplus \lambda_2 A = (\lambda_1 + \lambda_2)A$
- $(A \otimes B)^\lambda = A^\lambda \otimes B^\lambda$ (Power of product)
- $A^{\lambda_1} \otimes A^{\lambda_2} = A^{\lambda_1 + \lambda_2}$

Proof. Straightforward from the definitions of \oplus , \otimes , and A^λ in the N-Cubic fuzzy framework. □

Definition 3.6 (N-Cubic Fuzzy Weighted Arithmetic Mean (NCFWAM)). Let

$$A_i = \langle [(e_i^-, \omega_i^-), (e_i^+, \omega_i^+)], (c_i, \omega_i^c) \rangle,$$

and $w_i \in [0, 1]$ with $\sum_{i=1}^n w_i = 1$.

The NCFWAM is defined as

$$\text{NCFWAM}_w(A_1, \dots, A_n) = w_1 A_1 \oplus w_2 A_2 \oplus \dots \oplus w_n A_n$$

Where each component is computed as

$$\begin{aligned} e^- &= \sqrt{1 - \prod_{i=1}^n (1 - (e_i^-)^2)^{w_i}}, \\ e^+ &= \sqrt{1 - \prod_{i=1}^n (1 - (e_i^+)^2)^{w_i}}, \\ c &= \sqrt{1 - \prod_{i=1}^n (1 - (c_i)^2)^{w_i}}, \\ \omega^- &= \sum_{i=1}^n w_i \cdot \omega_i^-, \quad \omega^+ = \sum_{i=1}^n w_i \cdot \omega_i^+, \quad \omega^c = \sum_{i=1}^n w_i \cdot \omega_i^c. \end{aligned}$$

Hence, the result is

$$\langle [(e^-, \omega^-), (e^+, \omega^+)], (c, \omega^c) \rangle$$

Example 3.1 (Example of NCFWAM Aggregation). Consider three N-Cubic Fuzzy Sets

$$A_1 = \langle [(0.30, 0.25), (0.60, 0.20)], (0.40, 0.30) \rangle,$$

$$A_2 = \langle [(0.50, 0.20), (0.70, 0.15)], (0.60, 0.25) \rangle,$$

$$A_3 = \langle [(0.40, 0.30), (0.80, 0.10)], (0.50, 0.20) \rangle,$$

with weights

$$w_1 = 0.3, \quad w_2 = 0.4, \quad w_3 = 0.3.$$

Using the N-Cubic Fuzzy Weighted Arithmetic Mean (NCFWAM), we compute each component as

$$e^- = \sqrt{1 - [(1 - 0.30^2)^{0.3} \cdot (1 - 0.50^2)^{0.4} \cdot (1 - 0.40^2)^{0.3}]} = 0.447,$$

$$e^+ = \sqrt{1 - [(1 - 0.60^2)^{0.3} \cdot (1 - 0.70^2)^{0.4} \cdot (1 - 0.80^2)^{0.3}]} = 0.721,$$

$$c = \sqrt{1 - [(1 - 0.40^2)^{0.3} \cdot (1 - 0.60^2)^{0.4} \cdot (1 - 0.50^2)^{0.3}]} = 0.535,$$

$$\omega^- = 0.3 \cdot 0.25 + 0.4 \cdot 0.20 + 0.3 \cdot 0.30 = 0.245,$$

$$\omega^+ = 0.3 \cdot 0.20 + 0.4 \cdot 0.15 + 0.3 \cdot 0.10 = 0.145,$$

$$\omega^c = 0.3 \cdot 0.30 + 0.4 \cdot 0.25 + 0.3 \cdot 0.20 = 0.247.$$

Thus, the aggregated result is

$$\text{NCFWAM}_w(A_1, A_2, A_3) = \langle [(0.447, 0.245), (0.721, 0.145)], (0.535, 0.247) \rangle.$$

Definition 3.7 (N-Cubic Fuzzy Weighted Geometric Mean (NCFWGM)). Let

$$A_i = \langle [(e_i^-, \omega_i^-), (e_i^+, \omega_i^+)], (c_i, \omega_i^c) \rangle,$$

and $w_i \in [0, 1]$ with $\sum_{i=1}^n w_i = 1$.

