

Collective Aggregation Method Based on EVAMIX, the Gini Mean and the Hybrid Operator of Zimmermann and Zysno

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Abstract. These days, our lives are governed by a large number of decisions. They are subject to consensus or disagreement among the players involved in the decision-making process. So it's important to make a decision that meets everyone's aspirations. Several decision-support methods have therefore been developed, but they all have their shortcomings. The EVAMIX method is one such aggregation method that has good properties but is used in the context of a single decision-maker. The aim of our work is to extend the EVAMIX method to group decision-making. To this end, we have proposed a collective aggregation method based on the EVAMIX method, incorporating the Gini mean and the hybrid operator of Zimmermann and Zysno. Finally, we conducted numerical experiments, which produced some interesting results.

1. INTRODUCTION

In all organisational structures, one of the tasks which has a major impact on its existence is decision-making and is considered to be the very essence of management [1]. This obliges the managers of these human societies to call on outside help. Numerous decision-making methods exist in the literature and have shown their effectiveness, but sometimes provide controversial results [5]. Other methods provide approximate results; they do not yield a single solution that satisfies all the problem parameters [11]. This has sparked a surge of interest in finding a suitable method that produces more satisfactory results. It is with this in mind that many believe that multi-criteria decision-making methods for solving collective decision-making problems are

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closer to the real thing [18]. These collective aggregation methods are used to aggregate the opinions of the various players involved in the decision-making process with a view to obtaining consensual results [2]. The EVAMIX method is a single-decision multi-criteria decision-making method with good properties that is widely used to solve these types of problems [15]. The aim of our work is to contribute to group decision making using the EVAMIX method. To do so, we will combine it with the Gini mean and the hybrid operator of Zimmermann and Zysno to propose the collective aggregation method. In this work, we will first present some group decision support methods and the description of the EVAMIX method. Then, we will propose the principles of the collective aggregation method using the EVAMIX method, the Gini mean and the mixed connector of Zimmermann and Zysno. Finally, we will apply and compare these results with some existing group decision methods in the literature.

2. PRELIMINARIES

2.1. Presentation of the harmonic mean applied to group decision: the Lon-Zo method. [17] In this section w_j^k designates the weight assigned by decision-maker k with regard to criterion j . the harmonic mean \bar{x}_h of the values x_1, \dots, x_n is defined as follows:

$$\bar{x}_h = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}} \quad (2.1)$$

Note:

- N the number of decision-makers and D the set $D = \{d_1, d_2, \dots, d_N\}$;
- m the number of criteria whose index set is $\{1, 2, \dots, m\}$;
- M the number of shares and their set $A = \{a_1, a_2, \dots, a_M\}$. Let G_k be the additive value aggregation function for the decision maker D_k . The collective aggregation function based on the harmonic mean is called the Lon-Zo (Longin-Zoinabo) method and is defined as follows:

$$U(a_i) = \frac{N}{\sum_{k=1}^N \frac{1}{G_k(a_i)}} \quad (2.2)$$

with

$$G_k(a_i) = \sum_{j=1}^{j=m} w_j^k g_j^k \quad i = 1, \dots, M; j = 1, \dots, m. \quad (2.3)$$

2.2. Application of the arithmetic mean to group decision: the MACASP method. This section is inspired by [17]. The arithmetic mean of x_1, \dots, x_n is defined as follows:

$$\bar{x} = \frac{\sum_{i=1}^{i=n} x_i}{n} \quad (2.4)$$

- Let N be the number of decision-makers and D the set $D = \{d_1, d_2, \dots, d_N\}$;

- b) Let m be the number of criteria whose index set is $\{1, 2, \dots, m\}$;
- c) Let M be the number of shares and their set $A = \{a_1, a_2, \dots, a_M\}$. Let G_k be the additive value aggregation function for the decision maker D_k . Thus, we obtain the arithmetic mean as follows:

$$U_j(a_i) = \sum_{i=1}^N G_j(a_i)/N \quad i = 1, \dots, N; j = 1, \dots, m. \tag{2.5}$$

$$G_k(a_i) = \sum_{j=1}^{j=m} w_j^k g_j^k(a_i) \tag{2.6}$$

The collective aggregation function based on the arithmetic mean is called MACASP (Collective Aggregation Model using the Weighted Sum), denoted U and defined by:

$$U(a_i) = \sum_{k=1}^{k=N} G_k(a_i)/N \tag{2.7}$$

and

$$U(a_i) = \sum_{k=1}^{k=N} \sum_{j=1}^{j=m} w_j^k g_j^k(a_i)/N \tag{2.8}$$

$$U(a_i) = \sum_{j=1}^{j=m} \sum_{k=1}^{k=N} w_j^k g_j^k(a_i)/N \tag{2.9}$$

At the end, each action is given a final score.

Although these previous methods can be used to solve multi-criteria multi-decision-makers problems, they are most effective when all the data in the problems is numerical or quantitative. Some types of these methods are studied in [8] and [14]. In the case of problems with mixed data(quantative and qualitative) or only qualitative data, it would first be necessary to convert the qualitative data into numerical values, which could lead to the loss of a lot of information and consequently to controversial results. Tis new aggregation model is less compensatory and has the good properties desired for a good ranking method.

2.3. Description of the EVAMIX method. Based on [2] and [6], the steps of the EVAMIX method are as follows:

Step 1: Calculation of the index of dominance of the actions(of action i in relation to action j) α_{ij} and β_{ij} respectively for the ordinal(O) and cardinal(C) criteria.

$$\left\{ \begin{array}{l} \alpha_{ij} = \left[\sum_{k \in O} \{W_k \times \text{sgn}(e_{ik} - e_{jk})\}^c \right]^{1/c} \\ \beta_{ij} = \left[\sum_{k \in C} \{W_k \times (e_{ik} - e_{jk})\}^c \right]^{1/c} \\ i, j \in \{1, \dots, m\}, k \in \{1, \dots, n\} \end{array} \right. \quad \text{with} \quad \text{sgn}(e_{ik} - e_{jk}) = \begin{cases} -1 & \text{for } e_{ik} < e_{jk} \\ 0 & \text{for } e_{ik} \simeq e_{jk} \\ 1 & \text{for } e_{ik} > e_{jk} \end{cases} \tag{2.10}$$

Step 2 Calculation of the normalised dominance index:

$$\begin{cases} \delta_{ij} = \frac{(\alpha_{ij} - \alpha^-)}{(\alpha^+ - \alpha^-)} & (\text{ordinal criteria}) \\ d_{ij} = \frac{(\beta_{ij} - \beta^-)}{(\beta^+ - \beta^-)} & (\text{cardinal criteria}) \\ i, j \in \{1, \dots, m\} \end{cases} \quad (2.11)$$

where α^+ and β^+ are the maximum quantities of the rates of superiority of the actions respectively of the ordinal and cardinal criteria, α^- and β^- are the minimum quantities of the rates of superiority of the actions respectively of the ordinal and cardinal criteria.

Step 3: Calculate the total dominance of action i over action j as follows:

$$\begin{cases} D_{ij} = W_O \delta_{ij} + W_C d_{ij} & i, j \in \{1, \dots, m\} \\ W_O = \sum_{k \in O} W_k \quad \text{et} \quad W_C = \sum_{k \in C} W_k \end{cases} \quad (2.12)$$

Step 4 Calculate the overall score per action as follows:

$$S(a_i) = S_i = \left[\sum_j \frac{D_{ji}}{D_{ij}} \right]^{-1} \quad i, j \in \{1, \dots, m\} \quad (2.13)$$

$S(a_i) > S(a_j)$ means that action a_i is better than action a_j .

The EVAMIX method presented above exists in the literature but unfortunately it is only used on multi-criteria problems in the context of a single decision-maker, which limits its field of application. We then propose an extension of this method in order to adapt it to group decision.

