Int. J. Anal. Appl. (2026), 24:21

International Journal of Analysis and Applications

New Categories of Generalized Fixed Point Theorems in New Generalization of Complete Metric Spaces with an Application

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Abstract. The major objective of this manuscript is to present another novel extension of metric spaces namely; mapping weighted $\mathfrak{D}$-complete metric space. In the process, the uniqueness of various strictures of common and common coupled fixed point theorems have been investigated and verified for two pairs of self commutative maps under influence other improved types of extended contractive conditions in these spaces. On the other hand, a novel extended notion of Hausdorff distance namely; Hausdorff ∇^*-distance has been defined in these spaces. Our second main results of various novel types of improved coincidence fixed point theorems have been obtained by applying the conception of generalized Hausdorff ∇^*-distance. Additionally, various practical implementations of the existence fixed point for two pairs of self commutative maps as a solution for certain non-linear Volterra integral equations have been presented and investigated in the framework of map weighted $\mathfrak{D}$-complete metric spaces.

1. Introduction

The uniqueness of coincidence and common coupled fixed points for hybrid maps which satisfactory various certain contractive conditions in context of extended metric spaces is essential outcome in fixed point theory which has been extended and improved in numerous different directions in the literature, additionally coincidence and common coupled fixed point theorems for single and multi-valued maps have acquired formidable implementations in several sciences such as differential equation, optimization, control theory, approximation theory and discrete dynamics. In 1978 J. Dugundji and A. Granas [1] verified a uniqueness fixed point for a single-valued weakly contractive map in complete metric spaces. There were many

Received Nov. 13, 2025

2020 Mathematics Subject Classification. Primary 47H10, 54H20; Secondary 54H25.

Key words and phrases. $\mathcal{D}^*$-metric space; map weighted $\mathfrak{D}$-metric space; Hausdorff ∇^*-distance; coincidence fixed points; common coupled fixed point.

https://doi.org/10.28924/2291-8639-24-2026-21

ISSN: 2291-8639

authors presented various extensions of metric spaces such as (D-metric spaces) via B. C. Dhage [2, 3]. Afterward Z. Mustafa and B. Sims [4] detected that most of the constructions relating to essential topological properties of D-metric spaces are incorrect. Then, they presented an extenuation of metric spaces; called (G-metric spaces). Consequently, D. El-Moutawakil [5] gave a novel extension of renowned multi-valued contraction fixed point in the setting of symmetric spaces. After that, the notions of a coupled fixed point and mixed monotone maps have been presented via T. Bhaskar and V. Lakshmikantham [6], and obtained several coupled fixed point outcomes in partially ordered metric space. Different extensions of metric spaces have been offered via numerous authors in literature. In particular one of these spaces is $\mathcal{D}^*$-metric spaces which has been presented via S. Shaban, et al. [7] which as a one of probable modification of $\mathcal{D}$-metric spaces. Later, M. Abbas, et al. [8] Verified various fixed point theorems for multi-valued maps under extend contractive conditions in ordered extended metric spaces. Afterward, K. S. Eke and J. O. Olaleru, [9] expand the idea of symmetric spaces to G-symmetric, as well the existence of common fixed points for singled-valued occasionally faintly compatible maps has been verified in G-symmetric spaces. Motivated by this fact, they are [10] verified the existence of common fixed points for pair of hybrid contractive maps in G-symmetric-spaces. In addition, numerous outcomes related to common coupled and coupled coincidence fixed points have been manifested in the literature [11, 12]. In 2017, Z. Mustafa, et al [13] described novel notions of multi-valued contraction maps and established various coincidence and common fixed points outcomes in complete G-spaces. On the other hand the fixed point theory is an extremely active branch of pure and applied mathematics and is at the essence of nonlinear analysis which has experienced numerous implementations in the last century. One of these implementations is the introduction of various extended metric spaces and is the evidence of fixed point outcomes in these spaces along with its implementations in other sconces. One of these spaces is function weighted metric spaces which was described and presented via M. Jleli and B. Samet [14]. After that, E. L. Ghasab and H. Majani [15] presented a novel extended space as an extension of function weighted metric spaces which was presented via [14], as well they were defined the idea of Hausdorff Δ-distance in these spaces. Newly, N. A. Majid, et al. [16] introduced and verified various novel fixed point outcomes for monotone multi-valued maps in partially ordered complete $\mathcal{D}^*$-metric spaces, as well various existence and uniqueness of coupled fixed point outcomes of maps gratifying contractive condition have been investigated. Afterward, A. H. Abed and A. M. F. Al.Jumaili [17] described a novel category of extended metric spaces namely, $\mathcal{D}^*_d$-symmetric space and established numerous common fixed point outcomes for maps gratifying extended contractive conditions in $\mathcal{D}^*_d$-symmetric space. Also, in recent times, Oklah and A. Al.Jumaili [18] offered and investigated various practical implementations for pairs of self maps which are satisfactory extended contractive conditions of integral type in $\mathcal{D}^*$-metric spaces and obtained various common and coupled fixed point outcomes in such spaces. For future studies we could expand and generalize our outcomes to other spaces such as [19-21].