Then the NCFWGM is defined as

$$\text{NCFWGM}_w(A_1, \dots, A_n) = A_1^{w_1} \otimes A_2^{w_2} \otimes \dots \otimes A_n^{w_n}$$

Where each component is computed as

$$\begin{aligned} e^- &= \prod_{i=1}^n (e_i^-)^{w_i}, \\ e^+ &= \prod_{i=1}^n (e_i^+)^{w_i}, \\ c &= \prod_{i=1}^n (c_i)^{w_i}, \\ \omega^- &= \prod_{i=1}^n (\omega_i^-)^{w_i}, \quad \omega^+ = \prod_{i=1}^n (\omega_i^+)^{w_i}, \quad \omega^c = \prod_{i=1}^n (\omega_i^c)^{w_i}. \end{aligned}$$

Hence, the result is

$$\langle [(e^-, \omega^-), (e^+, \omega^+)], (c, \omega^c) \rangle$$

Example 3.2 (Example of NCFWGM Aggregation). Consider three N-Cubic Fuzzy Sets

$$A_1 = \langle [(0.30, 0.25), (0.60, 0.20)], (0.40, 0.30) \rangle,$$

$$A_2 = \langle [(0.50, 0.20), (0.70, 0.15)], (0.60, 0.25) \rangle,$$

$$A_3 = \langle [(0.40, 0.30), (0.80, 0.10)], (0.50, 0.20) \rangle,$$

with corresponding weights

$$w_1 = 0.3, \quad w_2 = 0.4, \quad w_3 = 0.3.$$

Applying the NCFWGM operator

$$e^- = (0.30)^{0.3} \cdot (0.50)^{0.4} \cdot (0.40)^{0.3} = 0.401,$$

$$\omega^- = (0.25)^{0.3} \cdot (0.20)^{0.4} \cdot (0.30)^{0.3} = 0.242,$$

$$e^+ = (0.60)^{0.3} \cdot (0.70)^{0.4} \cdot (0.80)^{0.3} = 0.696,$$

$$\omega^+ = (0.20)^{0.3} \cdot (0.15)^{0.4} \cdot (0.10)^{0.3} = 0.145,$$

$$c = (0.40)^{0.3} \cdot (0.60)^{0.4} \cdot (0.50)^{0.3} = 0.503,$$

$$\omega^c = (0.30)^{0.3} \cdot (0.25)^{0.4} \cdot (0.20)^{0.3} = 0.247.$$

Hence, the aggregated result is

$$NCFWGM_w(A_1, A_2, A_3) = \langle [(0.401, 0.242), (0.696, 0.145)], (0.503, 0.247) \rangle.$$

4. PROPOSED N-CUBIC FUZZY DEMATEL METHOD

Step 1: Identifying attributes and selecting decision-makers.

The set of decision-makers D^k ($k = 1, 2, \dots, m$) and the set of evaluation criteria or attributes C_i ($i, j = 1, 2, \dots, n$) are identified. Each decision-maker will evaluate the direct influence between every pair of attributes.

Step 2: Selecting an evaluation measure for linguistic assessment.