3. MAIN RESULTS

3.1. Aggregation functions used in our extension.

3.1.1. *Gini mean.* This section is from [4]. Consider two parameters r and s . The Gini mean of parameters r and s for the n values of x_k is defined by:

$$G_{r,s}(x) = \left(\frac{\sum_{k=1}^n x_k^{r+s}}{\sum_{k=1}^n x_k^s} \right)^{\frac{1}{r}} \quad \text{si } r \neq s \quad (3.1)$$

3.1.2. *Hybrid operator of Zimmermann and Zysno.* This section is inspired by [10].

. Consider n values of x_k such that $\forall x_k \in [0; 1]$. The hybrid exponential operator of Zimmermann and Zysno γ is defined by:

$$\gamma(x_1, \dots, x_n) = \left(\prod_{k=1}^n x_k \right)^{1-\alpha} \left(1 - \prod_{k=1}^n (1 - x_k) \right)^{\alpha} \quad \text{with } \alpha \in [0; 1] \quad (3.2)$$

Definition 3.1. *Symbols*

- d_k : Criterion k

- c_l : Criterion l
- a_i : Action i
- p_j^k : the weight assigned to criterion c_l by decision-maker k P_l
- P_l : Lthe total weight of each criterion
- α_{ij}^k : the dominance index of action a_i compared to a_j according to decision-maker d_k at the level of ordinal criteria.
- β_{ij}^k :the dominance index of action a_i compared to a_j according to decision-maker d_k at the level of cardinal criteria.
- $g_l^k(a_i)$:The evaluation of action a_i with regard to criterion c_l according to decision-maker d_k
- δ_{ij}^k : Normalized dominance index of stock a_i relative to a_j according to decision-maker d_k at the ordinal criteria level
- d_{ij}^k :Normalized dominance index of stock a_i relative to a_j according to decision-maker d_k at the cardinal criteria level
- α_{ij} : The normalized total dominance index of a_i relative to a_j in the ordinal criteria
- d_{ij} : The normalized total dominance index of a_i relative to a_j in the cardinal criteria
- D_{ij} : Total dominance of action a_i over a_j
- $S(a_i)$: Overall score for each action.

3.2. Principle of the collective aggregation method based on EVAMIX, the Gini mean and the hybrid operator of Zimmermann and Zysno(MACEGOHZZ method). This extension follows these steps:

Step 1:Global weight per criterion of the k decision-makers To obtain the global weight, we take $r=4$ and $s=2$:

$$\left\{ \begin{array}{l} P_j = \frac{\sum_{k=1}^K (p_j^k)^4}{\sum_{k=1}^K (p_j^k)^2}, ; j = 1, \dots, M \\ P = \{p_1, p_1, \dots, p_M\} \end{array} \right. \quad (3.3)$$

With p_j^k the weight assigned to criterion g_j by decision-maker k . For example, for three(03) decision-makers, we have $P_1 = \sqrt{\frac{(p_1^1)^4+(p_1^2)^4+(p_1^3)^4}{(p_1^1)^2+(p_1^2)^2+(p_1^3)^2}}$

Step 2: Calculation of the rate of superiority of actions α_{ij}^l and β_{ij}^l for a given decision-maker d_l .

$$\left\{ \begin{array}{l} \alpha_{ij}^l = \left[\sum_{k \in O} \{p_k^l \times \text{sgn}(g_k^l(a_i) - g_k^l(a_j))\}^c \right]^{1/c} \\ \beta_{ij}^l = \left[\sum_{k \in C} \{p_k^l \times (g_k^l(a_i) - g_k^l(a_j))\}^c \right]^{1/c} \\ i, j \in \{1, \dots, N\}, k \in \{1, \dots, M\}, l \in \{1, \dots, S\} \end{array} \right. \quad \text{sgn}(g_k^l(a_i) - g_k^l(a_j)) = \begin{cases} -1 & \text{si } g_k^l(a_i) < g_k^l(a_j) \\ 0 & \text{si } g_k^l(a_i) \simeq g_k^l(a_j) \\ 1 & \text{si } g_k^l(a_i) > g_k^l(a_j) \end{cases} \quad (3.4)$$

c being any real number, in our application we take $c=1$.

. **Step 3:** Normalisation of the rate of superiority α_{ij}^l and β_{ij}^l of the actions.

$$\begin{cases} \delta_{ij}^l = \frac{(\alpha_{ij}^l - \alpha^-)}{(\alpha^+ - \alpha^-)} & (\text{criteria ordinal}) \\ d_{ij}^l = \frac{(\beta_{ij}^l - \beta^-)}{(\beta^+ - \beta^-)} & (\text{cardinal criteria}) \\ i, j \in \{1, \dots, N\}; l \in \{1, \dots, S\} \end{cases} \quad (3.5)$$

where α^+ and β^+ are the maximum quantities of the rates of superiority of the actions respectively of the ordinal and cardinal criteria, α^- and β^- are the minimum quantities of the rates of superiority of the actions respectively of the ordinal and cardinal criteria.

Step 4: Normalised overall superiority rates δ_{ij} and d_{ij} of the actions.

Taking $\alpha = 0.5$, we obtain the following formula:

$$\begin{cases} \alpha_{ij} = \sqrt{\prod_{l=1}^S \delta_{ij}^l \times \left(1 - \prod_{l=1}^S (1 - \delta_{ij}^l)\right)} \\ d_{ij} = \sqrt{\prod_{l=1}^S d_{ij}^l \times \left(1 - \prod_{l=1}^S (1 - d_{ij}^l)\right)} \\ i, j = 1, \dots, N \end{cases} \quad (3.6)$$

Step 5 Calculating the total dominance of action i over action j.

$$\begin{cases} D_{ij} = P_O \times \alpha_{ij} + P_C \times d_{ij} & i, j \in \{1, \dots, M\} \\ P_O = \sum_{j \in O} P_j \quad \text{et} \quad P_C = \sum_{j \in C} P_j \end{cases} \quad (3.7)$$

Step 6 Calculation of the global score per action.

$$S_i = \left[\sum_j \frac{D_{ji}}{D_{ij}} \right]^{-1} \quad i, j \in \{1, \dots, M\} \quad (3.8)$$

$S(a_i) > S(a_j)$ means that action a_i is better than action a_j .

This new method generates middle compensation between weak and strong criteria and makes it possible to solve problems with mixed data. Which is not possible with many group decision methods see [8]. With a view to implementation and making it easier to take account of large-scale data, the following section presents an algorithm for our method.

3.3. MACEGOHZZ algorithm.

Algorithm 3.1. ENTRATES

- $\{a_1, \dots, a_N\}$ the set of N actions

- $\{c_1, \dots, c_M\}$ the set of M criteria

- $\{d_1, \dots, d_K\}$ the set of K decision-makers

$\{-p_1^1, \dots, p_j^k\}$ the weights of the criteria given by the decision-makers k

$\{-g_{11}^1, \dots, g_{ij}^k\}$ the evaluations of actions i with regard to criteria j by decision-makers k .

OUT:

- Ranking of actions from best to worst.