The main intent of this manuscript is to present a novel idea of expansion metric spaces namely; map weighted ∇^*-complete metric space. In the process, the uniqueness of various extended categories of common and common coupled fixed point theorems have been discussed and verified for two pairs of self commutative maps under influence improved types of extended contractive conditions in these spaces. Additionally, a novel extended concept of Hausdorff distance called; Hausdorff ∇^*-distance has been described, and various novel kinds of improved coincidence fixed point theorems have been obtained through utilizing the conception of Hausdorff ∇^*-distance. On the other hand, various practical implementations of the existence fixed point for two pairs of self commutative maps as a solution for certain non-linear Volterra integral equations have been presented and investigated in the structure of map weighted $\mathfrak{D}$-complete metric spaces. Moreover, some suitable appropriate examples that support our major outcomes have been equipped. Our suggested results are developing of various recognized analogous outcomes related to these categories of fixed point theorems in the context of extended complete metric spaces in the literature.

2. Materials and Methods

This segment, devoted to recall various conceptions and motivations which needed in the sequel and that will help us in the outcomes that follow and play indispensable role in this study for verifying our major outcomes.

Definition 2.1: [7] Suppose that $\mathcal{D}^*: \mathcal{X} \times \mathcal{X} \times \mathcal{X} \longrightarrow \mathbb{R}^+$, is a map described on $\mathcal{X} \neq \emptyset$ and gratifying the next conditions $\forall\, \varkappa^*, y^*, z^*, w^* \in \mathcal{X}$:

$(\mathcal{D}^*_1)\mathcal{D}^*(\varkappa^*, y^*, z^*) \geq 0, \forall\, \varkappa^*, y^*, z^* \in \mathcal{X}$;

$(\mathcal{D}^*_2)\mathcal{D}^*(\varkappa^*, y^*, z^*) = 0$ *iff* $\varkappa^* = y^* = z^*$;

$(\mathcal{D}^*_3)\mathcal{D}^*(\varkappa^*, y^*, z^*) = \mathcal{D}^*(\mathcal{P}\{\varkappa^*, y^*, z^*\})$,(symmetry) (s. t) $\mathcal{P}$ is a permutation map,

$(\mathcal{D}^*_4)\mathcal{D}^*(\varkappa^*, y^*, z^*) \leq \mathcal{D}^*(\varkappa^*, y^*, w^*) + \mathcal{D}^*(w^*, z^*, z^*)$.

In this case, the map $\mathcal{D}^*$ is called $\mathcal{D}^*$-metric and $(\mathcal{X}, \mathcal{D}^*)$ is called a $\mathcal{D}^*$-metric space.

Definition 2.2: [7] A $\mathcal{D}^*$-metric space $(\mathcal{X}, \mathcal{D}^*)$is said to be a symmetric if $\mathcal{D}^*(\varkappa^*, \varkappa^*, y^*) = \mathcal{D}^*(\varkappa^*, y^*, y^*), \forall\, \varkappa^*, y^* \in \mathcal{X}$, and is called non-symmetric if it's not symmetric.

Definition 2.3: [6] An element $(\varkappa^*, y^*) \in \mathcal{X} \times \mathcal{X}$, when $\mathcal{X} \neq \emptyset$ is said to be a coupled fixed point of a map $\mathcal{F}: \mathcal{X} \times \mathcal{X} \longrightarrow \mathcal{X}$ if $\mathcal{F}(\varkappa^*, y^*) = \varkappa^*$ with $\mathcal{F}(y^*, \varkappa^*) = y^*$.

