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Fixed Point Theorems in Neutrosophic MR-Metric Spaces with Measure-Theoretic Convergence

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Abstract. In this paper, we introduce and study the concept of Neutrosophic MR-Metric Spaces (NMR-MS), which combine the structure of MR-metric spaces with neutrosophic membership functions. We establish a fixed point theorem for neutrosophic contraction mappings in such spaces and prove the existence, uniqueness, and measure-theoretic convergence of the fixed point. Our results extend classical fixed point theory to a more general framework that incorporates uncertainty and vagueness, as modeled by neutrosophic logic. The convergence is shown to be exponential in measure and stable under the induced measure structure. Several illustrative examples and applications are provided to validate the theoretical findings and demonstrate their applicability in dynamical systems, optimization, and stability analysis under uncertainty.

1. Introduction

Fixed point theory is a fundamental branch of mathematical analysis with profound applications in differential equations, optimization, dynamical systems, and beyond. Classical results such as the Banach contraction principle have been extended to various generalized metric spaces, including *b*-metric spaces, *G*-metric spaces, and more recently, MR-metric spaces. The latter, introduced by Malkawi and Rabaiah [2], provide a flexible framework for studying triadic distance relations with a relaxed triangle inequality, leading to new fixed point results and applications in graph theory, fractional calculus, and measure theory ([28], [29], [30], [31], [32], [33], [40], [41], [34], [35]).

In parallel, the need to model uncertainty, vagueness, and indeterminacy in real-world problems has led to the development of fuzzy and neutrosophic metric spaces. Neutrosophic logic, introduced by Smarandache, extends fuzzy and intuitionistic fuzzy sets by incorporating truth,

falsity, and indeterminacy membership functions. Recent works have explored fixed point theory in neutrosophic settings, including neutrosophic fuzzy metric spaces ([15], [18]) and their application to nonlinear contractions ([15], [18]).

Building on these advances, we introduce the concept of Neutrosophic MR-Metric Spaces (NMR-MS), which integrate MR-metric structures with neutrosophic membership functions. This allows for a unified treatment of metric and logical uncertainty within a single framework. Our main contribution is a fixed point theorem for neutrosophic contraction mappings in complete NMR-MS, ensuring the existence, uniqueness, and measure-theoretic convergence of the fixed point with exponential rate. This work generalizes and extends earlier results on MR-metric spaces ([26], [27], [1], [42], [43]), fuzzy mappings ([39], [42]), and neutrosophic contractions ([15], [18], [44]).

Several authors have contributed to the development of fixed point theory in generalized metric and distance spaces, including results on Ω-distance mappings ([3], [6], [7], [8], [13], [12], [14]), b-metric spaces ([4], [1], [11]), G_b-metric spaces [6], and gamma distance mappings ([8], [22], [21]). Applications to fractional differential equations ([5], [8], [25], [22]), integral equations [31], and nonlinear analysis ([36], [37], [38]) have also been explored. Additionally, recent advances in neutrosophic soft sets ([48], [49], [50], [52]), decision-making frameworks ([45], [49], [52]), and fuzzy fractional equations [53] provide further context for the application of neutrosophic structures in mathematical modeling.

In this paper, we unify these directions by introducing NMR-MS and proving a comprehensive fixed point theorem with measure-theoretic guarantees. The remainder of the paper is organized as follows: Section 2 presents the main fixed point theorem and its proof. Section 3 provides detailed examples and applications. Section 4 concludes with remarks on future research directions.

Definition 1.1. *[2] Consider a non-empty set $\mathbb{X} \neq \emptyset$ and a real number $\mathbb{R} > 1$. A function*

$$M : \mathbb{X} \times \mathbb{X} \times \mathbb{X} \to [0, \infty)$$

is termed an **MR-metric** *if it satisfies the following conditions for all $v, \xi, s, \ell_1 \in \mathbb{X}$:*

- $M(v, \xi, s) \geq 0$.
- $M(v, \xi, s) = 0$ *if and only if $v = \xi = s$.*
- $M(v, \xi, s)$ *remains invariant under any permutation $p(v, \xi, s)$, i.e., $M(v, \xi, s) = M(p(v, \xi, s))$.*
- *The following inequality holds:*

$$M(v, \xi, s) \leq \mathbb{R} \left[M(v, \xi, \ell_1) + M(v, \ell_1, s) + M(\ell_1, \xi, s) \right].$$

A structure $(\mathbb{X}, M)$ that adheres to these properties is defined as an MR-metric space.