START: Declaration of variables

-PG $[n, n']$: Table of real

-K $[n]$: Table of real

-K' $[n]$: Table of real

-PG $[n]$: Table of real

-D $[n, n']$: Table of real

-alphk $[n, n']$: Table of real

-bet $[n, n']$: Table of real

-AGO $[n, n']$: Table of real

-AGC $[n, n']$: Table of real

-AC^k $[n, n']$: Table of real

bullet Main action

for j from 1 to n do

PG $[j] \leftarrow 1, K [j] \leftarrow 0, K' [j] \leftarrow 0$

for k from 1 to K do

$K' [j] \leftarrow K [j] + (p_j^k)^4; K' [j] \leftarrow K' [j] + (p_j^k)^2$

End for

PG $[j] \leftarrow (K [j] / K' [j])^{1/2}$

End for

for i from 1 to N do

for i' from 1 to N do

for k from 1 to K do

AC^k $[i, i'] \leftarrow 0$

for j from 1 to M' do (M' is the number of qualitative criteria)

AC^k $[i, i'] \leftarrow g_{ij}^k - g_{i'j}^k$

End for

End for

End for

End for

Function $\text{sgn}(AC^k [i, i'])$

If $AC^k [i, i'] > 0$ then

returns 1

otherwise if $AC^k [i, i'] < 0$ then

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    returns -1
  otherwise returns 0
  End If
for i from 1 to N return
  for i' from 1 to N do
    for k from 1 to K do
      alphk [i, i'] ← 0
      for j from 1 to M' do
        alphk [i, i'] ← alphk [i, i'] + pjksgn(ACk [i, i'])
      End for
    End for
  End for
End for
for i from 1 to N do
  for i' from 1 to N do
    for k from 1 to K do
      betk [i, i'] ← 0
      for j from 1 to M'' do (M'' is the number of quantitative criteria)
        betk [i, i'] ← betk [i, i'] + pjk(gijk - gi'jk)
      End for
    End for
  End for
End for
for k from 1 to K do
  alphmink ← max(alphk [i, i'])
  alphmaxk ← max(alphk [i, i'])
  betmink ← min(betk [i, i'])
  betmaxk ← max(betk [i, i'])
  Ecartalphk ← alphmaxk - alphmink
  Ecartbetk ← betmaxk - betmink
end for
for k from 1 to K do
  for i from 1 to N do
    for i' from 1 to N do
      for i from 1 to K do
        deltk [i, i'] ← (alphk [i, i'] - alphmink) / Ecartalphk
        dk [i, i'] ← (betk [i, i'] - betmink) / ecartbetk
      End for
    End for
  End for

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    End for
End for

for i from 1 to N do
    for i' from 1 to N do
        AGC [i, i'] ← 1
        AGO [i, i'] ← 1
        prod [i, i'] ← 1
        prod' [i, i'] ← 1
        pour k allant de 1 à K faire
            prod [i, i'] ← prod [i, i'] × deltk [i, i']
            prod' [i, i'] ← prod' [i, i'] × [1 - (1 - deltk [i, i'])]
        End for
        AGO [i, i'] ← prod [i, i'] × prod' [i, i']
    End for
End for

for i from 1 to N do
    for i' from 1 to N do
        AGC [i, i'] ← 1
        prodC [i, i'] ← 1
        prodC' [i, i'] ← 1
        for k from 1 to K do
            prodC [i, i'] ← prodC [i, i'] × dk [i, i']
            prodC' [i, i'] ← prodC' [i, i'] × [1 - (1 - dk [i, i'])]
        end for
        AGC [i, i'] ← prodC [i, i'] × prodC' [i, i']
    end for
end for

PAO ← 0
for j from 1 to M' do
    PAO ← PAO + pj sum of the weights of the ordinal criteria
end for

for i from 1 to N do
    for i' from 1 to N do
        D [i, i'] ← PAO × AGO [i, i'] + PAC × AGC [i, i']
    end for
end for