Definition 2.4: [22] Assume that $\mathcal{F}, \mathcal{G}: (\mathcal{X}, \mathcal{D}^*) \longrightarrow (\mathcal{X}, \mathcal{D}^*)$ are single-valued self-maps on $\mathcal{X}$. If $\mathcal{F}(\varkappa^*) = \mathcal{G}(\varkappa^*) = \varkappa^*$ for some $\varkappa^* \in \mathcal{X}$, in this case $\varkappa^*$ is called a common fixed point (Concisely, C.F.P) of $\mathcal{F}$ & $\mathcal{G}$.

Definition 2.5: [23] Suppose $\mathcal{F}, \mathcal{G}: (\mathcal{X}, \mathcal{D}^*) \longrightarrow (\mathcal{X}, \mathcal{D}^*)$ are self-maps on $\mathcal{X} \neq \emptyset$. If $\mathcal{F}(\varkappa^*) = \mathcal{G}(\varkappa^*) = \mathcal{w}^*$ for some $\varkappa^* \in \mathcal{X}$, in this case $\varkappa^*$ is called a coincidence point of $\mathcal{F}\& G$, and $\mathcal{w}^*$ is called a point of coincidence of $\mathcal{F}$ & G.

Definition 2.6: [24] An element $(\varkappa^*, \mathcal{y}^*) \in \mathcal{X} \times \mathcal{X}$ is called a common coupled fixed point (Concisely, C.C.F.P) of $\mathcal{F}: \mathcal{X}^2 \to \mathcal{X}$ & $G: X \to X$. *If* $\mathcal{F}(\varkappa^*, \mathcal{y}^*) = \mathcal{G}\varkappa^* = \varkappa^* \& \mathcal{F}(\mathcal{y}^*, \varkappa^*) = \mathcal{G}\mathcal{y}^* = \mathcal{y}^*$.

Definition2.7: [24] An element $(\varkappa^*, \mathcal{y}^*) \in \mathcal{X} \times \mathcal{X}$ is said to be a coupled coincidence point of a maps $\mathcal{F}: \mathcal{X}^2 \longrightarrow \mathcal{X}$ and $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{X}$ if $\mathcal{F}(\varkappa^*, \mathcal{y}^*) = \mathcal{G}\varkappa^*$ with $\mathcal{F}(\mathcal{y}^*, \varkappa^*) = \mathcal{G}\mathcal{y}^*$.

Definition2.8: [24] Assume that$\mathcal{X} \neq \emptyset$. We say that the maps $\mathcal{F}: \mathcal{X} \times \mathcal{X} \longrightarrow \mathcal{X}$ and $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{X}$ are commutative if $\mathcal{G}(\mathcal{F}(\varkappa^*, \mathcal{y}^*)) = \mathcal{F}(\mathcal{G}\varkappa^*, \mathcal{G}\mathcal{y}^*)\ \forall\, \varkappa^*, \mathcal{y}^* \in \mathcal{X}$.

Definition 2.9: [25, 14] A mapping $\mathcal{G}: (0, +\infty) \longrightarrow \mathbb{R}$ is said to be:

(i) Logarithmic-like if $\forall\, \{\varkappa_s^*\} \subset (0, +\infty)$ satisfies $\lim_{s\to\infty} \varkappa_s^* = 0$ iff $\lim_{s\to\infty} \mathcal{G}(\varkappa_s^*) = -\infty$.

(ii) Nondecreasing map if $\forall\, \mathcal{p}, \mathcal{q} \in (0, +\infty)$ (s. t), $0 < \mathcal{p} \leq \mathcal{q}$ we have $\mathcal{G}(\mathcal{p}) \leq \mathrm{g}(\mathcal{q})$.

Remark 2.10: We will indicate via Γ for the collection of each logarithmic-like and nondecreasing maps which gratifying the conditions in Definition-2-9.

Definition 2.11: [14] Assume that $\mu: \mathcal{X} \times \mathcal{X} \longrightarrow [0, +\infty)$ is a map gratifying the next conditions and there exists a map $\mathcal{G} \in \Gamma$ and constant $\mathcal{K} \in [0, +\infty)$:

$(\boldsymbol{\mu_1})$ $\mu(\varkappa^*, \mathcal{y}^*) = 0 \Leftrightarrow \varkappa^* = \mathcal{y}^*, \forall\, \varkappa^*, \mathcal{y}^* \in \mathcal{X}$;

$(\boldsymbol{\mu_2})$ $\mu(\varkappa^*, \mathcal{y}^*) = \mu(\mathcal{y}^*, \varkappa^*), \forall\, \varkappa^*, \mathcal{y}^* \in \mathcal{X}$;

$(\boldsymbol{\mu_3})$ For each $(\varkappa^*, \mathcal{y}^*) \in \mathcal{X}^2$, as well $\forall\, e \in \mathbb{N}$ and $e \geq 2$, obtain

$\mu(\varkappa^*, \mathcal{y}^*) > 0 \Longrightarrow \mathcal{G}\big(\mu(\varkappa^*, \mathcal{y}^*)\big) \leq \mathcal{G}(\sum_{i=1}^{e-1} \mu(\mathcal{q}_i, \mathcal{q}_{i+1})) + \mathcal{K}$.