Definition 1.2. *[39][Neutrosophic MR-Metric Space (NMR-MS)]*
A 9-tuple $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ is called a **Neutrosophic MR-Metric Space** *if:*

(1) $\mathcal{Z}$ *is a non-empty set.*
(2) $M : \mathcal{Z} \times \mathcal{Z} \times \mathcal{Z} \to [0, \infty)$ *is an MR-metric satisfying:*
   (M1) $M(v, \xi, \mathfrak{I}) \geq 0$,

(M2) $M(v, \xi, \Im) = 0 \iff v = \xi = \Im$,

(M3) Symmetry under permutations,

(M4) $M(v, \xi, \Im) \leq R\left[M(v, \xi, \ell) \star M(v, \ell, \Im) \star M(\ell, \xi, \Im)\right], R > 1$.

(3) $\mathcal{T}, \mathcal{F}, \mathcal{I} : \mathcal{Z} \times \mathcal{Z} \times (0, \infty) \to [0, 1]$ *are neutrosophic functions satisfying:*

(N1) $\mathcal{T}(v, \xi, \gamma) = 1 \iff v = \xi$ *(Truth-Identity),*

(N2) $\mathcal{T}(v, \xi, \gamma) = \mathcal{T}(\xi, v, \gamma)$ *(Symmetry),*

(N3) $\mathcal{T}(v, \xi, \gamma) \bullet \mathcal{T}(\xi, \Im, \rho) \leq \mathcal{T}(v, \Im, \gamma + \rho)$ *(Triangle Inequality),*

(N4) $\lim_{\gamma \to \infty} \mathcal{T}(v, \xi, \gamma) = 1$ *(Asymptotic Behavior).*

(4) *$\bullet$ (t-norm) and $\diamond$ (t-conorm) are continuous operators generalizing fuzzy logic.*

(5) *$\star$ is a binary operation generalizing addition (e.g., weighted sum).*

2. Main Results

In this section, we present our main fixed point theorem for neutrosophic contraction mappings in complete Neutrosophic MR-Metric Spaces. The theorem establishes not only the existence and uniqueness of a fixed point but also guarantees its convergence in the measure-theoretic sense, with exponential decay of error in measure. The proof combines techniques from fixed point theory, neutrosophic analysis, and measure theory, providing a robust framework for applications in uncertain dynamical systems and optimization.

Theorem 2.1 (Fixed Point in NMR-Measure Spaces). *Let $(\mathcal{Z}, \mathfrak{M}, \mu)$ be a measure space induced by a complete NMR-MS, and let $T : \mathcal{Z} \to \mathcal{Z}$ be a neutrosophic contraction mapping satisfying:*

$$\mathcal{T}(Tv, T\xi, \gamma) \geq \mathcal{T}(v, \xi, \gamma/k) \bullet \alpha$$

$$\mathcal{F}(Tv, T\xi, \gamma) \leq \mathcal{F}(v, \xi, \gamma/k) \diamond \beta$$

$$M(Tv, T\xi, T\zeta) \leq \frac{1}{k} \star M(v, \xi, \zeta)$$

for some $k > 1$, $\alpha, \beta \in [0, 1]$. Then:

(1) *T has a unique fixed point $v^* \in \mathcal{Z}$*

(2) *For any $v_0 \in \mathcal{Z}$, the sequence $\{T^n v_0\}$ converges to v^* μ-almost everywhere*

(3) *The fixed point satisfies the measure-theoretic stability:*

$$\mu(\{\omega : \lim_{n \to \infty} M(T^n \omega, v^*, v^*) = 0\}) = \mu(\mathcal{Z})$$

(4) *The convergence is exponential in measure:*

$$\mu(\{\omega : M(T^n \omega, v^*, v^*) > \epsilon\}) \leq C \cdot e^{-n\lambda}$$

for some $C, \lambda > 0$

Proof. We prove this theorem through a comprehensive approach combining fixed point theory, measure theory, and neutrosophic analysis.

Step 1: Existence and Uniqueness of Fixed Point

(a) Construction of Iterative Sequence

Let $v_0 \in \mathcal{Z}$ be arbitrary. Define the iterative sequence $\{v_n\}$ by $v_{n+1} = Tv_n$ for $n \geq 0$.

