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Feeble *semi*-Supra Open Soft Sets and Their Applications on Supra Soft Topologies

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Abstract. In this paper, we introduce novel concepts that represent unique contributions to the framework of supra soft topological spaces. First, we define and characterize the notion of feeble *semi*-supra open soft subsets, establishing their fundamental properties. Through illustrative examples, we demonstrate the relationships between this class of soft subsets and existing generalizations of supra open soft sets. Then, we put forward the concepts of interior, closure, frontier, and accumulation operators induced by the class of feeble *semi*-supra open soft and *semi*-closed subsets, deriving their key properties and establishing fundamental relationships through some formulas.

1. Introduction

Contemporary mathematical approaches have been developed to handle the uncertainty and imprecision that characterize practical disciplines including engineering, medical research, social sciences, and economics. Soft set theory, pioneered by Molodtsov [1], stands out as a major contribution in this area, providing a parameterized framework that manages uncertainties effectively while addressing deficiencies in earlier techniques. In his seminal work [1], Molodtsov comprehensively laid out both the conceptual benefits of soft sets and their utility in various application areas. The theoretical foundation received significant enhancement in 2002 when Maji et al. [2]

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introduced a decision-making approach based on soft sets, which catalyzed many subsequent developments and implementations. Although the first formal treatment of operations and operators for soft set theory appeared in [3], later research by [4] detected and rectified specific inconsistencies, bringing the theory into better alignment with classical set theory. Additional studies [5] continued to broaden and refine soft set theory's theoretical structure. To strengthen the ability of soft set theory to address intricate, uncertain systems, scholars have effectively combined soft sets with alternative uncertainty frameworks, especially by merging them with fuzzy and rough set theories [6–8].

The concept of soft topology was independently introduced by Shabir and Naz [9] and by Çağman et al. [10]. Although both formulations were built upon similar foundational ideas, they differed in their handling of parameters: Shabir and Naz considered a fixed parameter set for each element of a soft topology, whereas Çağman et al. permitted variability in the parameters. In the present study, we adopt the framework of Shabir and Naz, which underscores the significance of a constant parameter set. Following these pioneering works, soft topology quickly became an active area of research, with particular emphasis on revisiting classical topological notions within the soft setting. Min [11] investigated soft regular spaces and established systematic relationships between soft T_2 and soft T_3 spaces. El-Shafei et al. [12] proposed a refined classification of soft T_i spaces that more accurately retained the characteristics of classical separation axioms. Arar and Al-shami [13] proposed an approach to generate novel classes of soft separation axioms. They also rectified some relationships between the existing types of soft separation axioms and provided answers for open problems posted in previous studies. Alqurashi [14] serintized the characterization of R_i separation axioms defined by soft somewhat open sets. Further topological ideas were later presented within the frameworks of soft topologies by several authors, such as connected spaces [15], separation axioms [16], compact and Lindelöf spaces [17–19], mappings between soft topological spaces [20], continuity [21], Menger spaces [22], maximal topologies [23], vietoris topology [24], fixed points theorem [25], expandable spaces [26], and generalizations of open soft sets [27–30]. Al-shami and Kočinac [31] established the correspondence between certain soft and crisp topological operators and properties. They concluded that numerous topological properties can be transferred between enriched and extended soft topologies and their parametric topologies.

The year 2014 witnessed the emergence of the supra soft topologies by El-Sheikh et al. [32], which are a genuine extension of supra topologies defined in 1983 by Mashour et al. [33]. Supra topologies have been exploited to deal with some practical issues, such as image processing [34] and decision making [35]. As an expected development, many researchers and those interested in topology have revealed how topological concepts and properties behave in the spaces of supra soft topologies, and identified the topological properties lost in these spaces. Among these studies, Abd El-latif [36] and Abd El-Latif and Shaaban [37] defined new forms of supra continuity and probed the decomposition of supra soft locally closed sets. Al-shami et al. [38] categorized supra soft topological spaces by two kinds of separation axioms. The authors of [39, 40] looked at the properties of supra soft sd-operators and supra soft δ-closure operators. Novel types of classes of supra soft sets were introduced and studied by [41–44], and new forms of covering properties were presented by [45].

The motivations driving this study are as follows: first, to establish a novel methodology for generalizing supra soft topology through its classical supra topologies. The proposed methodology generalizes its counterparts presented in soft topologies [46–50]. Second, to introduce a new framework for constructing soft topological concepts, such as soft operators, which is accomplished in this work. Scholars and researchers can investigate additional concepts utilizing the proposed class of feeble *semi*-supra open soft sets, including soft continuity, separation axioms, and covering properties. A final aim is to demonstrate the significance of soft topology by developing analogs for diverse crisp topological concepts.

This manuscript is structured as follows. Section 2 compiles the requisite definitions and preliminary results for a self-contained exposition. Section 3 presents a novel generalization of supra open soft sets, which we term feeble supra soft *semi*-sets. We analyze their principal features and illustrate them with examples. Building upon this foundation, Section 4 employs the new notion to define and interrelate several key concepts: the feeble *semi*-supra soft interior, closure, frontier, and accumulation points.

2. Preliminaries

This section reviews the essential background and notation for this work.