for i from 1 to N do

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S [i] ← 0
for i' from 1 to N do
    S [i] ← [S [i] + D [i', i] / D [i, i']]-1
end for
end for
Sort(S [1], S [2], ..., S [n] )
END

```

3.4. Study of complexity. Time complexity analysis is an essential step in the evaluation of an algorithmic model. It facilitates a simple understanding and monitoring of the evolution of a method's execution time as the data size changes. It also allows for comparison of the method in question with other existing methods.

- **step 1:** Element-wise operation:

▷ Calculation of $\sqrt{\frac{\sum_{k=1}^K (p_j^k)^4}{\sum_{k=1}^K (p_j^k)^2}}$: Global weight per criterion c_j of the k decision-makers →

$\mathcal{O}(m \times k)$

▷ $\left[\sum_{k \in \mathcal{O}} \{p_k^l \times \text{sgn}(g_k^l(a_i) - g_k^l(a_j))\}^c \right]^{1/c}$: Calculation of the rate of superiority of actions α_{ij}^k and β_{ij}^k for a given decision-maker d_k → $\mathcal{O}(n^2 \times m \times k)$

▷ $\left[\sum_{c \in \mathcal{C}} \{p_k^l \times (g_k^l(a_i) - g_k^l(a_j))\}^c \right]^{1/c}$ → $\mathcal{O}(n^2 \times m \times k)$

▷ $\frac{(\alpha_{ij}^l - \alpha^-)}{(\alpha^+ - \alpha^-)}$: Normalisation of the rate of superiority α_{ij}^l and β_{ij}^l of the actions → $\mathcal{O}(n^2 \times m \times k)$

▷ $\frac{(\beta_{ij}^l - \beta^-)}{(\beta^+ - \beta^-)}$: Normalisation of the rate of superiority α_{ij}^l and β_{ij}^l of the actions → $\mathcal{O}(n^2 \times m \times k)$

▷ $\sqrt{\prod_{l=1}^S \delta_{ij}^l \times \left(1 - \prod_{l=1}^S (1 - \delta_{ij}^l)\right)}$: Normalised overall superiority rates δ_{ij} and d_{ij} of the actions → $\mathcal{O}(n^2 \times k)$

▷ $\sqrt{\prod_{l=1}^S d_{ij}^l \times \left(1 - \prod_{l=1}^S (1 - d_{ij}^l)\right)}$: Normalised overall superiority rates δ_{ij} and d_{ij} of the actions → $\mathcal{O}(n^2 \times k)$

▷ $P_O \times \alpha_{ij} + P_C \times d_{ij}$: Calculating the total dominance of action i over action j → $\mathcal{O}(n^2)$.

▷ $\left[\sum_j \frac{D_{ji}}{D_{ij}} \right]^{-1}$: Calculating the score for each action → $\mathcal{O}(n^2)$

- **Step 2:** Total complexity

Conclusion: The time complexity of the proposed new method, after a rigorous study of each part of the algorithm, is: $\mathcal{O}(\max(n^2 \times m \times k, n^2 \times k, n^2 \times m))$.

TABLE 1. Multiple criteria methods and their complexities [21]

Method	Reference	Complexity	Multiple Decision-maker
TOPSIS	Hwang and Yoon (1981)	$O(m.n)$	no
ELECTRE (I, II, III)	Roy (1968, 1978a, 1978b)	$O(m.n^2)$	no
PROMETHEE (I,II III)	Brans,Vincke (1985) Brans,Mareschal(1986;1994)	$O(m.n^2)$	no
AHP	Saaty (1980)	$O(m^3 + m.n^3)$	no
Group AHP	Ossadnik et al. (2016)	$O(l.m.n^2)$	yes
DS/AHP	7Beynon et al. (2000)	$O(l.m.n^2)$	yes
AHP/VAHP	Soltanifar and Kamyabi (2024).	$O(l.m.n^2)$	yes
AHP-SWARA	Zolfani and Saparauskas (2013)	$O(m.n + m^2)$	no
SWARA-VAHP	Keršulienė et al. (2010)	$O(l.n^2.m)$	yes
AHP-BWM	Rezaei (2015)	$O(m.n + m^2)$	no
BWM-VAHP	Mi et al. (2019)	$O(l.m.n + m^2)$	yes
WM-AHP	Dong et al. (2010)	$O(l.m.n^2)$	yes
BM-AHP	Ishizaka and Labib (2011)	$O(m.n^2)$	no
CAHP	Ngoie et al. (2022)	$O(\max(n.m.l, m.n^3))$	yes

In view of all the above, the MACEGOHZZ method can be used to solve certain group decision problems.

3.5. **Desired properties.** In [3], it is shown that all classification methods verify these various properties and their mathematical formulae are developed in [12].

- (1) **Anonymity:** If the aggregation model verifies this property, it must treat the decision-makers involved in the decision-making process equally. When two decision-makers swap their different judgement alue, the final result should remain intact. With the new method each time the decision-markers' evaluation values are swapped, the MACEGOHZZ method always provides the same winner. The aggregation concerns only the evaluation values, so a random exchange of decision-makers has no influence on the final choice.
- (2) **Neutrality:** Its ensures that the grading method provides equal treatment of alternatives. Indeed, if two actions swap positions in each decision-maker's judgment matrix then the final result would always respect this change of order. The MACEGOHZZ method respects this, especially when calculating total dominance and score for each action.
- (3) **Pareto effeciency:** Let's assume that one action is preferred by all decision-makers, so this action will have a higher judgment value than all other actions. The MACEGOHZZ method will give it a high final score and it wil be identified as the best action.

- (4) **Independance from irrelevant actions:** Let's consider two action a and b such that $a \overset{P^k}{j} b$ (read a is preferred to b for all decision-makers d_k according to all criteria c_j), then the addition or absence of another action c would not influence the final ranking of a over b. the MACEGOHZZ method respects that.
- (5) **Monotonicity:** Let's consider the set $A = \{a_1, a_2, \dots, a_n\}$, $d = \{d_1, d_2, \dots, d_s\}$, $C = \{c_1, c_2, \dots, c_m\}$ actions, decision-makers and criteria respectively, and $E = (e_{ij}^k)$ the judgments of action i by each decision-maker d_k according to criterion j. Suppose that $\text{MACEGOHZZ}(e_{ij}^k) = a_e$. If the evaluation e_{ij}^k , for all $k=1, \dots, s$ is swapped by g_{ij}^k with $g_{ij}^k \geq e_{ij}^k$ then the MACEGOHZZ method would identify the new winner a_g .

Theorem 3.1. *The MACEGOHZZ method respects the properties of anonymity, neutrality, Pareto efficiency, Independance from irrelevant actions and Monotonicity.*

Proof

- *anonymity:* Consider the following sets of actions, decision-makers and criteria respectively: $A = \{a_1, a_2, \dots, a_n\}$, $d = \{d_1, d_2, \dots, d_s\}$, $C = \{c_1, c_2, \dots, c_m\}$ and $E = (e_{ij}^k)$ the judgments of action i by each decision-maker d_k according to criterion j. Let's take $E(a_i) = (e_j^{d_l}(a_i))$ and $E(a_i) = (e_j^{d_z}(a_i))$ the decision matrix of any two decision-makers d_l and d_z . Suppose that $S(a_t) > S(a_i)$ with $t=1, \dots, n$; that is to say a_t is a winner. Now, if the two decision-makers swap their judgments values that is to say $e_j^{d_l}(a_i) \leftarrow e_j^{d_z}(a_i)$ and $e_j^{d_z}(a_i) \leftarrow e_j^{d_l}(a_i)$ then $S(a_t) = \left[\sum_j \frac{D_{jt}}{D_{ij}} \right]^{-1} > S(a_i) = \left[\sum_j \frac{D_{ji}}{D_{ij}} \right]^{-1}$. So $\text{MACEGOHZZ}(e_j^{d_l}(a_i); e_j^{d_z}(a_i)) = a_t$ with $l, z=1, \dots, s$. Hence the anonymity of the MACEGOHZZ method.