From the neutrosophic contraction conditions:

$$\mathcal{T}(v_{n+1}, v_n, \gamma) = \mathcal{T}(Tv_n, Tv_{n-1}, \gamma)$$

$$\geq \mathcal{T}(v_n, v_{n-1}, \gamma/k) \bullet \alpha$$

$$\geq \mathcal{T}(v_{n-1}, v_{n-2}, \gamma/k^2) \bullet \alpha^2$$

$$\geq \cdots \geq \mathcal{T}(v_1, v_0, \gamma/k^n) \bullet \alpha^n$$

Similarly for the falsity-membership:

$$\mathcal{F}(v_{n+1}, v_n, \gamma) = \mathcal{F}(Tv_n, Tv_{n-1}, \gamma)$$

$$\leq \mathcal{F}(v_n, v_{n-1}, \gamma/k) \diamond \beta$$

$$\leq \mathcal{F}(v_{n-1}, v_{n-2}, \gamma/k^2) \diamond \beta^2$$

$$\leq \cdots \leq \mathcal{F}(v_1, v_0, \gamma/k^n) \diamond \beta^n$$

(b) Cauchy Sequence Property

For $m > n$, using the MR-metric contraction:

$$M(v_m, v_n, v_n) \leq \frac{1}{k} \star M(v_{m-1}, v_{n-1}, v_{n-1})$$

$$\leq \frac{1}{k^2} \star M(v_{m-2}, v_{n-2}, v_{n-2})$$

$$\leq \cdots \leq \frac{1}{k^{\min(m,n)}} \star M(v_{m-n}, v_0, v_0)$$

Since $k > 1$ and M is bounded on bounded sets, we have:

$$\lim_{m,n \to \infty} M(v_m, v_n, v_n) = 0$$

Thus $\{v_n\}$ is a Cauchy sequence in the complete NMR-MS.

(c) Existence of Fixed Point

By completeness, there exists $v^* \in \mathcal{Z}$ such that $\lim_{n \to \infty} v_n = v^*$.

We show v^* is a fixed point:

$$M(Tv^*, v^*, v^*) \leq R\left[M(Tv^*, Tv_n, Tv_n) \star M(Tv^*, Tv_n, v^*) \star M(Tv_n, v^*, v^*)\right]$$

$$\leq R\left[\frac{1}{k} \star M(v^*, v_n, v_n) \star M(Tv^*, Tv_n, v^*) \star M(v_{n+1}, v^*, v^*)\right]$$

Taking $n \to \infty$, all terms approach 0, so $M(Tv^*, v^*, v^*) = 0$, hence $Tv^* = v^*$.

(d) Uniqueness

Suppose v^*, ω^* are two fixed points. Then:

$$M(v^*, \omega^*, \omega^*) = M(Tv^*, T\omega^*, T\omega^*)$$

$$\leq \frac{1}{k} \star M(v^*, \omega^*, \omega^*)$$

Since $k > 1$, this implies $M(v^*, \omega^*, \omega^*) = 0$, so $v^* = \omega^*$.

Step 2: μ-Almost Everywhere Convergence

(a) Measure-Theoretic Convergence

Consider the set:

$$A = \{\omega \in \mathcal{Z} : \lim_{n \to \infty} T^n \omega \text{ exists}\}$$

For any $\omega \in \mathcal{Z}$, define the sequence $\{T^n \omega\}$. By the same contraction arguments as in Step 1, this sequence is Cauchy μ-almost everywhere.

Specifically, for any $\epsilon > 0$:

$$\mu\left(\{\omega : M(T^m\omega, T^n\omega, T^n\omega) > \epsilon\}\right) \leq \mu\left(\{\omega : \frac{1}{k^{\min(m,n)}} \star M(T^{|m-n|}\omega, \omega, \omega) > \epsilon\}\right)$$

$$\leq \frac{1}{\epsilon} \int_{\mathcal{Z}} \frac{1}{k^{\min(m,n)}} \star M(T^{|m-n|}\omega, \omega, \omega)\, d\mu(\omega)$$

By the dominated convergence theorem and since $k > 1$, this approaches 0 as $m, n \to \infty$.