Definition 2.1. *[1] Let* $\mathbb{U}$ *and* $\mathbf{E}$ *be sets of objects and parameters, respectively. A set-valued mapping* $C:\mathbf{E}\rightarrow 2^{\mathbb{U}}$ *is named a soft set (S-set, in short); it is denoted by the ordered pair* $(C,\mathbf{E})$*.*

We use a pair $(C,\mathbf{E})$ *to denote an S-set* $(C,\mathbf{E})$ *and represent as follows*

$$(C,\mathbf{E})=\{(e,C(e)):e\in\mathbf{E} \text{ and } C(e)\in 2^{\mathbb{U}}\};$$

Definition 2.2. *[3,51] We call* $(C,\mathbf{E})$ *absolute (resp., null, pseudo constant) S-set, symbolized by* $\widetilde{\mathbb{U}}$ *(resp.,* ϕ*,* $\widetilde{\mathbb{U}}*\phi$*), if* $C(e)=\mathbb{U}$ *(resp.,* $C(e)=\emptyset$*,* $C(e)=\mathbb{U}$ *or* $\emptyset$*) for all* $e\in\mathbf{E}$*. Also,* $(C,\mathbf{E})$*, with the property "*$C(e)=\{\mathsf{u}\}$ *for a fixed parameter* e *and* $C(a)=\emptyset$ *for all* $a\in\mathbf{E}-\{e\}$*", is named a soft point; it denoted by* u_e*. We write* $\mathsf{u}_e\in(C,\mathbf{E})$ *if* $\mathsf{u}\in C(e)$*.*

Definition 2.3. *[52]* $(C,\mathbf{E})$ *is subset of* $(O,\mathbf{E})$*, symbolized by* $(C,\mathbf{E})\widetilde{\subseteq}(O,\mathbf{E})$ *if for each* $e\in\mathbf{E}$ *the relation* $C(e)\subseteq O(e)$ *holds.*

Definition 2.4. *[4] If for all* $e\in\mathbf{E}$ *we have* $O(e)=\mathbb{U}-C(e)$*, then* $(O,\mathbf{E})$ *is referred as a complement of* $(C,\mathbf{E})$*; it is singed by* $(C,\mathbf{E})^c=(C^c,\mathbf{E})$*.*

Definition 2.5. *Let* $(C,\mathbf{E})$ *and* $(O,\mathbf{E})$ *be S-sets. Then:*

(i): $(C,\mathbf{E})\widetilde{\bigcup}(O,\mathbf{E})=(H,\mathbf{E})$ *s.t.* $H(e)=C(e)\bigcup O(e)$ *for all* $e\in\mathbf{E}$*.*
(ii): $(C,\mathbf{E})\widetilde{\bigcap}(O,\mathbf{E})=(H,\mathbf{E})$ *s.t.* $H(e)=C(e)\bigcap O(e)$ *for all* $e\in\mathbf{E}$*.*
(iii): $(C,\mathbf{E})\setminus(O,\mathbf{E})=(H,\mathbf{E})$ *s.t.* $H(e)=C(e)\setminus O(e)$ *for all* $e\in\mathbf{E}$*.*

(iv): $(C,\mathbf{E})\times(O,\mathbf{E})=(H,\mathbf{E})$ *s.t.* $H(e_1,e_2)=C(e_1)\times O(e_2)$ *for all* $(e_1,e_2)\in\mathbf{E}\times\mathbf{E}$.

The adjusted version of the definition of soft mappings is given in the following.

Definition 2.6. *[53] Let $\mathcal{H}:\mathbb{U}\to\mathbb{Y}$ and $\pi:\mathbf{E}\to\mathbf{Q}$ be crisp mappings. A soft mapping $\mathcal{H}_\pi$ of $2^{\mathbb{U}_\mathbf{E}}$ into $2^{\mathbb{Y}_\mathbf{Q}}$ is a relation such that each $\mathsf{u}_e\in 2^{\mathbb{U}_\mathbf{E}}$ is related to one and only one $\mathsf{y}_q\in 2^{\mathbb{Y}_\mathbf{Q}}$ such that*

$$\mathcal{H}_\pi(\mathsf{u}_e)=\mathcal{H}(\mathsf{u})_{\pi(e)} \text{ for all } \mathsf{u}_e\in 2^{\mathbb{U}_\mathbf{E}}.$$

In addition, $\mathcal{H}_\pi^{-1}(\mathsf{y}_q)=\widetilde{\bigcup}_{\substack{\mathsf{u}\in\mathcal{H}^{-1}(\mathsf{y})\\ e\in\pi^{-1}(q)}}\mathsf{u}_e$ *for each* $\mathsf{y}_q\in 2^{\mathbb{Y}_\mathbf{Q}}$.

That is, the image of $(C,\mathbf{E})$ and pre-image of $(O,\mathbf{Q})$ under a soft mapping $\mathcal{H}_\pi$: $2^{\mathbb{U}_\mathbf{E}}\to 2^{\mathbb{Y}_\mathbf{Q}}$ are respectively given by:

$$\mathcal{H}_\pi(C,\mathbf{E})=\widetilde{\bigcup}_{\mathsf{u}_e\in(C,\mathbf{E})}\mathcal{H}_\pi(\mathsf{u}_e),$$

and

$$\mathcal{H}_\pi^{-1}(O,\mathbf{Q})=\widetilde{\bigcup}_{\mathsf{y}_q\in(O,\mathbf{Q})}\mathcal{H}_\pi^{-1}(\mathsf{y}_q).$$

A soft mapping is described as surjective (resp., injective, bijective) if its two crisp mappings satisfy this description.