- *Neutrality:* Let's take two actions a_i and a_j evaluated by k decision-makers $\{d_1, d_2, \dots, d_s\}$ and with evaluation matrices such as

$$d_k = \begin{bmatrix} g_1^k(a_x) & g_2^k(a_x) & \dots & g_m^k(a_x) \\ g_1^k(a_y) & g_2^k(a_y) & \dots & g_m^k(a_y) \end{bmatrix}$$

Let's assume $S(a_x) = \left[\sum_j \frac{D_{jx}}{D_{xj}} \right]^{-1} > S(a_y) = \left[\sum_j \frac{D_{jy}}{D_{yj}} \right]^{-1}$ using the MACEGOHZZ $a_x P a_y$ (read " a_x is better than a_y by MACEGOHZZ "). Now if the decision-makers d_k The evaluation scores for the two actions are substituted a_x and a_y i.e. $e_1^k(a_y), e_2^k(a_y), \dots, e_m^k(a_y)$ to action a_x and vice versa $S(a_y) > S(a_x)$ this means that by MACEGOHZZ a_y is better than a_x . This justifies the neutrality of MACEGOHZZ.

- *Pareto efficiency:* To do this, let's assume that the decision-makers' judgment matrices

$$d_k = \begin{bmatrix} e_1^k(a_1) & e_2^k(a_1) & \dots & e_m^k(a_1) \\ e_1^k(a_2) & e_2^k(a_2) & \dots & e_m^k(a_2) \\ \dots & \dots & \dots & \dots \\ e_1^k(a_n) & e_2^k(a_n) & \dots & e_m^k(a_n) \end{bmatrix}$$

for all criteria c_j and actions $a_i, a_2 P_j^k a_i \Rightarrow g_j^k(a_2) \geq g_j^k(a_i)$ for all decision-makers d_k with $k=1, \dots, s$.

We have $g_j^k(a_2) \geq g_j^k(a_i) \Rightarrow \left[\sum_j \frac{D_{2i}^k}{D_{2i}^k} \right]^{-1} > \left[\sum_j \frac{D_{ij}^k}{D_{ij}^k} \right]^{-1} \Rightarrow S(a_2) > S(a_i)$ so by MACEGOHZZ a_2 is the best from where Pareto efficiency.

- *Independance from irrelevant actions:* Consider $A = \{a_1, a_2, \dots, a_n\}$ A set of actions, then take any two actions a_r and a_n in this set. I all decision-makers as a whole rank a_r compared to a_n i.e. $e_j^k(a_r) \geq e_j^k(a_n) \Rightarrow a_r P_j^k a_n$. MACEGOHZZ will naturally classify a_r front a_n . When you delete or add any action in the set of actions predefined by MACEGOHZZ $S(a_r) > S(a_n)$ so a_r will always be ranked higher than a_n .

- *Monotonicity:* Let be a set $A = \{a_1, a_2, \dots, a_n\}$, a set of decision-makers $d = \{d_1, d_2, \dots, d_s\}$ a set of criteria $C = \{c_1, c_2, \dots, c_m\}$ and $E = (e_{ij}^k)$. Suppose that MACEGOHZZ(e_{ij}^k)= a_p .

MACEGOHZZ(e_{ij}^k)= $a_p \Rightarrow \left[\sum_j \frac{D_{jp}^k}{D_{jp}^k} \right]^{-1} > \left[\sum_j \frac{D_{ij}^k}{D_{ij}^k} \right]^{-1} \Rightarrow S(a_p) > S(a_i)$. Now, let's substitute e_{ij}^k by e'_{ij}^k such as $e'_{ij}^k > e_{ij}^k$.

$e'_{ij}^k > e_{ij}^k \Rightarrow \left[\sum_j \frac{D_{iq'}^k}{D_{iq'}^k} \right]^{-1} > \left[\sum_j \frac{D_{ij}^k}{D_{ij}^k} \right]^{-1} \Rightarrow S(a'_q) > S(a_i)$ so MACEGOHZZ(e'_{ij}^k)= a'_q .

3.6. Comparative table of existing methods and the MACEGOHZZ method. This comparative study will clearly show the strengths and weaknesses of the collective methods LON-ZO, MACASP and ELECTRE I and the capacity of the MACEGOHZZ method to progress beyond each of them.

TABLE 2. Comparative table of LON-ZO, MACASP, ELECTRE I and the MACEGOHZZ method

	forces	weakness	the importance of MACEGOHZZ on each of these methods
LON-ZO	effectively adapted on cardinal criteria; easy to use	compensatory; failure to comply with the direction of optimization of the criteria	less compensatory; suitable for mixed data (ordinal and cardinal); takes into account the direction of optimization of the criteria
MACASP	effectively adapted on cardinal criteria; easy to use	compensatory; inefficient application on ordinal criteria; failure to comply with the direction of optimization of the criteria	less compensatory; suitable for mixed data (ordinal and cardinal); takes into account the direction of optimization of the criteria
ELETRE I	Non-compensatory; Takes into account data imprecision	robust calculation; requires conversion of ordinal to numeric evaluations=loss of real information	simple calculation; polynomial time complexity; does not require conversion of ordinal evaluations into numerical values = real information

This table clearly presents the novelty of the MACEGOHZZ method and its ability to produce satisfactory results for certain group decision problems.

4. NUMERICAL SIMULATION

4.1. **Example 1.** A higher education institution wanted to recruit a director of studies. After examining the applications, four applications were selected for the final phase. These were to be evaluated and selected by a panel comprising the founder, the rector and the president of the institute on the basis of selection criteria such as: leadership skills (fair, fairly good, good, very good), number of years of professional experience, interpersonal skills (not very important, less important, important, very very important) and the monthly cost of each candidate's service. These three decision-makers wanted to have a ranking of the candidates to make their choice easier. Using the following family of criteria

$C = \{c_1, c_2, c_3, c_4\}$ where c_1 is leadership skills, c_2 number of years of professional experience, c_3 relational and interpersonal skills, c_4 monthly cost of service for each candidate and a family of alternatives:

$A = \{\text{folder 1, folder 2, folder 3, folder 4}\}$.

Each decision-maker evaluates the various files in the light of the criteria, and we obtain the following judgement matrices:

TABLE 3. decision maker 1(founder)

	c_1	c_2	c_3	c_4
weight	6	5	4	7
folder 1	good	5	not very important	7
folder 2	quite good	6	important	7
folder 3	fair	7	very important	1
folder 4	fair	4	less important	2

TABLE 4. decision maker 2(rector)

	c_1	c_2	c_3	c_4
weight	4	3	7	5
folder 1	quite good	2	very important	4
folder 2	fair	7	very important	3
folder 3	good	8	not very important	1
folder 4	fairly good	1	not very important	2

TABLE 5. decision maker 3(president)

	c_1	c_2	c_3	c_4
weight	1	5	2	4
folder 1	good	2	not very important	6
folder 2	very good	8	less important	3
folder 3	fair	4	very important	7
folder 4	good	3	less important	4

4.1.1. *Resolution using the LON-ZO method.* First of all, let's convert the characters into numerical values, using Saarty's scale as a guide.