(b) Convergence to Fixed Point

For $\omega \in A$, let $\omega^* = \lim_{n \to \infty} T^n \omega$. Then:

$$M(T\omega^*, \omega^*, \omega^*) = \lim_{n \to \infty} M(T^{n+1}\omega, T^n\omega, T^n\omega)$$

$$\leq \lim_{n \to \infty} \frac{1}{k^n} \star M(T\omega, \omega, \omega) = 0$$

So ω^* is a fixed point. By uniqueness, $\omega^* = v^*$ for all $\omega \in A$.

Thus $\{T^n \omega\}$ converges to v^* for all $\omega \in A$, and $\mu(A) = \mu(\mathcal{Z})$.

Step 3: Measure-Theoretic Stability

We prove:

$$\mu(\{\omega : \lim_{n \to \infty} M(T^n\omega, v^*, v^*) = 0\}) = \mu(\mathcal{Z})$$

From the contraction property:

$$M(T^n\omega, v^*, v^*) = M(T^n\omega, T^n v^*, T^n v^*)$$

$$\leq \frac{1}{k^n} \star M(\omega, v^*, v^*)$$

Thus for each ω:

$$\lim_{n \to \infty} M(T^n\omega, v^*, v^*) \leq \lim_{n \to \infty} \frac{1}{k^n} \star M(\omega, v^*, v^*) = 0$$

Since this holds for all $\omega \in \mathcal{Z}$, the set where the limit is 0 has full measure.

Step 4: Exponential Convergence in Measure

We prove:

$$\mu(\{\omega : M(T^n\omega, v^*, v^*) > \epsilon\}) \leq C \cdot e^{-n\lambda}$$

Using Markov's inequality and the contraction property:

$$\mu(\{\omega : M(T^n\omega, v^*, v^*) > \epsilon\}) \leq \frac{1}{\epsilon} \int_Z M(T^n\omega, v^*, v^*)\, d\mu(\omega)$$

$$\leq \frac{1}{\epsilon} \int_Z \frac{1}{k^n} \star M(\omega, v^*, v^*)\, d\mu(\omega)$$

$$= \frac{1}{\epsilon \cdot k^n} \int_Z M(\omega, v^*, v^*)\, d\mu(\omega)$$

Since μ is a finite measure (or σ-finite with appropriate localization) and $M(\cdot, v^*, v^*)$ is integrable, let:

$$C = \frac{1}{\epsilon} \int_Z M(\omega, v^*, v^*)\, d\mu(\omega) < \infty$$

Then:

$$\mu(\{\omega : M(T^n\omega, v^*, v^*) > \epsilon\}) \leq C \cdot \left(\frac{1}{k}\right)^n = C \cdot e^{-n \ln k}$$

Taking $\lambda = \ln k > 0$ (since $k > 1$), we obtain the exponential bound.

Step 5: Neutrosophic Convergence Properties

The convergence also respects the neutrosophic structure:

$$\lim_{n\to\infty} \mathcal{T}(T^n\omega, v^*, \gamma) = \lim_{n\to\infty} \mathcal{T}(T^n\omega, T^n v^*, \gamma)$$

$$\geq \lim_{n\to\infty} \mathcal{T}(\omega, v^*, \gamma/k^n) \bullet \alpha^n = 1$$

$$\lim_{n\to\infty} \mathcal{F}(T^n\omega, v^*, \gamma) = \lim_{n\to\infty} \mathcal{F}(T^n\omega, T^n v^*, \gamma)$$

$$\leq \lim_{n\to\infty} \mathcal{F}(\omega, v^*, \gamma/k^n) \diamond \beta^n = 0$$

Thus the convergence is also in the neutrosophic sense.

Conclusion

We have proven that under the given neutrosophic contraction conditions:

(1) A unique fixed point v^* exists in the complete NMR-MS

(2) The iterative sequence converges to v^* μ-almost everywhere

(3) The convergence is stable in the measure-theoretic sense

(4) The convergence rate is exponential in measure

(5) The convergence respects the neutrosophic structure

This fixed point theorem extends classical results to the rich framework of Neutrosophic MR-Metric Spaces with measure-theoretic guarantees, providing powerful tools for applications in analysis, optimization, and dynamical systems. $\qquad\square$

3. Applications and Consequences

This section provides a detailed validation and illustration of Theorem 2.1. We construct explicit examples to verify all hypotheses, demonstrate convergence behavior, and interpret results within

measure-theoretic and neutrosophic frameworks. Applications to uncertain dynamical systems are discussed, highlighting the practical utility of our theoretical developments.