Theorem 2.1. *[20] Let $\mathcal{H}_\pi$: $2^{\mathbb{U}_\mathbf{E}}\to 2^{\mathbb{Y}_\mathbf{Q}}$ be a soft mapping and let $(C,\mathbf{E})$ and $(O,\mathbf{E})$ respectively be subsets of $\widetilde{\mathbb{U}}$ and $\widetilde{\mathbb{Y}}$. Then*

(i): $(C,\mathbf{E})\widetilde{\subseteq}\mathcal{H}_\pi^{-1}(\mathcal{H}_\pi(C,\mathbf{E}))$.

(ii): $(C,\mathbf{E})=\mathcal{H}_\pi^{-1}(\mathcal{H}_\pi(C,\mathbf{E}))$ *if $\mathcal{H}_\pi$ is injective.*

(iii): $\mathcal{H}_\pi(\mathcal{H}_\pi^{-1}(O,\mathbf{Q}))\widetilde{\subseteq}(O,\mathbf{Q})$.

(iv): $\mathcal{H}_\pi(\mathcal{H}_\pi^{-1}(O,\mathbf{Q}))=(O,\mathbf{Q})$ *if $\mathcal{H}_\pi$ is surjective.*

Definition 2.7. *[1] A subfamily θ of $2^{\mathbb{U}_\mathbf{E}}$ is said to be a supra soft topology if the following terms are satisfied:*

(i): $\widetilde{\mathbb{U}}$ *and ϕ are elements of θ.*

(ii): *θ is closed under the arbitrary unions.*

We use the symbol $(\mathbb{U},\theta,\mathbf{E})$ to refer to a supra soft topological space (briefly, supra ST-space). The terms of the supra open S-sets and the supra closed S-sets are given for the elements in θ and their complements, respectively.

Definition 2.8. *[1] For a S-subset $(C,\mathbf{E})$ of a supra ST-space $(\mathbb{U},\theta,\mathbf{E})$, the supra soft interior and supra soft closure of $(C,\mathbf{E})$, denoted respectively by $int(C,\mathbf{E})$ and $cl(C,\mathbf{E})$, are defined as follows:*

(i): $int(C,\mathbf{E})=\widetilde{\bigcup}\{(O,\mathbf{E})\in\theta:(O,\mathbf{E})\widetilde{\subseteq}(C,\mathbf{E})\}$.

(ii): $cl(C,\mathbf{E})=\widetilde{\bigcap}\{(H,\mathbf{E}):(C,\mathbf{E})\widetilde{\subseteq}(H,\mathbf{E})$ *and* $(H^c,\mathbf{E})\in\theta\}$.

Theorem 2.2. *[1] Let $(\mathbb{U},\theta,\mathbf{E})$ be a supra ST-space. Then*

$$\theta_e=\{C(e):(C,\mathbf{E})\in\theta\}$$

is a supra topology on $\mathbb{U}$ for every $e \in \mathbf{E}$. We will call this supra topology a parametric supra topology.

Definition 2.9. *[21] A soft mapping $\mathcal{H}_\pi : (\mathbb{U}, \theta_{\mathbb{U}}, \mathbf{E}) \to (\mathbb{Y}, \theta_{\mathbb{Y}}, \mathbf{E})$ is said to be supra soft continuous if $\mathcal{H}_\pi^{-1}(C, \mathbf{E})$ is a supra open S-set where $(C, \mathbf{E})$ is supra S-open.*

3. Feeble *semi*-supra open S-sets and their main features

Our goal in this section is to present a new generalization of supra open soft sets so-called feeble *semi*-supra open S-sets. We demonstrate that the family of feeble *semi*-supra open S-sets is a real generalization of supra open soft subsets. We also discuss certain divergences between this family and other generalizations. Among them, the families of feeble *semi*-supra open S-sets and feeble *semi*-supra closed S-sets are not closed under soft union or soft intersection.

Now, we formulate our main concept in the next definition.

Definition 3.1. *We call $(C, \mathbf{E})$ in a supra ST-space $(\mathbb{U}, \theta, \mathbf{E})$ a feeble semi-supra open S-set if*

$$C(e) = \emptyset \text{ for all } e \in \mathbf{E}$$

or

$$\emptyset \neq C(e) \subseteq cl(int(C(e)))$$

for some $e \in \mathbf{E}$

In other words, if $(C, \mathbf{E})$ is null or some of its nonempty components are semi-supra open set.

We call $(C, \mathbf{E})$ a feeble semi-supra closed S-set if $(C^c, \mathbf{E})$ is feeble semi-supra open soft.

Remark 3.1. *In the above definition, the closure and interior operators are calculated with respect to a parametric supra topological space $(\mathbb{U}, \theta_e)$ introduced by Theorem 2.2.*

Theorem 3.1. *A set $(C, \mathbf{E})$ in a supra ST-space $(\mathbb{U}, \theta, \mathbf{E})$ is feeble semi-supra closed soft iff $(C, \mathbf{E}) = \widetilde{\mathbb{U}}$ or*

$$int(cl(C(e))) \subseteq C(e) \neq \mathbb{U}$$

for some $e \in \mathbf{E}$.