To facilitate expert judgments in the N-Cubic Fuzzy DEMATEL process, we define a set of linguistic terms (e.g., Strong, Moderate, Weak, No Influence), each represented by an N-Cubic fuzzy number of the form:

$$\tilde{d}_{ij} = \left\langle \left[(e_{ij}^-, \omega_{ij}^-), (e_{ij}^+, \omega_{ij}^+) \right], (c_{ij}, \omega_{ij}^c) \right\rangle.$$

We adopt a score function specifically designed for N-Cubic fuzzy numbers to translate linguistic terms into numeric score indices SI . The proposed score index for \tilde{d}_{ij} is defined as

$$SI(\tilde{d}_{ij}) = \left| 50 \times \left[(e_{ij}^+ - c_{ij})^2 - (\omega_{ij}^+ - \omega_{ij}^c)^2 + (e_{ij}^- - c_{ij})^2 - (\omega_{ij}^- - \omega_{ij}^c)^2 \right] \right|^{1/2}$$

This formula captures the intensity of influence based on the distance between membership degrees (e^-, e^+) and non-membership degree (c), while incorporating uncertainty via ($\omega^-, \omega^+, \omega^c$).

We then define four linguistic terms commonly used in pairwise influence judgments, along with their corresponding N-Cubic fuzzy values and SI values as shown in Table 1.

TABLE 1. Linguistic Terms and Corresponding N-Cubic Fuzzy Values with Score Indices (6 Levels)

Definition	Abbr	e^-	ω^-	e^+	ω^+	c	ω^c	$SI(\tilde{d}_{ij})$
Very Strong	VS	0.80	0.10	0.98	0.05	0.60	0.10	5
Strong	S	0.70	0.15	0.90	0.10	0.55	0.15	4
Moderate	M	0.55	0.20	0.75	0.20	0.45	0.25	3
Weak	W	0.35	0.25	0.55	0.25	0.35	0.30	2
Very Weak	VW	0.15	0.30	0.30	0.30	0.25	0.35	1
No Influence	NI	0.00	0.20	0.10	0.20	0.15	0.30	0

The SI values computed from the above formula yield crisp equivalents of the fuzzy linguistic terms, enabling further matrix operations and normalization in subsequent steps of the NCFSD-DEMATEL process.

Step 3: Defining decision-maker weights.

Initial linguistic importance \hat{w}_k to each decision-maker D^k using a linguistic-to-numeric scale (e.g., Equally Important = 0.6, Very Important = 1.0) is assigned. The weights are normalized as follows:

$$w_k = \frac{\hat{w}_k}{\sum_{k=1}^m \hat{w}_k}, \quad \text{so that} \quad \sum_{k=1}^m w_k = 1.$$

Step 4: Aggregating the direct influence evaluations from multiple decision-makers.

Let d_{ij}^k be the NCFS assigned by decision-maker D^k to the influence of attribute C_i on C_j . The aggregated NCFS is computed via the N-Cubic Weighted Arithmetic Mean (NCFWAM)

$$d_{ij} = \text{NCFWAM}_w(d_{ij}^1, d_{ij}^2, \dots, d_{ij}^m) = \left\langle \left[(e_{ij}^-, \omega_{ij}^-), (e_{ij}^+, \omega_{ij}^+) \right], (c_{ij}, \omega_{ij}^c) \right\rangle.$$

Step 5: Calculating the NCFS-based score function matrix $S = [s_{ij}]$.

For each aggregated NCFS d_{ij} , its score value is computed using a proposed score function for NCFS:

$$s_{ij} = \frac{1}{2} \left[(e_{ij}^+ - c_{ij})^2 - (\omega_{ij}^+ - \omega_{ij}^c)^2 + (e_{ij}^- - c_{ij})^2 - (\omega_{ij}^- - \omega_{ij}^c)^2 \right]$$

Step 6: Normalizing the score matrix.

Normalize S is normalized into a matrix X using the normalization factor q

$$X = qS, \quad \text{where} \quad q = \min \left\{ \frac{1}{\max_i \sum_{j=1}^n |s_{ij}|}, \frac{1}{\max_j \sum_{i=1}^n |s_{ij}|} \right\}$$

Step 7: Constructing the total-relation matrix.

$$T = X(I - X)^{-1}$$

where $T = [t_{ij}]$ reflects both direct and indirect influences between all attribute pairs.