For all $(\mathcal{q}_i)_{i=1}^{e} \subset \mathcal{X}$ with $(\mathcal{q}_1, \mathcal{q}_e) = (\varkappa^*, \mathcal{y}^*)$.

In that case, the map μ is called a map weighted metric or an Γ-metric on $\mathcal{X}$, and $(\mathcal{X}, \mu)$ is called a map weighted metric or Γ-metric space.

3. Some Fixed Points Results in Mapping Weighted 𝔇-Complete Metric Spaces

Our incentive of present this segment is to define a novel idea of expansion metric spaces namely; map weighted 𝔇-complete metric spaces. Various extended categories of common and common coupled fixed point theorems have been offered and verified for two pairs of self commutative maps and present the definition of convergent in these spaces.

In the beginning we present the following vital definition.

Definition 3.1: Let $\nabla^*: \mathcal{X}^3 \longrightarrow [0, +\infty)$ be given map. Presume that $\exists\, \mathcal{F} \in \Gamma$ with a constant $\mathcal{K} \in [0, +\infty)$ (s. t)

$(\boldsymbol{\nabla_1^*})$ $\nabla^*(\varkappa^*, \varkappa^*, \mathcal{y}^*) > 0, \forall\, \varkappa^*, \mathcal{y}^* \in \mathcal{X}$ with $\varkappa^* \neq \mathcal{y}^*$;

$(\boldsymbol{\nabla_2^*})$ $\nabla^*(\varkappa^*, \mathcal{y}^*, z^*) = 0$ *iff* $\varkappa^* = \mathcal{y}^* = z^*, \forall \varkappa^*, \mathcal{y}^*, z^* \in \mathcal{X}$;

$(\boldsymbol{\nabla_3^*})$ $\nabla^*(\varkappa^*, \mathcal{y}^*, z^*) = \nabla^*(\mathcal{y}^*, \varkappa^*, z^*) = \cdots \forall\, \varkappa^*, \mathcal{y}^*, z^* \in \mathcal{X}$;

$(\boldsymbol{\nabla_4^*})$ $\forall\, (\varkappa^*, \mathcal{y}^*, z^*) \in \mathcal{X}^3$, and $\forall\, e \in \mathbb{N}$ with $e \geq 2$,we have

$$\begin{Bmatrix} \nabla^*(\varkappa^*,\mathcal{y}^*,z^*) > 0 \Longrightarrow \mathcal{F}\big(\nabla^*(\varkappa^*,\mathcal{y}^*,z^*)\big) \leq \\ \mathcal{F}(\sum_{i=3}^{e-1} \nabla^*(\wp_i,\wp_{i+1},\wp_{i-1}) + \nabla^*(\wp_{i-1},\wp_i,\wp_i)) + \mathcal{K} \end{Bmatrix}$$

$\forall\ (\wp_i)_{i=1}^{e} \subset \mathcal{X}$ with $(\wp_1,\wp_{e-1},\wp_e) = (\varkappa^*,\mathcal{y}^*,z^*)$.

In this case, the map ∇^* is called a map weighted extended metric or $\Gamma\mathfrak{D}$ metric on $\mathcal{X}$, and $(\mathcal{X},\nabla^*)$ is called a map weighted extended metric or $\Gamma\mathfrak{D}$-metric space.

Next, present the next example which is extending to that of [15], in $\Gamma\mathfrak{D}$-metric spaces which demonstrates the validity of Definition(3.1).