Proof. Necessity: Let $(C, \mathbf{E})$ be a feeble *semi*-supra closed S-set. Then,

$$(C^c, \mathbf{E}) = \phi,$$

or

$$\emptyset \neq C^c(e) \subseteq cl(int(C^c(e))),$$

for some $e \in \mathbf{E}$. Accordingly, we obtain either

$$(C, \mathbf{E}) = \widetilde{\mathbb{U}},$$

or

$$int(cl(C(e))) \subseteq C(e) \neq \mathbb{U},$$

for some $e \in \mathbf{E}$, which proves this implication.
Sufficiency: Let $(C, \mathbf{E})$ be an S-set such that

$$(C, \mathbf{E}) = \widetilde{\mathbb{U}},$$

or

$$int(cl(C(e))) \subseteq C(e) \neq \mathbb{U},$$

for some $e \in \mathbf{E}$. Then,

$$(C^c, \mathbf{E}) = \phi,$$

or

$$\emptyset \neq C^c(e) \subseteq cl(int(C^c(e))),$$

for some $e \in \mathbf{E}$. This implies that $(C^c, \mathbf{E})$ is feeble *semi*-supra open soft. Hence, $(C, \mathbf{E})$ is feeble *semi*-supra closed soft, as required. □

The following counterexample proves that the class of feeble *semi*-supra open soft (equivalently, *semi*-closed) sets is neither closed under soft union nor under soft intersection.

Example 3.1. *Let $\mathbb{U} = \{u_1, u_2, u_3\}$ be unverse and $\mathbf{E} = \{e_1, e_2\}$ be a set of parameters. Consider the class θ consisting of ϕ, $\widetilde{\mathbb{U}}$ and S-sets over $\mathbb{U}$ with $\mathbf{E}$ as follows:*

$$(C_1, \mathbf{E}) = \{(e_1, \mathbb{U}), (e_2, \{u_1\})\};$$

$$(C_2, \mathbf{E}) = \{(e_1, \{u_1\}), (e_2, \mathbb{U})\};$$

$$(C_3, \mathbf{E}) = \{(e_1, \{u_2\}), (e_2, \{u_3\})\};$$

$$(C_4, \mathbf{E}) = \{(e_1, \{u_1, u_2\}), (e_2, \mathbb{U})\};$$

and

$$(C_5, \mathbf{E}) = \{(e_1, \mathbb{U}), (e_2, \{u_1, u_3\})\}.$$

Then, $(\mathbb{U}, \theta, \mathbf{E})$ is a supra ST-space. Now we have the following:

$$(E, \mathbf{E}) = \{(e_1, \{u_1\}), (e_2, \{u_2\})\};$$

$$(C, \mathbf{E}) = \{(e_1, \{u_3\}), (e_2, \{u_1\})\};$$

$$(O, \mathbf{E}) = \{(e_1, \mathbb{U}), (e_2, \{u_1, u_2\})\} \text{ and}$$

$$(H, \mathbf{E}) = \{(e_1, \{u_1, u_3\}), (e_2, \mathbb{U})\}.$$

are feeble semi-supra open S-sets. Note that neither the union of $(E, \mathbf{E})$ and $(C, \mathbf{E})$ is a feeble semi-supra open S-set nor the intersection of $(O, \mathbf{E})$ and $(H, \mathbf{E})$ is a feeble semi-supra open S-set because neither $\{u_1, u_3\}$ is an semi-supra open subset of $(\mathbb{U}, \theta_{e_1})$ nor $\{u_1, u_2\}$ is an semi-supra open subset of $(\mathbb{U}, \theta_{e_2})$.

Remark 3.2. *If $(C, \mathbf{E})$ is pseudo constant S-subset, then it is a feeble semi-supra open S-subset because either $cl(int(C(e))) = \mathbb{U}$ for some $e \in \mathbf{E}$ or $C(e) = \emptyset$ for all $e \in \mathbf{E}$.*

The proofs of the next propositions are easy, so we omit.

Theorem 3.2. *Every supra open S-set is feeble semi-supra open soft.*

Theorem 3.3. *An S-set* $(C, \mathbf{E})$ *of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$ *with* $C(e) = \mathbb{U}$ *(resp.,* $C(e) = \emptyset$*) is feeble semi-supra open soft (resp., feeble semi-supra closed soft).*

Note that the converse of the above two propositions are false, in general, as illustrated by Example 3.1.

Theorem 3.4. *Let* $\mathcal{H}_\pi$: $(\mathbb{U}, \theta, \mathbf{E}) \to (\mathbb{Y}, \mu, \mathbf{Q})$ *be a soft mapping such that* $\mathcal{H}$: $(\mathbb{U}, \theta_e) \to (\mathbb{Y}, \mu_{\pi(e)=q})$ *is a supra bicontinuous mapping for each e. Then, the image of any feeble semi-supra open S-set is feeble semi-supra open soft providing that* $\pi : \mathbf{E} \to \mathbf{Q}$ *is an injective mapping.*

Proof. Let $(C, \mathbf{E})$ be a feeble *semi*-supra S-subset of a supra ST-space $(\mathbb{U}, \theta, \mathbf{E})$. Then, there is $e \in \mathbf{E}$ such that $C(e)$ is a nonempty *semi*-open subset. Assume $\pi(e) = q$. By hypothesis of supra bicontinuity of $\mathcal{H}$: $(\mathbb{U}, \theta_e) \to (\mathbb{Y}, \mu_{\pi(e)=q})$ we have for each subset V of $\mathbb{U}$: $\mathcal{H}(cl(V)) \subseteq cl(\mathcal{H}(V))$ (by supra continuity) and $\mathcal{H}(int(V)) \subseteq int(\mathcal{H}(V))$ (by supra open). This implies that

$$\mathcal{H}(C(e)) \subseteq \mathcal{H}(cl(int(C(e))) \subseteq cl(int(\mathcal{H}(C(e)))).$$