fair	not very important	3
quite good	less important	5
good	important	7
very good	very important	9

Applying the LON-ZO method, we get this:

	$\sum_{j=1}^{j=4} w_j^1 g_j^1$	$\sum_{j=1}^{j=4} w_j^2 g_j^2$	$\sum_{j=1}^{j=4} w_j^3 g_j^3$	$U(a_i) = \frac{3}{\sum_{k=1}^3 \frac{1}{G_k(a_i)}}$	Rang
folder 1	128	109	47	78.40	3e
folder 2	137	111	71	98.70	1er
folder 3	96	78	69	79.51	2e
folder 4	72	45	48	56.34	4e

Solving with the LON-ZO method gives us the **folder 2** as the best choice.

	$\sum_{k=1}^3 w_1^k g_1^k(a_i)$	$\sum_{k=1}^3 w_2^k g_2^k(a_i)$	$\sum_{k=1}^3 w_3^k g_3^k(a_i)$	$\sum_{k=1}^3 w_4^k g_4^k(a_i)$
folder 1	69	41	81	93
folder 2	51	91	101	76
folder 3	49	79	75	40
folder 4	45	38	51	40

	$\frac{\sum_{j=1}^4 \sum_{k=1}^4 w_j^k g_j^k(a_i)}{N}$	Rank
folder 1	284/3	2 nd
folder 2	319/3	1 st
folder 3	243/3	3 rd
folder 4	174/3	4 th

4.1.2. *Resolution using the MACASP method.* With the MACASP method, the best choice is **folder 2**.

4.1.3. *Resolution by the MACEGOHZZ method.* In this example 1, the qualitative criteria are leadership skills and interpersonal skills. The quantitative criteria are the number of years of professional experience and the monthly cost of service for each candidate.

TABLE 6. overall criteria weight matrix

	d_1	d_2	d_3	$P_j = \sqrt{\frac{\sum_{k=1}^3 (p_j^k)^4}{\sum_{k=1}^3 (p_j^k)^2}}$
c_1	6	4	1	4.81
c_2	5	3	5	4.53
c_3	4	7	2	5.30
c_4	7	5	4	5.62

TABLE 7. Index of dominance of actions in the founder's cardinal criteria)

	folder 1	folder 2	folder 3	folder 4
folder 1	0	-5	32	40
folder 2	5	0	37	45
folder 3	-32	-37	0	8
folder 4	-40	-45	-8	0

TABLE 8. (Index of dominance of actions in the founder's ordinal criteria

	folder 1	folder 2	folder 3	folder 4
folder 1	0	2	2	2
folder 2	-2	0	2	10
folder 3	-2	-2	0	4
folder 4	-2	-10	-4	0

TABLE 9. Index of dominance of actions in the rector's cardinal criteria

	folder 1	folde 2	folde 3	folde 4
folde 1	0	-10	-3	13
folde 2	10	0	7	23
folde 3	3	-7	0	16
folde 4	-13	-23	-16	0

TABLE 10. Index of dominance of actions in the rector's ordinal criteria

	folder 1	folde 2	folde 3	folde 4
folder 1	0	4	3	7
folder 2	-4	0	3	3
folder 3	-3	-3	0	4
folder 4	-7	-3	-4	0

TABLE 11. **Index of dominance of actions in the President's cardinal criteria**

	folder 1	folde 2	folde 3	folde 4
folder 1	0	-18	-14	3
folder 2	18	0	4	21
folder 3	14	-4	0	17
folder 4	-3	-21	-17	0

TABLE 12. **Index of dominance of actions in the President's ordinal criteria**

	folder 1	folde 2	folde 3	folde 4
folder 1	0	-3	-1	-2
folder 2	3	0	-1	1
folder 3	1	1	0	2
folder 4	2	-1	-2	0

TABLE 13. **Differential of the dominance index in the founder's ordinal criteria**

	folder 1	folde 2	folde 3	folde 4
folder 1	0,5	0.6	0.6	0.6
folder 2	0.4	0.5	0.6	1
folder 3	0.4	0.4	0.5	0.7
folder 4	0.4	0	0.3	0.5

TABLE 14. **Differential of the dominance index in the ordinal criteria of the rector**

	folder 1	folde 2	folde 3	folde 4
folder 1	0,5	0.78	0.71	1
folder 2	0.21	0.5	0.71	0.71
folder 3	0.28	0.28	0.5	0.78
folder 4	0	0.28	0.21	0.5

TABLE 15. **Differential of the dominance index in the president's ordinal criteria**

	folder 1	folde 2	folde 3	folde 4
folder 1	0,5	0	0.33	0.16
folder 2	1	0.5	0.33	0.66
folder3	0.66	0.66	0.5	0.83
folder 4	0.83	0.33	0.16	0.5

TABLE 16. Differential of the dominance index in the founder's cardinal criteria

	folder 1	folde 2	folde 3	folde 4
folder 1	0.5	0.44	0.85	0.94
folder 2	0.55	0.5	0.91	1
folder 3	0.14	0.08	0.5	0.58
folder 4	0.05	0	0.41	0.5

TABLE 17. Differential of the dominance index in the rector's cardinal criteria

	folder 1	folde 2	folde 3	folde 4
folder 1	0.5	0.28	0.43	0.78
folder 2	0.71	0.5	0.65	1
folder 3	0.56	0.34	0.5	0.84
folder 4	0.21	0	0.15	0.5

TABLE 18. Differential of the dominance index in the President's cardinal criteria

	folder 1	folde 2	folde 3	folde 4
folder 1	0.5	0.07	0.16	0.57
folder 2	0.92	0.5	0.59	1
folder 3	0.83	0.40	0.5	0.90
folder 4	0.42	0	0.09	0.5

TABLE 19. (Differential of the global dominance index of actions in the cardinal criteria)

	d_1	d_2	d_3	$\alpha_{ij} = \sqrt{\prod_{l=1}^3 \delta_{ij}^l \times \left(1 - \prod_{l=1}^3 (1 - \delta_{ij}^l)\right)}$
α_{11}	0.5	0.5	0.5	0.5
α_{12}	0.44	0.28	0.07	0.03
α_{13}	0.85	0.43	0.16	0.14
α_{14}	0.94	0.78	0.57	0.57
α_{21}	0.55	0.71	0.92	0.55
α_{22}	0.5	0.5	0.5	0.5
α_{23}	0.91	0.65	0.59	0.59
α_{24}	1	1	1	1
α_{31}	0.14	0.56	0.83	0.14
α_{32}	0.08	0.34	0.40	0.05
α_{33}	0.5	0.5	0.5	0.5
α_{34}	0.58	0.84	0.90	0.58
α_{41}	0.05	0.21	0.42	0.02
α_{42}	0	0	0	0
α_{43}	0.41	0.15	0.09	0.02
α_{44}	0.5	0.5	0.5	0.5

TABLE 20. (Differential of the overall dominance index of actions in the ordinal criteria)

	d_1	d_2	d_3	$d_{ij} = \sqrt{\prod_{l=1}^3 d_{ij}^l \times \left(1 - \prod_{l=1}^3 (1 - d_{ij}^l)\right)}$
α_{11}	0.5	0.5	0.5	0.5
α_{12}	0.6	0.78	0.78	0.6
α_{13}	0.6	0.71	0.71	0.6
α_{14}	0.6	1	1	0.6
α_{21}	0.4	0.21	0.21	0.07
α_{22}	0.5	0.5	0.5	0.5
α_{23}	0.6	0.71	0.71	0.6
α_{24}	1	0.71	0.71	0.71
α_{31}	0.4	0.28	0.28	0.13
α_{32}	0.4	0.28	0.28	0.13
α_{33}	0.5	0.5	0.5	0.5
α_{34}	0.7	0.78	0.78	0.7
α_{41}	0.4	0	0	0
α_{42}	0	0.28	0.28	0
α_{43}	0.3	0.21	0.21	0.05
α_{44}	0.5	0.5	0.5	0.5

TABLE 21. Total dominance

D11	D12	D13	D14	D21	D22	D23	D24	D31	D32	D33	D34	D41	D42	D43	D44
10.14	6.44	7.54	11.88	6.39	10.14	10.12	17.39	2.79	1.83	10.14	13.07	0.21	0	0.80	10.14

TABLE 22. The overall score and ranking of actions

S1(folder 1)	S2(folder 2)	S3(folder 3)	S4(folder 4)
0.42	0.46	0.09	0.01
2 nd	1 st	3 rd	4 th

Solving using the MACEGOHZZ method gave us the following arrangement: S2>S1>S3>S4 so **folder 2** is the best choice.

4.2. **Example 2.** This example is taken from [9].

This problem concerns the choice of a better partner. To do this, we have:

- Five partners : Nippon Paint KK, Courtaoulds Coating Holding, Kansai Paint, International Pint, US Sec of navy ;
- Four criteria: C_1, C_2, C_3, C_4 ;
- three decision-makers: D1, D2, D3.

The decision-maker matrices are given below:

TABLE 23. Decision-maker's judgement matrix 1/D1

	quality(c_1)	Technology(c_2)	time(c_3)	Cost(c_4)
poids	3	4	3	5
Nippon Paint KK	6	8	9	4
Courtaoulds Coating Holding	4	5	6	7
Kansai Paint	7	6	8	4
International Pint	6	8	4	7
US Sec of navy	5	4	7	6

TABLE 24. Decision-maker's judgement matrix 2/D2

	quality(c_1)	Technology(c_2)	time(c_3)	Cost(c_4)