Step 8: Computing Prominence and Relation; drawing the cause-effect diagram.

Let:

$$D_i = \sum_{j=1}^n t_{ij}, \quad (\text{influence given}) \quad R_i = \sum_{j=1}^n t_{ji}, \quad (\text{influence received})$$

$$D_i + R_i = \text{Prominence} \quad D_i - R_i = \text{Relation}$$

- If $D_i - R_i > 0$, then C_i belongs to the cause group.

- If $D_i - R_i < 0$, then C_i belongs to the effect group.

Plotting $(D_i + R_i, D_i - R_i)$ on a 2D chart enables the identification of strategic factors in the system.

The proposed N-Cubic Fuzzy DEMATEL methodology integrates the concept of N-Cubic fuzzy sets into the classical DEMATEL framework in order to handle complex uncertainty and subjective judgments in multi-criteria decision-making. The method consists of four major as follows:

- (1) computing the score matrix based on the N-Cubic fuzzy score function,
- (2) normalizing the score matrix using a dynamic scaling factor,

5. APPLICATION: EVALUATING ELDERLY QUALITY OF LIFE USING NCFS-DEMATEL

To illustrate the applicability of the proposed N-Cubic Fuzzy DEMATEL (NCFS-DEMATEL) methodology, a case study was conducted to evaluate the decision to continue studying in model schools for the elderly in Mueang District, Phitsanulok Province, Thailand. The analysis employed a 27-item questionnaire structured based on four key dimensions. The constructed questionnaire was validated by three experts in related fields using the Index of Item-Objective Congruence (IOC). Additional, the research project was approved by the Human Research Ethics of Rajamangala University of Technology Lanna (RMUTL-IRB 022/2025).

The research participants included seven personnel working in the selected prototype elderly school in the prototype elderly school in Mueang Phitsanulok District. The selection was based on personnel with expertise working with the elderly for at least 10 years in the prototype elderly school in Mueang Phitsanulok District who received awards from the selection of the Department of Older Persons Affairs, Ministry of Social Development and Human Security. The complete list of 27 quality of life assessment items used in this study is as follows.

5.1. Evaluation Criteria and Study Variables.

- **Personal Factors** (Items 1–10): capturing the motivations, psychological needs, and inter-personal behaviors of elderly individuals.
- **Social Factors** (Items 11–15): reflecting familial support, peer influence, and community involvement.
- **Learning Factors** (Items 16–21): covering aspects related to curriculum quality, instructor competence, and learning formats.
- **Environmental Factors** (Items 22–27): involving access to infrastructure, accessibility of information, and ease of transportation.

A panel of experts was asked to evaluate the causal relationships among these 27 indicators using linguistic terms, which were mapped to predefined N-Cubic fuzzy numbers. These values capture the range and confidence of influence between each pair of factors as follows

5.1.1. *Personal Factors.*

- C1 Older adults desire to learn new things.
- C2 Older adults want to use their free time productively.
- C3 Older adults wish to meet, talk, or participate in activities with other elderly people at the school.
- C4 Older adults can share or pass on good experiences and practices to others or younger generations, such as children, grandchildren, or close relatives.
- C5 Older adults can apply the knowledge or skills gained from the elderly school to solve problems or improve their daily lives.
- C6 On important days or festivals, older adults receive blessings, gifts, or participate in activities that demonstrate care and attention.

- C7 Older adults feel proud to be part of the elderly school community.
- C8 When older adults express their opinions, others listen and respect them, including mutual respect among classmates.
- C9 Older adults can request cooperation from others to accomplish what they desire.
- C10 Older adults communicate politely, with consideration and empathy toward their conversation partners or classmates.

5.1.2. *Social Factors.*

- C11 The families of older adults participate in the decision-making process regarding their enrollment in the elderly school.
- C12 The families of older adults provide support and encouragement during their studies at the elderly school.
- C13 Older adults enroll in the elderly school through the invitation of friends or close acquaintances.
- C14 Older adults are inspired and motivated to be role models when they see friends or peers of the same age enrolling in the elderly school.
- C15 Older adults wish to be part of community activities and participate in various events.