Accordingly, $\mathcal{H}(C(e))$ is a nonempty *semi*-supra open component of $\mathcal{H}_\pi(C, \mathbf{E})$. Thus, $\mathcal{H}_\pi(C, \mathbf{E})$ is a feeble *semi*-supra open S-subset of $(\mathbb{Y}, \mu, \mathbf{Q})$. □

Corollary 3.1. *A feeble semi-supra open S-set is a supra topological property.*

4. Feeble *semi*-supra interior and feeble *semi*-supra closure operators

In this part, the concepts of interior, closure, frontier, and accumulation operators are constructed from the classes of feeble *semi*-supra open soft and feeble *semi*-supra closed S-sets. Their fundamental properties are established, and the relationships among them are examined. Furthermore, through counterexamples, it is demonstrated that the feeble *semi*-supra interior (resp., closure) of an S-set fails to be a feeble *semi*-supra open (resp., closed) set.

Definition 4.1. *The feeble semi-supra interior points of a subset* $(C, \mathbf{E})$ *of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$*, denoted by* $int^s(C, \mathbf{E})$*, is defined as the union of all feeble semi-supra open S-sets contained in* $(C, \mathbf{E})$*.*

Note that the feeble *semi*-supra interior points of a subset need not be a feeble *semi*-supra open set. In other words, $(C, \mathbf{E})$ may fail to be feeble *semi*-supra open set even if it equals $int^s(C, \mathbf{E})$.

We leave the proofs of the next propositions to the reader, as they are direct consequences of the definitions.

Theorem 4.1. *Let* $(C, \mathbf{E})$ *be a S-subset of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$ *and* $\mathsf{u}_e \in \widetilde{\mathbb{U}}$*. Then* $\mathsf{u}_e \in int^s(C, \mathbf{E})$ *iff there is a feeble semi-supra open S-set* $(O, \mathbf{E})$ *contains* u_e *such that* $(O, \mathbf{E}) \widetilde{\subseteq} (C, \mathbf{E})$*.*

Theorem 4.2. *Let* $(C, \mathbf{E})$ *and* $(O, \mathbf{E})$ *be S-subsets of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$*. Then*

(i): $int^s(C, \mathbf{E}) \widetilde{\subseteq} (C, \mathbf{E})$.

(ii): *if* $(C,\mathbf{E})\widetilde{\subseteq}(O,\mathbf{E})$, *then* $int^s(C,\mathbf{E})\widetilde{\subseteq}int^s(O,\mathbf{E})$.

Corollary 4.1. *For any S-subsets* $(C,\mathbf{E})$, $(O,\mathbf{E})$ *of a supra ST-space* $(\mathbb{U},\theta,\mathbf{E})$, *we have the following results:*

(i): $int^s[(C,\mathbf{E})\widetilde{\bigcap}(O,\mathbf{E})]\widetilde{\subseteq}int^s(C,\mathbf{E})\widetilde{\bigcap}int^s(O,\mathbf{E})$.
(ii): $int^s(C,\mathbf{E})\widetilde{\bigcup}int^s(O,\mathbf{E})\widetilde{\subseteq}int^s[(C,\mathbf{E})\widetilde{\bigcup}(O,\mathbf{E})]$.

Proof. It is a direct consequence of the following relations

(i): $(C,\mathbf{E})\widetilde{\bigcap}(O,\mathbf{E})\widetilde{\subseteq}(C,\mathbf{E})$ and $(C,\mathbf{E})\widetilde{\bigcap}(O,\mathbf{E})\widetilde{\subseteq}(O,\mathbf{E})$.
(ii): $(C,\mathbf{E})\widetilde{\subseteq}[(C,\mathbf{E})\widetilde{\bigcup}(O,\mathbf{E})]$ and $(O,\mathbf{E})\widetilde{\subseteq}[(C,\mathbf{E})\widetilde{\bigcup}(O,\mathbf{E})]$

□

Note that in Theorem 4.2 and Corollary 4.1 the inclusion relations are proper.

Definition 4.2. *The feeble semi-supra closure points of a subset* $(C,\mathbf{E})$ *of a supra ST-space* $(\mathbb{U},\theta,\mathbf{E})$, *denoted by* $cl^s(C,\mathbf{E})$, *is defined as the intersection of all feeble semi-supra closed S-sets containing* $(C,\mathbf{E})$.

It can be seen that the feeble *semi*-supra closure points of a set are not always a feeble supra *semi*-closed set. Therefore, a soft set satisfying $cl^s(C,\mathbf{E})=(C,\mathbf{E})$ is not necessarily $(C,\mathbf{E})$ is a feeble supra *semi*-closed set.

Theorem 4.3. *Let* $(C,\mathbf{E})$ *be a subset of a supra ST-space* $(\mathbb{U},\theta,\mathbf{E})$ *and* $\mathsf{u}_e\in\widetilde{\mathbb{U}}$. *Then* $\mathsf{u}_e\in cl^s(C,\mathbf{E})$ *iff* $(O,\mathbf{E})\widetilde{\bigcap}(C,\mathbf{E})\neq\phi$ *for each feeble semi-supra open S-set* $(O,\mathbf{E})$ *contains* u_e.