poids	4	3	2	5
Nippon Paint KK	7	5	3	8
Courtaoulds Coating Holding	3	6	8	4
Kansai Paint	6	8	4	3
International Pint	5	4	6	7
US Sec of navy	2	3	7	5

TABLE 25. Decision-maker's judgement matrix 2/D2

	quality(c_1)	Technology(c_2)	time(c_3)	Cost(c_4)
poids	4	5	3	5
Nippon Paint KK	8	3	6	7
Courtaoulds Coating Holding	6	5	7	3
Kansai Paint	5	8	4	2
International Pint	4	7	3	6
US Sec of navy	7	6	5	8

4.2.1. Resolution by the LON-ZO method. Using the LON-ZO method, we obtain this:

	$\sum_{j=1}^{j=5} w_j^1 g_j^1$	$\sum_{j=1}^{j=5} w_j^2 g_j^2$	$\sum_{j=1}^{j=5} w_j^3 g_j^3$	$U(a_i) = \frac{3}{\sum_{k=1}^3 \frac{1}{G_k(a_i)}}$
Nippon Paint KK	97	89	100	95.10
Courtaoulds Coating Holding	85	66	85	77.55
Kansai Paint	89	71	82	79.96
International Pint	97	79	90	88.03
US Sec of navy	82	56	113	77.11

	Overall scores	Rank
Nippon Paint KK	95.10	1 st
Courtaoulds Coating Holding	77.55	4 th
Kansai Paint	79.96	3 rd
International Pint	88.03	2 nd
US Sec of navy	77.11	5 th

This gives us the partner **Nippon Paint KK** as the best choice.

4.2.2. Resolution using the MACASP method. .

	$\sum_{k=1}^3 w_1^k g_1^k(a_i)$	$\sum_{k=1}^3 w_2^k g_2^k(a_i)$	$\sum_{k=1}^3 w_3^k g_3^k(a_i)$	$\sum_{k=1}^3 w_4^k g_4^k(a_i)$	$\sum_{k=1}^3 w_5^k g_5^k(a_i)$
Nippon Paint KK	78	62	51	95	286/3
Courtaoulds Coating Holding	48	63	55	70	236/3
Kansai Paint	65	88	44	45	242/3
International Pint	54	79	33	100	266/3
US Sec of navy	51	55	50	95	251/3

	Overall scores	Rank
Nippon Paint KK	286/3	1 st
Courtaoulds Coating Holding	236/3	5 th
Kansai Paint	242/3	4 th
International Pint	266/3	2 nd
US Sec of navy	251/3	3 rd

Using this method, we also obtain Nippon **Nippon Paint KK** as our best partner.

4.2.3. Resolution using the MACEGOHZZ. method In this example 2, the qualitative criteria are quality(c_1) and technology(c_2). The quantitative criteria are time(c_3) and cost(c_4).

TABLE 26. matrix of the global weights of the criteria

	d_1	d_2	d_3	$P_j = \sqrt{\frac{\sum_{k=1}^3 (p_j^k)^4}{\sum_{k=1}^3 (p_j^k)^2}}$
c_1	3	4	4	3.80
c_2	4	3	5	4.38
c_3	3	2	3	2.84
c_4	5	5	5	5

TABLE 27. Index of dominance of actions in the D1's cardinal criteria

	a_1	a_2	a_3	a_4	a_5
Nippon Paint KK (a_1)	0	-6	3	0	-4
Courtaoulds Coating Holding (a_2)	6	0	9	6	2
Kansai Paint(a_3)	-3	-9	0	-3	-7
International Pint(a_4)	0	-6	3	0	-4
US Sec of navy(a_5)	4	-2	7	4	0

TABLE 28. Index of dominance of actions in the D1's ordinal criteria

	a_1	a_2	a_3	a_4	a_5
a_1	0	7	1	0	7
a_2	-7	0	-7	-7	1
a_3	-1	7	0	-1	7
a_4	0	7	1	0	7
a_5	-7	-1	-7	-7	0

TABLE 29. Index of dominance of actions in the D2's cardinal criteria

	a_1	a_2	a_3	a_4	a_5
a_1	0	10	23	-1	7
a_2	-10	0	13	-11	-3
a_3	-23	-13	0	-24	-16
a_4	1	11	24	0	8
a_5	-7	3	16	-8	0

TABLE 30. Index of dominance of actions in the D2's ordinal criteria

	a_1	a_2	a_3	a_4	a_5
a_1	0	1	1	7	7
a_2	-1	0	-7	-1	7
a_3	-1	7	0	-1	7
a_4	-7	1	-7	0	7
a_5	-7	-7	-7	-7	0

TABLE 31. Index of dominance of actions in the cardinal criteria of D3

	a_1	a_2	a_3	a_4	a_5
a_1	0	17	31	14	-2
a_2	-17	0	14	-3	-19
a_3	-31	-14	0	-17	-33
a_4	-14	3	17	0	-16
a_5	2	19	33	16	0

TABLE 32. Index of dominance of actions in the ordinal criteria in D3

	a_1	a_2	a_3	a_4	a_5
a_1	0	-1	-1	-1	-1
a_2	1	0	-1	-1	-9
a_3	1	1	0	9	1
a_4	1	1	-9	0	1
a_5	1	9	-1	-1	0

TABLE 33. Index of differential dominance of actions in the ordinal criteria of D1

	a_1	a_2	a_3	a_4	a_5
a_1	0.5	1	0.57	0.5	1
a_2	0	0.5	0	0	0.57
a_3	0.42	1	0.5	0.42	1
a_4	0.5	1	0.57	0.5	1
a_5	0	0.42	0	0	0.5

TABLE 34. Index of differential dominance of actions in the ordinal criteria of D2

	a_1	a_2	a_3	a_4	a_5
a_1	0.5	0.57	0.57	1	1
a_2	0.42	0.5	0	0.42	1
a_3	0.42	1	0.5	1	1
a_4	0	0.57	0	0.5	1
a_5	0	0	0	0	0.5

TABLE 35. Index of differential dominance of actions in the ordinal criteria of D3

	a_1	a_2	a_3	a_4	a_5
a_1	0.5	0.44	0.44	0.44	0.44
a_2	0.55	0.5	0.44	0.44	0
a_3	0.55	0.55	0.5	1	0.55
a_4	0.55	0.55	0	0.5	0.55
a_5	0.55	1	0.44	0.44	0.5

TABLE 36. Index of differential dominance of actions in the cardinal criteria of D1

	a_1	a_2	a_3	a_4	a_5
a_1	0.5	0.16	0.66	0.5	0.27
a_2	0.83	0.5	1	0.83	0.61
a_3	0.33	0	0.5	0.33	0.11
a_4	0.5	0.16	0.66	0.5	0.27
a_5	0.72	0.38	0.88	0.72	0.5

TABLE 37. Index of differential dominance of actions in the cardinal criteria of D2

	a_1	a_2	a_3	a_4	a_5
a_1	0.5	0.70	0.97	0.47	0.64
a_2	0.29	0.5	0.77	0.27	0.43
a_3	0.02	0.22	0.5	0	0.16
a_4	0.52	0.72	1	0.5	0.66
a_5	0.35	0.56	0.83	0.33	0.5

TABLE 38. Index of differential dominance of actions in the cardinal criteria of D3

	a_1	a_2	a_3	a_4	a_5
a_1	0.5	0.75	0.96	0.71	0.46
a_2	0.24	0.5	0.71	0.45	0.21
a_3	0.03	0.28	0.75	0.24	0
a_4	0.28	0.54	0.75	0.5	0.25
a_5	0.53	0.78	1	0.74	0.5

TABLE 39. Index of overall differential dominance of cardinal criteria

	d_1	d_2	d_3	$\alpha_{ij} = \sqrt{\prod_{l=1}^3 \delta_{ij}^l \times \left(1 - \prod_{l=1}^3 (1 - \delta_{ij}^l)\right)}$
α_{11}	0.5	0.5	0.5	0.33
α_{12}	0.16	0.70	0.75	0.29
α_{13}	0.66	0.97	0.96	0.79
α_{14}	0.5	0.47	0.71	0.39
α_{15}	0.27	0.64	0.46	0.26
α_{21}	0.83	0.29	0.24	0.23
α_{22}	0.5	0.5	0.5	0.33
α_{23}	1	0.77	0.71	0.74
α_{24}	0.83	0.27	0.45	0.30
α_{25}	0.61	0.43	0.21	0.21
α_{31}	0.33	0.02	0.03	0.008
α_{32}	0	0.22	0.28	0
α_{33}	0.5	0.5	0.5	0.33
α_{34}	0.33	0	0.24	0
α_{35}	0.11	0.16	0	0
α_{41}	0.5	0.52	0.28	0.24
α_{42}	0.16	0.72	0.54	0.24
α_{43}	0.66	1	0.75	0.71
α_{44}	0.5	0.5	0.5	0.33
α_{45}	0.27	0.66	0.55	0.30
α_{51}	0.72	0.35	0.53	0.35
α_{52}	0.38	0.56	0.78	0.40
α_{53}	0.88	0.83	1	0.86
α_{54}	0.72	0.33	0.74	0.41
α_{55}	0.5	0.5	0.5	0.33

TABLE 40. Index of overall differential dominance of ordinal criteria

	d_1	d_2	d_3	$d_{ij} = \sqrt{\prod_{l=1}^3 d_{ij}^l} \times \left(1 - \prod_{l=1}^3 (1 - d_{ij}^l)\right)$
α_{11}	0.5	0.5	0.5	0.33
α_{12}	1	0.57	0.44	0.50
α_{13}	0.57	0.57	0.44	0.36
α_{14}	0.5	1	0.44	0.47
α_{15}	1	1	0.44	0.66
α_{21}	0	0.42	0.55	0
α_{22}	0.5	0.5	0.5	0.33
α_{23}	0	0	0.44	0
α_{24}	0	0.42	0.44	0
α_{25}	0.57	1	0	0
α_{31}	0.42	0.42	0.55	0.29
α_{32}	1	1	0.55	0.74
α_{33}	0.5	0.5	0.5	0.33
α_{34}	0.42	1	1	0.65
α_{35}	1	1	0.55	0.74
α_{41}	0.5	0	0.55	0
α_{42}	1	0.57	0.55	0.56
α_{43}	0.57	0	0	0
α_{44}	0.5	0.5	0.5	0.33
α_{45}	1	1	0.55	0.74
α_{51}	0	0	0.55	0
α_{52}	0.42	0	1	0
α_{53}	0	0	0.44	0
α_{54}	0	0	0.44	0
α_{55}	0.5	0.5	0.5	0.33

TABLE 41. Total dominance

D11	D12	D13	D14	D15	D21	D22	D23	D24	D25	D31	D32
5.30	6.40	9.19	6.97	7.57	1.81	5.30	5.81	2.42	1.69	2.48	6.10