5.1.3. *Learning Factors.*

- C16 The curriculum and activities are interesting and applicable to real life.
- C17 The curriculum and activities support the holistic development of learners, including aspects such as health, emotions, and technology.
- C18 The personnel or staff at the elderly school possess knowledge and skills in working effectively with older adults.
- C19 The personnel or staff at the elderly school are friendly, humble, understanding, and approachable.
- C20 Teachers or facilitators in each subject or activity possess knowledge and the ability to communicate and transfer learning effectively.
- C21 Teachers or facilitators organize teaching methods or activities appropriate to the learning topics and the potential of older adults.

5.1.4. *Environmental Factors.*

- C22 Older adults can follow news and activity updates from the school through various channels such as staff, the LINE application, Facebook, or public announcement boards.
- C23 The elderly school has suitable restrooms and ramps for the use of older adults.
- C24 Desks, chairs, and other learning support equipment are appropriate for the use of older adults.
- C25 The distance from the home or residence of older adults to the school is reasonable.
- C26 Transportation to the elderly school is convenient and accessible.

C27 The school or related agencies provide transportation assistance for older adults who cannot travel by themselves.

Following the NCFs-DEMATEL process described in Algorithm 1, we performed the following key steps:

- (1) **Expert Evaluation:** Experts provided pairwise influence ratings between indicators using linguistic terms such as “Strong”, “Moderate”, “Weak”, and “No influence”, converted into N-Cubic fuzzy values.
- (2) **Aggregation:** The fuzzy judgments of multiple experts were aggregated using the N-Cubic Weighted Arithmetic Mean (NCFWAM).
- (3) **Scoring and Normalization:** Each aggregated N-Cubic value was converted into a score using a predefined NCFs score function and normalized to obtain the direct influence matrix.
- (4) **Total Influence and Analysis:** The total relation matrix was constructed, and the prominence and relation indices were computed to identify key causal and effect factors.

This approach allows for a systematic identification of key factors affecting the quality of life of older adults, including the factors that primarily influence research findings, as follows:

5.2. Results of DEMATEL Analysis. The Decision-Making Trial and Evaluation Laboratory (DEMATEL) method was applied to investigate the causal interrelationships among the N-Cubic fuzzy criteria used to evaluate the quality of life of elderly participants in senior schools. The results are illustrated through the cause–effect diagram and the direct-influence heatmap.

Cause–Effect Analysis. Figure 1 shows the cause–effect diagram, where the horizontal axis (prominence, $r + c$) indicates the degree of importance of each criterion in the overall system, while the vertical axis (relation, $r - c$) differentiates between the cause group (positive values) and the effect group (negative values).

Heatmap of Direct Influences. Figure 2 depicts the heatmap of the direct-influence matrix. Darker shades indicate stronger interactions between criteria. The visualization confirms that learning-related and social-oriented factors serve as central transmitters of influence across the system, supporting the classification of these criteria into the cause group.

Discussion of Causal Analysis Results. Overall, the DEMATEL results demonstrate that *learning and social criteria act as key causal drivers*, while *personal and environmental criteria represent the outcomes*. This finding implies that policies and interventions should prioritize enhancing educational engagement, peer interaction, and knowledge exchange within senior schools. By focusing on these causal factors, it becomes possible to indirectly improve critical outcomes such as emotional security, adaptability, and overall quality of life. In practical terms, senior schools and policymakers should design programs that encourage collaborative learning, strengthen community participation, and provide accessible, safe facilities, thereby fostering a holistic improvement