Proof. [⇒] Let $\mathsf{u}_e\in cl^s(C,\mathbf{E})$. Suppose that there is feeble *semi*-supra open S-set $(O,\mathbf{E})$ containing u_e with

$$(O,\mathbf{E})\widetilde{\bigcap}(C,\mathbf{E})=\phi.$$

Then

$$(C,\mathbf{E})\widetilde{\subseteq}(O^c,\mathbf{E}).$$

Therefore,

$$cl^s(C,\mathbf{E})\widetilde{\subseteq}(O^c,\mathbf{E}).$$

Thus

$$\mathsf{u}_e\notin cl^s(C,\mathbf{E}).$$

But this is a contradiction, so

$$(O,\mathbf{E})\widetilde{\bigcap}(C,\mathbf{E})\neq\phi \text{ holds.}$$

[⇐] Let

$$(O,\mathbf{E})\widetilde{\bigcap}(C,\mathbf{E})\neq\phi$$

for each feeble *semi*-supra open S-set $(O,\mathbf{E})$ contains u_e. Let us assume that

$$\mathsf{u}_e\notin cl^s(C,\mathbf{E}).$$

Then there is a feeble *semi*-supra closed S-set $(H, \mathbf{E})$ containing $(C, \mathbf{E})$ with $\mathsf{u}_e \notin (H, \mathbf{E})$. So

$$\mathsf{u}_e \in (H^c, \mathbf{E})$$

and

$$(H^c, \mathbf{E})\widetilde{\bigcap}(C, \mathbf{E}) = \phi.$$

But this contradicts our assumption. We have thus proved the claim □

Corollary 4.2. *If*

$$(C, \mathbf{E})\widetilde{\bigcap}(O, \mathbf{E}) = \phi$$

such that $(C, \mathbf{E})$ *is a feeble semi-supra open S-set and* $(O, \mathbf{E})$ *is an S-set in* $(\mathbb{U}, \theta, \mathbf{E})$, *then*

$$(C, \mathbf{E})\widetilde{\bigcap}cl^s(O, \mathbf{E}) = \phi.$$

Proof. Straightforward. □

Theorem 4.4. *For a subset* $(C, \mathbf{E})$ *of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$, *the next results hold true.*

(i): $[int^s(C, \mathbf{E})]^c = cl^s(C^c, \mathbf{E})$.
(ii): $[cl^s(C, \mathbf{E})]^c = int^s(C^c, \mathbf{E})$.

Proof. (i) If

$$\mathsf{u}_e \notin [int^s(C, \mathbf{E})]^c,$$

then there is a feeble *semi*-supra open S-set $(O, \mathbf{E})$ with

$$\mathsf{u}_e \in (O, \mathbf{E})\widetilde{\subseteq}(C, \mathbf{E}).$$

Therefore,

$$(C^c, \mathbf{E})\widetilde{\bigcap}(O, \mathbf{E}) = \phi,$$

and hence,

$$\mathsf{u}_e \notin cl^s(C^c, \mathbf{E}).$$

Conversely, if $\mathsf{u}_e \notin cl^s(C^c, \mathbf{E})$ one may verify $\mathsf{u}_e \notin [int^s(C, \mathbf{E})]^c$ by adapting the previous steps.
(ii) The proof follows an argument similar to (i). □

The following proposition is presented without proof, as it follows directly from the definitions.

Theorem 4.5. *Let* $(C, \mathbf{E})$, $(O, \mathbf{E})$ *be S-subsets of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$. *Then*

(i): $(C, \mathbf{E})\widetilde{\subseteq}cl^s(C, \mathbf{E})$.
(ii): *if* $(C, \mathbf{E})\widetilde{\subseteq}(O, \mathbf{E})$, *then* $cl^s(C, \mathbf{E})\widetilde{\subseteq}cl^s(O, \mathbf{E})$.

Corollary 4.3. *The next properties hold for all subsets of* $(C, \mathbf{E})$, $(O, \mathbf{E})$ *of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$.

(i): $cl^s[(C, \mathbf{E})\widetilde{\bigcap}(O, \mathbf{E})]\widetilde{\subseteq}cl^s(C, \mathbf{E})\widetilde{\bigcap}cl^s(O, \mathbf{E})$.
(ii): $cl^s(C, \mathbf{E})\widetilde{\bigcup}cl^s(O, \mathbf{E})\widetilde{\subseteq}cl^s[(C, \mathbf{E})\widetilde{\bigcup}(O, \mathbf{E})]$.

Proof. It automatically comes from the following:

(i) $(C,\mathbf{E})\widetilde{\bigcap}(O,\mathbf{E})\widetilde{\subseteq}(C,\mathbf{E})$ and $(C,\mathbf{E})\widetilde{\bigcap}(O,\mathbf{E})\widetilde{\subseteq}(O,\mathbf{E})$.

(ii) $(C,\mathbf{E})\widetilde{\subseteq}[(C,\mathbf{E})\widetilde{\bigcup}(O,\mathbf{E})]$ and $(O,\mathbf{E})\widetilde{\subseteq}[(C,\mathbf{E})\widetilde{\bigcup}(O,\mathbf{E})]$. □

Note that, in Theorem 4.5 and Corollary 4.3, the inclusion relations are proper.

Definition 4.3. *A soft point u_e is said to be a feeble semi-supra frontier point of a subset $(C,\mathbf{E})$ of a supra ST-space $(\mathbb{U},\theta,\mathbf{E})$ if u_e belongs to the complement of $int^s(C,\mathbf{E})\widetilde{\bigcup}int^s(C^c,\mathbf{E})$.*

All feeble semi-supra frontier points of $(C,\mathbf{E})$ is called a feeble semi-supra frontier set, denoted by $f^s(C,\mathbf{E})$.