D33	D34	D35	D41	D42	D43	D44	D45	D51	D52	D53	D54	D55
5.30	5.36	6.10	1.95	6.52	5.57	5.30	8.48	2.76	3.16	6.75	3.23	5.30

TABLE 42. Overall score and ranking of actions

S1	S2	S3	S4	S5
0.45	0.09	0.12	0.36	0.13
1 st	5 th	4 th	2 nd	3 rd

With $S1=S(\text{Nippon Paint KK})$; $S1=S(\text{Courtaoulds Coating Holding})$; $S3(\text{Kansai Paint})$; $S4=S(\text{International Pint})$; $S5=S(\text{US Sec of navy})$.

Using the MACEGOHZZ method, we obtain the following order: $S1>S4>S5>S3>S2$ so the best action is the partner **Nippon Paint KK**.

4.3. Comparison.

TABLE 43. Example 1

	LON-ZO	MACASP	MACEGOHZZ
folder 1	3 rd	2 nd	2 nd
folder 2	1 st	1 st	1 st
folder 3	2 nd	3 rd	3 rd
folder 4	4 th	4 th	4 th

TABLE 44. Example 2

	LON-ZO	MACASP	MACEGOHZZ
Nippon Paint KK	1 st	1 st	1 st
Courtaoulds Coatind Holding	4 th	5 th	5 th
Kansai Paint	3 rd	4 th	4 th
International Pint	2 nd	2 nd	2 nd
US Sec of navy	5 th	3 rd	3 rd

4.3.1. *Theoretical comparison.* From these two examples, we can see that the LON-ZO, MACASP and MACEGOHZZ methods all identified the same action as the best choice: **Lefolder 2** and **Nippon Paint KK** in example 1 and example 2 respectively. In addition, the MACASP and MACEGOHZZ methods provided a common ranking of actions and disagreed with the rest of the action ranking provided by the LON-ZO method. Some collective aggregation methods such as EMETOP, MACUQ and the extension of the ELECTRE II method developed respectively in [13], [7] and [16] were used to solve these problems and also gave the same result as the MACEGOHZZ method.

To further justify the scientific validity of our new method, we also compared its results with those obtained using the MACBEV method (Collective Aggregation Model based on the Hybridization of the EVAMIX Method and the VMAVA+). The MACBEV method is a very recent group decision-making method specializing in solving mixed-data decision-making problems. We observed that it provides the same rankings as the MACBEV method [19]. It surpasses the MACBEV method in its simplicity and the polynomial nature of its worst-case time complexity. It is easily extensible to fuzzy sets and will contribute to efficiently solving these types of problems in [20].

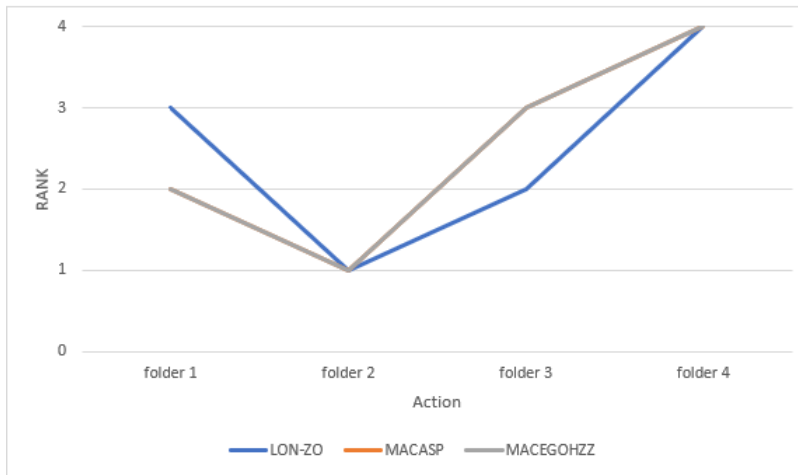


FIGURE 1. Graphical comparison of methods in example 1

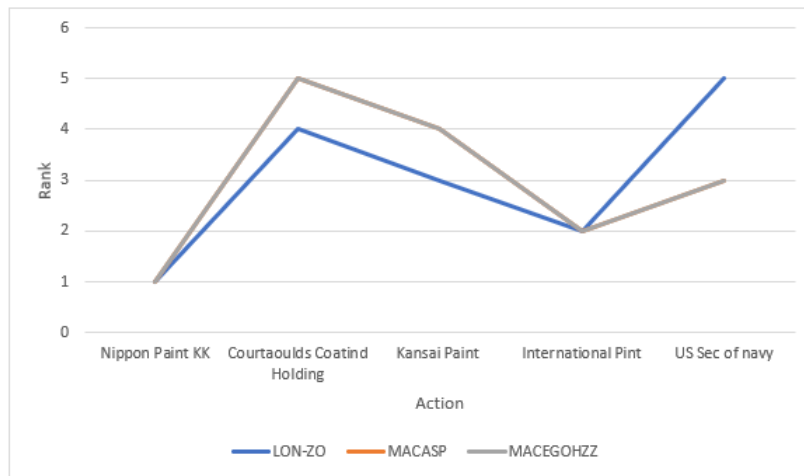


FIGURE 2. Graphical comparison of methods in example 2

4.3.2. *Graphical comparison.* Figure 1 and 2 clearly show the different coincidences of results provided by the three methods, so the new method could be used in spite of these two methods to solve certain selection problems especially those involving mixed data (ordinal and cardinal).

4.4. **Sensitivity analysis.** In this section, we present a sensitivity analysis of the method by varying the weightings of the criteria and strictly following the steps of the MACEGOHZZ method. The results are presented in the tables below:

TABLE 45. first variation in the weights of the criteria

	c_1	c_2	c_3	c_4
d_1	6	5	4	7
d_2	4	3	7	5
d_3	1	5	2	4
Rank $a_2 > a_1 > a_3 > a_4$				

TABLE 46. Second variation in the weights of the criteria

	c_1	c_2	c_3	c_4
d_1	7	8	5	6
d_2	6	5	8	6
d_3	5	3	4	6
Rank $a_1 > a_2 > a_3 > a_4$				

TABLE 47. third variation in the weights of the criteria

	c_1	c_2	c_3	c_4
d_1	8	1	2	6
d_2	8	2	1	7
d_3	5	4	6	9
Rank $a_1 > a_3 > a_2 > a_4$				

TABLE 48. Fourth variation in the weights of the criteria

	c_1	c_2	c_3	c_4
d_1	7	6	5	4
d_2	8	7	6	5
d_3	5	4	3	2
Rank $a_2 > a_1 > a_3 > a_4$				

TABLE 49. Fifth variation in the weights of the criteria

	c_1	c_2	c_3	c_4
d_1	8	7	1	9
d_2	1	2	7	8
d_3	7	6	8	2
Rank $a_2 > a_1 > a_3 > a_4$				

The study shows that stock a_2 is almost robust in most cases. In other cases, there is slight variation in stock rankings. This demonstrates that the weighting of criteria influences the final decision. The method can be stable with respect to some criteria and also exhibit slight variation with respect to others. Consequently, the MACEGOHZZ method is not entirely stable.

4.5. Advantage. The MACEGOHZZ method presents major innovations. It allows for the efficient resolution of mixed-data decision problems, something many group decision methods cannot achieve. Furthermore, preserving ordinal evaluations without requiring conversion to numerical values during aggregation leads to fairer and more consensual results. Indeed, converting ordinal information to numerical values can result in information loss, often leading to unexpected outcomes. Finally, in addition to its quadratic time complexity in the worst case and its ability to account for the optimization direction of the criteria, its stability analysis has shown a relative stability in the rankings of different actions when the weightings vary. This is a significant advantage of the method.

4.6. limits. The MACEGOHZZ method, like all other methods, has limitations. It cannot solve decision-making problems involving weights and interval-based data, thus limiting its scope. These types of problems are as follows:

TABLE 50. problem with weights and data in interval form

	c_1	c_2	...	c_n
weight	$[p_1^l, p_1^u]$	$[p_2^l, p_2^u]$...	$[p_n^l, p_n^u]$
a_1	$[g_{11}^l, g_{11}^u]$	$[g_{12}^l, g_{12}^u]$...	$[g_{1n}^l, g_{1n}^u]$
a_2	$[g_{21}^l, g_{21}^u]$	$[g_{22}^l, g_{22}^u]$...	$[g_{2n}^l, g_{2n}^u]$
....
a_m	$[g_{m1}^l, g_{m1}^u]$	$[g_{m2}^l, g_{m2}^u]$...	$[g_{mn}^l, g_{mn}^u]$

5. CONCLUSION

In view of the previous results provided by the LON-ZO, MACASP and MACEGOHZZ collective aggregation methods based on EVAMIX, the Gini mean and the Hybrid operator of Zimmermann and Zysno, we can say that the MACEGOHZZ method can be used to solve group decision problems. The major advantage of the MACEGOHZZ method over previous method is that, it can solve mixed-data problems without converting ordinal evaluations into numerical values. Despite these satisfactory results, we have to recognise that the method has shortcomings, which is why our future research will focus mainly on studying the robustness of the results of the MACEGOHZZ method, then extend its application to problems with data and weights entirely with intervals.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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