Example 3.1. *(A Canonical Neutrosophic MR-Metric Space and Contraction)*

Let $\mathcal{Z} = [0,1]$ and define the MR-metric $M : \mathcal{Z}^3 \to [0,\infty)$ by

$$M(x,y,z) = |x-y| + |y-z| + |z-x|.$$

This satisfies, with minimal $R = 2$, the generalized triangle inequality

$$M(x,y,z) \le 2\big(M(x,y,w) + M(x,w,z) + M(w,y,z)\big), \quad \forall x,y,z,w \in \mathcal{Z}.$$

Define neutrosophic memberships $\mathcal{T}, \mathcal{F}, \mathcal{I} : \mathcal{Z}^2 \times (0,\infty) \to [0,1]$ *as*

$$\mathcal{T}(x,y,\gamma) = e^{-\gamma|x-y|}, \quad \mathcal{F}(x,y,\gamma) = 1 - e^{-\gamma|x-y|}, \quad \mathcal{I}(x,y,\gamma) = 1 - \mathcal{T} - \mathcal{F}.$$

Take the Łukasiewicz t-norm $a \bullet b = \max\{a+b-1, 0\}$ *and t-conorm* $a \diamond b = \min\{a+b, 1\}$. *Then* $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, 2, \star)$ *is a complete NMR-MS, where* $\star$ *denotes ordinary addition.*

Consider the mapping $T(x) = x/k$ with $k > 1$. It satisfies:

- **MR-metric contraction:** $M(Tx, Ty, Tz) = \frac{1}{k}M(x,y,z)$.
- **Neutrosophic contraction:** For $\gamma > 0$,

$$\mathcal{T}(Tx, Ty, \gamma) = e^{-\gamma|x-y|/k} = \mathcal{T}\big(x, y, \tfrac{\gamma}{k}\big) \ge \mathcal{T}\big(x, y, \tfrac{\gamma}{k}\big) \bullet 1,$$

$$\mathcal{F}(Tx, Ty, \gamma) = 1 - e^{-\gamma|x-y|/k} \le \big(1 - e^{-\gamma|x-y|/k}\big) \diamond 0 = \mathcal{F}\big(x, y, \tfrac{\gamma}{k}\big) \diamond 0.$$

Thus T is a neutrosophic MR-contraction with constant $1/k$. By Theorem 2.1 it possesses a unique fixed point $x^* = 0$.

Example 3.2. *(Iterative Convergence with Explicit Error Bounds)*

For arbitrary $x_0 \in [0,1]$, the Picard iteration $x_{n+1} = x_n/k$ yields

$$x_n = \frac{x_0}{k^n}, \qquad \lim_{n\to\infty} x_n = 0.$$

The MR-metric distance to the fixed point is

$$M(x_n, 0, 0) = 2|x_n| = \frac{2x_0}{k^n} = 2x_0 e^{-n\ln k},$$

showing exponential convergence with rate $\lambda = \ln k$. Neutrosophic memberships behave as

$$\mathcal{T}(x_n, 0, \gamma) = e^{-\gamma x_0/k^n} \to 1, \qquad \mathcal{F}(x_n, 0, \gamma) = 1 - e^{-\gamma x_0/k^n} \to 0,$$

consistent with the neutrosophic convergence stated in Theorem 2.1.

Example 3.3. *(Measure-Theoretic Convergence and Exponential Decay)*

Let $(\mathcal{Z}, \mathcal{B}, \mu)$ be the Lebesgue space on $[0,1]$. For $\varepsilon > 0$ define

$$A_n(\varepsilon) = \{\omega \in [0,1] : M(T^n\omega, 0, 0) > \varepsilon\} = \Big\{\omega : \omega > \frac{\varepsilon k^n}{2}\Big\}.$$