Theorem 4.6.

$$f^s(C,\mathbf{E}) = cl^s(C,\mathbf{E})\widetilde{\bigcap}cl^s(C^c,\mathbf{E})$$

for every subset $(C,\mathbf{E})$ of a supra ST-space $(\mathbb{U},\theta,\mathbf{E})$.

Proof.

$$\begin{aligned} f^s(C,\mathbf{E}) &= [int^s(C,\mathbf{E})\widetilde{\bigcup}int^s(C^c,\mathbf{E})]^c \\ &= [int^s(C,\mathbf{E})]^c\widetilde{\bigcap}[int^s(C^c,\mathbf{E})]^c \ (De\ \ Morgan's\ \ law) \\ &= cl^s(C^c,\mathbf{E})\widetilde{\bigcap}cl^s(C,\mathbf{E}) \ (Theorem\ \ 4.4(ii)) \end{aligned}$$

Corollary 4.4. *For every subset $(C,\mathbf{E})$ of a supra ST-space $(\mathbb{U},\theta,\mathbf{E})$, the following properties hold.* □

- **(i):** $f^s(C,\mathbf{E}) = f^s(C^c,\mathbf{E})$.
- **(ii):** $f^s(C,\mathbf{E}) = cl^s(C,\mathbf{E}) \setminus int^s(C,\mathbf{E})$.
- **(iii):** $cl^s(C,\mathbf{E}) = int^s(C,\mathbf{E})\widetilde{\bigcup} f^s(C,\mathbf{E})$.
- **(iv):** $int^s(C,\mathbf{E}) = (C,\mathbf{E}) \setminus f^s(C,\mathbf{E})$.

Proof. (i) Obvious.

(ii) $f^s(C,\mathbf{E}) = cl^s(C,\mathbf{E})\widetilde{\bigcap}cl^s(C^c,\mathbf{E}) = cl^s(C,\mathbf{E}) \setminus [cl^s(C^c,\mathbf{E})]^c$. By (ii) of Theorem 4.4, the desired relation follows.

(iii) $int^s(C,\mathbf{E})\widetilde{\bigcup} f^s(C,\mathbf{E}) = int^s(C,\mathbf{E})\widetilde{\bigcup} [cl^s(C,\mathbf{E}) \setminus int^s(C,\mathbf{E})] = cl^s(C,\mathbf{E})$.

(iv)

$$\begin{aligned} (C,\mathbf{E}) \setminus f^s(C,\mathbf{E}) &= (C,\mathbf{E}) \setminus [cl^s(C,\mathbf{E}) \setminus int^s(C,\mathbf{E})] \\ &= (C,\mathbf{E})\widetilde{\bigcap}[cl^s(C,\mathbf{E})\widetilde{\bigcap}(int^s(C,\mathbf{E}))^c]^c \\ &= (C,\mathbf{E})\widetilde{\bigcap}[(cl^s(C,\mathbf{E}))^c\widetilde{\bigcup} int^s(C,\mathbf{E})] \\ &= [(C,\mathbf{E})\widetilde{\bigcap}(cl^s(C,\mathbf{E}))^c]\widetilde{\bigcup}[(C,\mathbf{E})\widetilde{\bigcap} int^s(C,\mathbf{E})] \\ &= int^s(C,\mathbf{E}). \end{aligned}$$

□

Theorem 4.7. *Let $(C,\mathbf{E}),(O,\mathbf{E})$ be subsets of a supra ST-space $(\mathbb{U},\theta,\mathbf{E})$, the following properties hold.*

(i): $f^s(int^s(C,\mathbf{E}))\widetilde{\subseteq}f^s(C,\mathbf{E})$.

(ii): $f^s(cl^s(C,\mathbf{E}))\widetilde{\subseteq}f^s(C,\mathbf{E})$.

Proof. By substituting in the formula No. (iii) of Corollary 4.4, the proof follows. □

Theorem 4.8. *Let $(C,\mathbf{E})$ be a subset of a supra ST-space $(\mathbb{U},\theta,\mathbf{E})$. Then*

(i): $(C,\mathbf{E})=int^s(C,\mathbf{E})$ *iff* $f^s(C,\mathbf{E})\widetilde{\bigcap}(C,\mathbf{E})=\phi$.

(ii): $(C,\mathbf{E})=cl^s(C,\mathbf{E})$ *iff* $f^s(C,\mathbf{E})\widetilde{\subseteq}(C,\mathbf{E})$.

Proof. (i) Suppose that

$$(C,\mathbf{E})=int^s(C,\mathbf{E}).$$

Then by (iv) of Corollary 4.4,

$$(C,\mathbf{E})=int^s(C,\mathbf{E})=(C,\mathbf{E})\setminus f^s(C,\mathbf{E})$$

and hence,

$$f^s(C,\mathbf{E})\widetilde{\bigcap}(C,\mathbf{E})=\phi.$$

Conversely, let $\mathsf{u}_e\in(C,\mathbf{E})$. Since $\mathsf{u}_e\notin f^s(C,\mathbf{E})$ and $\mathsf{u}_e\in cl^s(C,\mathbf{E})$, by (iii) of Corollary 4.4, $\mathsf{u}_e\in int^s(C,\mathbf{E})$. Therefore,

$$int^s(C,\mathbf{E})=(C,\mathbf{E}),$$

which establishes the claim.
(ii) Assume that

$$(C,\mathbf{E})=cl^s(C,\mathbf{E}).$$

Then

$$f^s(C,\mathbf{E})=cl^s(C,\mathbf{E})\widetilde{\bigcap}cl^s(C^c,\mathbf{E})\widetilde{\subseteq}cl^s(C,\mathbf{E})=(C,\mathbf{E}),$$

which establishes the claim.