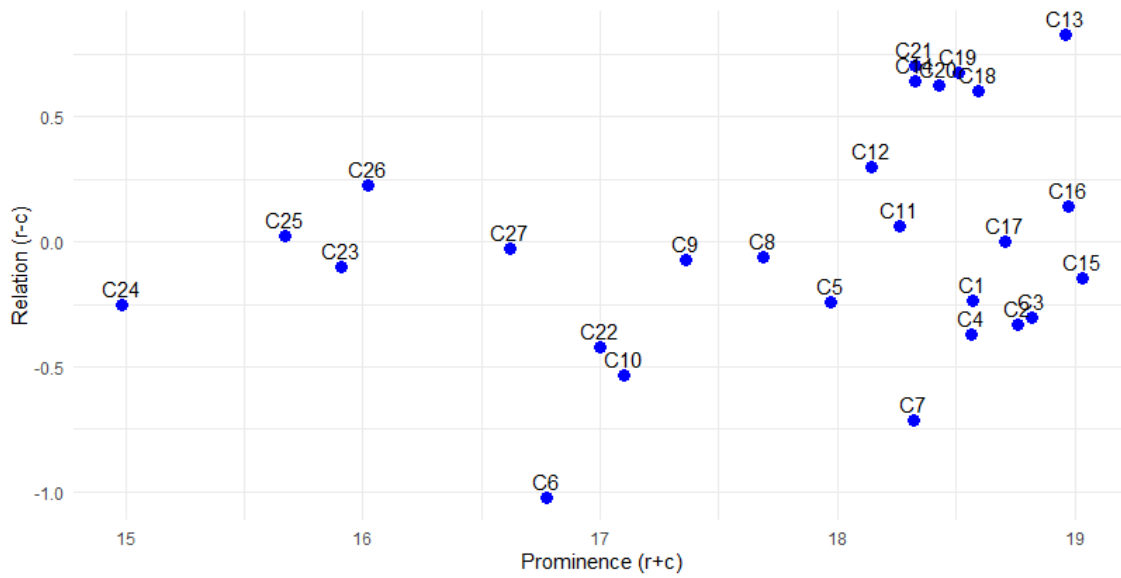


FIGURE 1. Cause-effect diagram of N-Cubic fuzzy criteria for elderly quality of life in senior schools. Prominence ($r + c$) reflects the overall importance of each criterion, while relation ($r - c$) separates cause and effect groups.