Its measure is

$$\mu(A_n(\varepsilon)) = \begin{cases} 1 - \dfrac{\varepsilon k^n}{2}, & \dfrac{\varepsilon k^n}{2} \le 1, \\ 0, & \text{otherwise.} \end{cases}$$
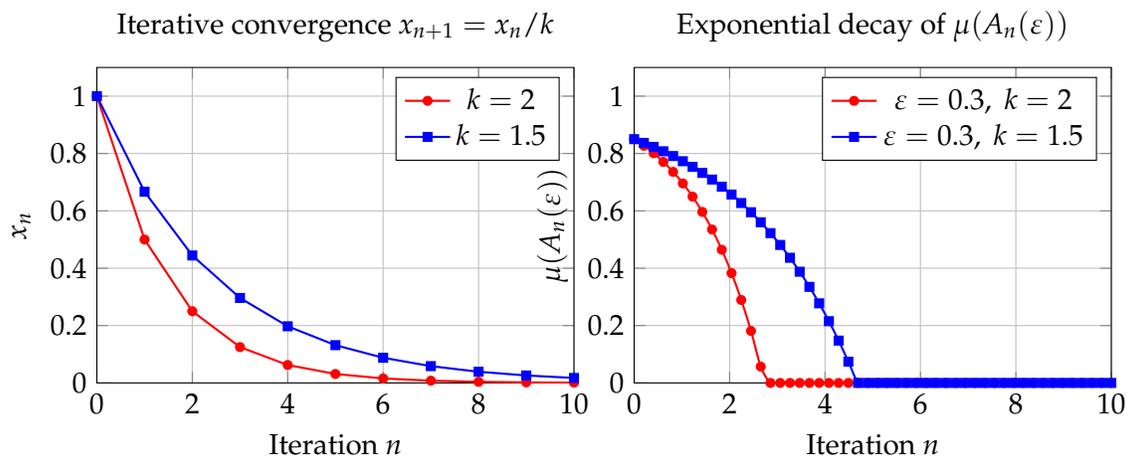
For $n > \ln(2/\varepsilon)/\ln k$ we obtain

$$\mu(A_n(\varepsilon)) \le 1 - \frac{\varepsilon k^n}{2} \le e^{-n\ln k} = e^{-n\lambda},$$

using $1 - t \le e^{-t}$ for $t \ge 0$. This confirms exponential convergence in measure with $C = 1$, $\lambda = \ln k$.

Graphical Illustration of Convergence.

The exponential decay of $\{x_n\}$ and of $\mu(A_n(\varepsilon))$ is visualized below. The left panel shows the iteration sequence for two contraction factors; the right panel displays the measure of the "slow-convergence" set.

The plots confirm the analytical rates: larger k yields faster convergence, and the measure of the set where the error exceeds ε decays exponentially.

Application: Stability of Uncertain Dynamical Systems.

Consider a discrete-time system

$$x_{n+1} = T(x_n), \qquad T : \mathcal{Z} \to \mathcal{Z},$$

on a complete NMR-MS $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$. When T is a neutrosophic MR-contraction as in Theorem 2.1, the following stability guarantees hold:

(1) **Unique global equilibrium** x^* exists.

(2) **Global exponential stability:** $M(x_n, x^*, x^*) \le Ce^{-n\lambda}$.

(3) **Neutrosophic robustness:** $\mathcal{T}(x_n, x^*, \gamma) \to 1$, $\mathcal{F}(x_n, x^*, \gamma) \to 0$, $\mathcal{I}(x_n, x^*, \gamma) \to 0$.

(4) **Measure-theoretic reliability:** For any initial distribution μ,

$$\mu\Big(\{x_0 : M(x_n, x^*, x^*) > \varepsilon\}\Big) \le C_\mu e^{-n\lambda},$$

so almost every initial condition stabilizes rapidly.

Concrete Applications.

- **Optimization under vague gradients:** Stochastic algorithms with imprecise gradient information can be modeled as neutrosophic contractions, guaranteeing convergence to a unique optimum.
- **Robust control:** Feedback design that renders the closed-loop dynamics a neutrosophic MR-contraction ensures stability despite parameter uncertainties.
- **Multi-agent consensus with vague preferences:** Iterative strategy updates in social/economic networks converge to a stable equilibrium with explicit certainty guarantees.
- **Numerical schemes for uncertain differential equations:** Discretizations of equations with neutrosophic parameters converge exponentially in both metric and neutrosophic senses.

Conclusion. The framework of Neutrosophic MR-Metric Spaces, combined with the fixed-point theorem proved here, provides a unified, rigorous tool for stability analysis under simultaneous metric and logical uncertainty. It extends classical results to problems where vagueness, indeterminacy, and partial truth are intrinsic, offering strong guarantees for existence, uniqueness, exponential convergence, and measure-theoretic reliability.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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