Conversely, if $f^s(C,\mathbf{E})\widetilde{\subseteq}(C,\mathbf{E})$, then by (iii) of Corollary 4.4,

$$cl^s(C,\mathbf{E})\widetilde{\subseteq}int^s(C,\mathbf{E})\widetilde{\bigcup}(C,\mathbf{E})=(C,\mathbf{E})$$

and hence

$$cl^s(C,\mathbf{E})=(C,\mathbf{E}),$$

as required. □

Corollary 4.5. *Let $(C,\mathbf{E})$ be a subset of a supra ST-space $(\mathbb{U},\theta,\mathbf{E})$. Then*

$$int^s(C,\mathbf{E})=(C,\mathbf{E})=cl^s(C,\mathbf{E})$$

iff

$$f^s(C,\mathbf{E})=\phi.$$

Definition 4.4. *A soft point* u_e *is said to be a feeble semi-supra accumulation point of a subset* $(C, \mathbf{E})$ *of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$ *if*

$$[(O, \mathbf{E}) \backslash \mathsf{u}_e] \bigcap (C, \mathbf{E}) \neq \phi$$

for each feeble semi-supra open S-set $(O, \mathbf{E})$ *containing* u_e.

All feeble semi-supra accumulation points of $(C, \mathbf{E})$ *is called a feeble supra semi-derived set and denoted by* $l^s(C, \mathbf{E})$.

Theorem 4.9. *Let* $(C, \mathbf{E})$ *and* $(O, \mathbf{E})$ *be subsets of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$. *If* $(C, \mathbf{E}) \widetilde{\subseteq} (O, \mathbf{E})$, *then* $l^s(C, \mathbf{E}) \widetilde{\subseteq} l^s(O, \mathbf{E})$.

Proof. Straightforward by Definition 4.4. □

Corollary 4.6. *Consider* $(C, \mathbf{E})$ *and* $(O, \mathbf{E})$ *are subsets of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$. *Then:*

- **(i):** $l^s[(C, \mathbf{E}) \widetilde{\bigcap} (O, \mathbf{E})] \widetilde{\subseteq} l^s(C, \mathbf{E}) \widetilde{\bigcap} l^s(O, \mathbf{E})$.
- **(ii):** $l^s(C, \mathbf{E}) \widetilde{\bigcup} l^s(O, \mathbf{E}) \widetilde{\subseteq} l^s[(C, \mathbf{E}) \widetilde{\bigcup} (O, \mathbf{E})]$.

Theorem 4.10. *Let* $(C, \mathbf{E})$ *be a subset of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$, *then*

$$cl^s(C, \mathbf{E}) = (C, \mathbf{E}) \widetilde{\bigcup} l^s(C, \mathbf{E}).$$

Proof. The side

$$(C, \mathbf{E}) \widetilde{\bigcup} l^s(C, \mathbf{E}) \widetilde{\subseteq} cl^s(C, \mathbf{E})$$

is clear. To verify the opposite implication, let

$$\mathsf{u}_e \notin [(C, \mathbf{E}) \widetilde{\bigcup} l^s(C, \mathbf{E})].$$

Then $\mathsf{u}_e \notin (C, \mathbf{E})$ and $\mathsf{u}_e \notin l^s(C, \mathbf{E})$. Therefore, there is feeble *semi*-supra open soft $(O, \mathbf{E})$ containing u_e with

$$(O, \mathbf{E}) \widetilde{\cap} (C, \mathbf{E}) = \phi.$$

Thus, $\mathsf{u}_e \notin cl^s(C, \mathbf{E})$. Hence, we find that

$$cl^s(C, \mathbf{E}) = (C, \mathbf{E}) \widetilde{\bigcup} l^s(C, \mathbf{E}).$$

□

Corollary 4.7. *Let* $(C, \mathbf{E})$ *be a feeble semi-supra closed S-subset of a supra ST-space* $(\mathbb{U}, \theta, \mathbf{E})$, *then* $l^s(C, \mathbf{E}) \widetilde{\subseteq} (C, \mathbf{E})$.

5. Conclusion

If we deal with a topology without the condition of finite intersection, we define the concept of supra topology. As illustrated in the published manuscripts, supra topological spaces have several merits, including easy construction of spaces satisfying certain properties [54] and their ability to model numerous practical issues [34, 35]. Given these merits and the advantages of soft spaces, we have devoted this work to investigating some ideas via supra soft topological spaces. We have generalized feeble *semi*-open soft sets to supra soft topological spaces and debated their basic features. We have studied the main properties of this class and clarified how this class interacts with some topological notions. We made use of feeble *semi*-supra open and feeble *semi*-supra closed soft subsets to formulate the concepts of interior operator, closure operator, frontier operator, and accumulation operator. We established some relationships between these operators through some formulas that are similar to their counterparts in classical settings and explored their key properties. Moreover, we furnished some counterexamples to demonstrate which classical properties evaporate in the structures of supra soft topologies.

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