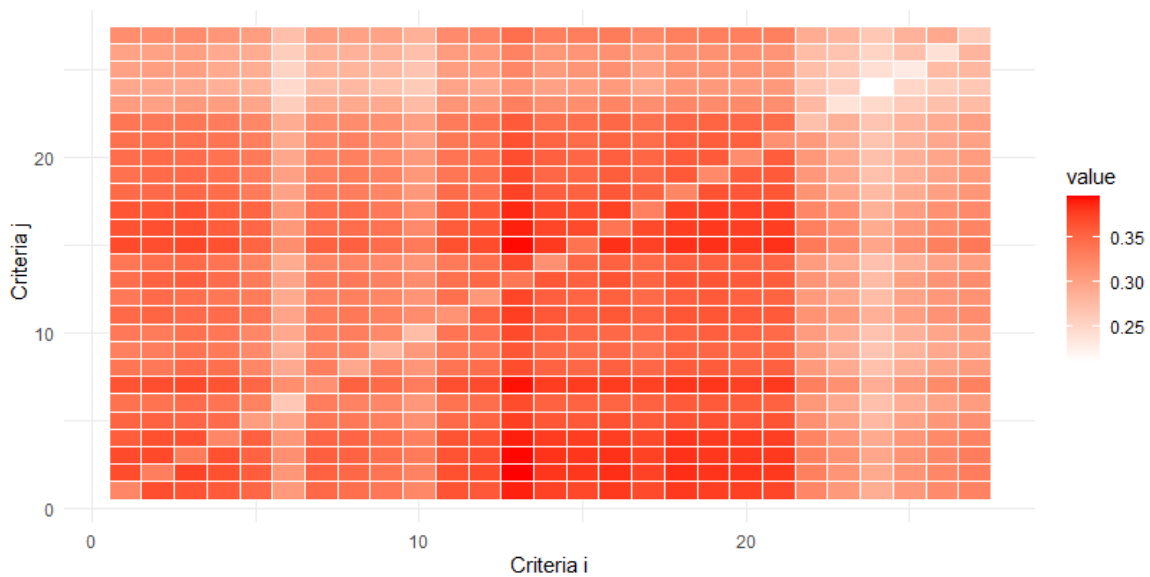


FIGURE 2. Heatmap of the direct-influence matrix showing the intensity of relationships among all N-Cubic fuzzy criteria.

in the quality of life of the elderly population. According to a 2024 survey by the National Statistical Office, Thailand has officially entered a complete aged society, with more than 20% of its population aged 60 and above. Key data on the elderly have been collected and are useful for both government and private agencies. These data can be used to design welfare programs, services, and environments that enable older adults to live stable, self-reliant lives with an improved quality

of life. One effective approach to enhancing the quality of life of the elderly is to create spaces for participation in community activities, regardless of personal priorities. In this regard, the Department of Older Persons Affairs, Ministry of Social Development and Human Security in 2017, developed a manual for the operation of elderly schools. This guide serves as a framework for local administrative organizations, elderly associations, and other agencies interested in establishing schools for the elderly. It aims to promote lifelong learning and provide education on topics of interest to older adults while fostering the development of essential life skills. Instruction is provided by volunteer teachers or relevant agencies. At the same time, the schools offer a platform for the elderly to demonstrate their potential by sharing knowledge, experience, and accumulated wisdom. Activities at elderly schools are organized on a clear weekly schedule, typically held one day per week. The locations of these schools vary and are often integrated into existing community facilities, such as school buildings, elderly clubs, temples, or local administrative offices whose missions include improving the quality of life for people of all ages.

A study was conducted on the factors contributing to continued enrollment in two model elderly schools in Mueang District, Phitsanulok Province: Rom Samo Witthaya School for the Elderly (established in 2016) and Plai Chumphon School for the Elderly (established in 2017). Both schools have received the Model School for the Elderly Award from the Department of Older Persons Affairs, Ministry of Social Development and Human Security. They are recognized for offering activities that enhance the quality of life of their students and for serving as models in the development of elderly care systems by local administrative organizations. Many other elderly schools have organized study visits to these two schools to learn and adapt their programs for their own communities.

Both schools have consistently maintained annual enrollment, attracting both returning and new students who wish to continue their studies and complete the courses. The analysis revealed that the primary causal factors driving enrollment in these elderly schools are social and learning factors. The most prominent social factors are C13 and C14. These factors reflect older adults' desire for a supportive social environment and opportunities to engage in group activities. Participation in such activities fosters a sense of acceptance and belonging, allowing older adults to perceive themselves as integral members of society.

Moreover, the development of intellectual abilities and essential life skills—applicable in daily living and shareable with others—enhances self-worth, life satisfaction, and mental well-being. This finding aligns with research by Nasamon Butwiset and Uparittha Inthasat in 2021 [16], who found that membership in social groups or clubs positively influences the quality of life of the elderly. Their study in Phra Nakhon Si Ayutthaya District indicated that club participation motivates the elderly to gather, gain knowledge, and exchange experiences. The diversity of lifestyles within such groups can inspire individuals to improve their own practices, benefiting both their bodies and minds. In addition, research by Sirikan Krajangpho et al. in 2018 [17] emphasized that activities for the elderly should meet their needs and align with their prior

experiences. This practice will lead to positive change, teamwork, trust, strong relationships, and a heightened sense of self-worth.

Another factor contributing to the continued enrollment of older adults in the education system for the elderly is learning-related factors of C21 and C19. These findings indicate that lifelong learning plays an important role in reducing the decline that may occur among the elderly due to changes in their physical and mental conditions. Physically, the condition of older adults naturally deteriorates with age, leading to issues such as neurocognitive disorders and non-chronic infectious diseases. Some elderly individuals may also experience poor mental health, often stemming from life transitions such as retirement—which may cause feelings of worthlessness—or the loss of a spouse or loved one, resulting in sadness and anxiety. Studying and participating in activities within the elderly education system, which are designed to suit their learning needs and capacities, help older adults develop themselves both physically and mentally, while also fostering awareness of changes within the social context of school activities. Furthermore, the understanding and supportive relationships among school personnel and staff ensure that the elderly do not feel lonely or devalued.

These insights are consistent with the research findings of Chettha Kaewprom et al. in 2020 [18], which highlight that school activities promoting relationships, health, acceptance, learning, achievement, mutual benefits, and good citizenship among seniors contribute to a greater sense of self-worth. Such activities are developed under the concept of lifelong learning, based on the principle that older adults possess inherent value and potential that should be nurtured holistically—physically, mentally, emotionally, intellectually, and socially.

Discussion of Impact Analysis Results. The research found that the factors with the greatest impact on continued enrollment in model schools for the elderly are personal factors of C6, C7, and C10. These findings indicate that when older adults attend schools for the elderly, they receive care and attention through activities organized by peers, staff, and the community. This fosters a sense of self-worth and belonging, reinforcing the feeling that they are not alone in the aging process. School activities, designed through appropriate curriculum planning, promote teamwork and experience-sharing, which in turn enhance communication skills, empathy, and understanding of individual differences.

Furthermore, opportunities for the elderly to develop new knowledge and skills—along with the chance to share their accumulated knowledge, experiences, and wisdom with the community or younger generations—instill a sense of pride in contributing to others. These findings are consistent with the research of Yurathorn Jeena in 2024 [19], who proposed an integrated model of home, temple, and school for strengthening self-worth among the elderly in communities. The study revealed that self-esteem issues in older adults often stem from generational misunderstandings within families, leading to weak communication and emotional distance. Therefore, integrated collaboration among homes, temples, schools, and government agencies should be established to promote activities that enable the elderly to take on meaningful roles, demonstrate their abilities,

and pass on local wisdom. Such cooperation helps reinforce their sense of self-worth. Self-worth can be developed from receiving both physical and emotional support—love, care, and acceptance—from others. This process of self-recognition and social acceptance is essential for older adults, as it helps cultivate positive attitudes, confidence, emotional stability, social adjustment, and the ability to cope with or accept disappointment in appropriate ways.

The second most significant factors affecting continued enrollment in model schools for the elderly are environmental factors. C22 Older adults can follow news and photos of school activities through various channels, such as staff communication, the Line application, Facebook, and public relations signs. It can be observed that, in addition to interpersonal communication and public relations signage, online media has become increasingly important for people of all ages. Most elderly individuals use online media for recreation and communication, as it offers convenient and rapid access to information. This indicates that older people are becoming more accepting of social and technological changes. Activities in schools for the elderly include courses on the use of various media to help develop essential skills for the modern era, raise awareness, and prevent online fraud that may target older adults. Furthermore, the diverse communication channels used by schools to share news and photos of activities allow older people with different levels of media literacy to access information comprehensively. These findings are consistent with Nawaphorn Sunonthanam's in 2020 [20] study on elderly self-esteem through the use of the LINE application. The study found that using the Line application enhances self-esteem among older adults by reinforcing their sense of importance through feelings of interest, acceptance, care, support, love, and belonging. Moreover, the government should provide appropriate internet infrastructure and communication devices. By developing a user-friendly, safe platform that promotes media literacy, seniors will be able to use the LINE application to receive health information, access news, communicate with others, and foster a stronger sense of self-worth. The use of LINE application also helps reduce loneliness, strengthen family relationships, and enhance self-esteem among the elderly in the digital age.

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