Example 3.2: Assume that $\mathcal{X} = \mathbb{R}$ and $\mathcal{F}(t) = \mathrm{Ln}(t)$ is a nondecreasing map. In this case, for each $\varkappa^* \neq \mathcal{y}^* \neq z^* \in \mathcal{X}$, & $\forall\ e \in \mathbb{N}$ with $e \geq 3$, with each $(\wp_i)_{i=1}^{e} \subset \mathcal{X}$ with $(\wp_1,\wp_{e-1},\wp_e) = (\varkappa^*,\mathcal{y}^*,z^*)$, and each ∇^*-metric on $\mathcal{X}$, obtain

$$\begin{Bmatrix} \nabla^*(\varkappa^*,\mathcal{y}^*,z^*) > 0 \Longrightarrow \mathrm{Ln}\big(\nabla^*(\varkappa^*,\mathcal{y}^*,z^*)\big) \leq \\ \mathrm{Ln}(\sum_{i=3}^{e-1} \nabla^*(\wp_i,\wp_{i+1},\wp_{i-1}) + \nabla^*(\wp_{i-1},\wp_i,\wp_i)). \end{Bmatrix}$$

Because, $\nabla^*(\varkappa^*,\mathcal{y}^*,z^*) \leq \sum_{i=3}^{e-1} \nabla^*(\wp_i,\wp_{i+1},\wp_{i-1}) + \nabla^*(\wp_{i-1},\wp_i,\wp_i))$. Now here, put $\mathcal{K} = 0$. therefore, $(\mathcal{X},\nabla^*)$ is $\Gamma\mathfrak{D}$-metric spac

Definition 3.3: Suppose $(\mathcal{X},\nabla^*)$ is $\Gamma\mathfrak{D}$ -metric space. A subset $\mathcal{M} \subseteq \mathcal{X}$ is called open if for each $\varkappa^* \in \mathcal{M}, \exists$ some $m > 0$ (s. t), $\mathcal{B}(\varkappa^*,m) \subset \mathcal{M}$, where

$$\mathcal{B}(\varkappa^*,m) = \{\mathcal{y}^* \in \mathcal{X} : \nabla^*(\varkappa^*,\varkappa^*,\mathcal{y}^*) < m\}.$$

We declare that $\mathcal{C} \subseteq \mathcal{X}$ closed if $\mathcal{X} - \mathcal{C}$ is open.

Currently, present the definitions of convergence, Cauchy and completeness of an $\Gamma\mathfrak{D}$-metric space.

Definition3.4: Assume that $(\mathcal{X},\nabla^*)$ is ∇^* -metric space, so:

(i) $\{\varkappa_s^*\}$ in $\mathcal{X}$ is called converges to $\varkappa^* \in \mathcal{X}$ *iff* $\nabla^*(\varkappa_s^*,\varkappa_s^*,\varkappa^*) = \nabla^*(\varkappa^*,\varkappa^*,\varkappa_s^*) \to 0$, as $s \to \infty$. i.e. $\forall\ \varepsilon > 0, \exists\ e \in \mathbb{Z}^+$(s. t),$\forall\ s \geq e \Rightarrow \nabla^*(\varkappa^*,\varkappa^*,\varkappa_s^*) < \varepsilon$. That is, equivalent with,$\forall\ \varepsilon > 0,, \exists\ e \in \mathbb{Z}^+$(s. t), $\forall\ s, r \geq e \Longrightarrow \nabla^*(\varkappa^*,\varkappa_s^*,\varkappa_r^*) < \varepsilon$.

(ii) $\{\varkappa_s^*\}$ is called $\mathcal{D}^*$-Cauchy *if* $\forall\ \varepsilon > 0, \exists\ e \in \mathbb{Z}^+$ (s. t), $\forall\ \nabla^*(\varkappa_s^*,\varkappa_s^*,\varkappa_r^*) < \varepsilon$. That is, *if* $\nabla^*(\varkappa_s^*,\varkappa_s^*,\varkappa_r^*) \to 0$ as , $s, r \to \infty$.

(iii) A space$(\mathcal{X},\nabla^*)$ is called ∇^*-complete metric space, *if* each ∇^*-Cauchy sequence in $(\mathcal{X},\nabla^*)$ is convergent.

Theorem 3.5: Let $(\mathcal{X},\nabla^*)$ be a map weighted $\mathfrak{D}$-complete metric space. As well, $S, \mathfrak{F}: \mathcal{X} \longrightarrow \mathcal{X}$ be any two commutative maps (s. t),$\mathfrak{F}(\mathcal{X}) \subset S(\mathcal{X})$,with $S(\mathcal{X})$ is closed. Assume $\exists\ \hbar \in (0,1)$ (s. t)

$$\nabla^*(\mathfrak{F}\varkappa^*,\mathfrak{F}\mathcal{y}^*,\mathfrak{F}z^*) \leq \hbar\nabla^*(S\varkappa^*,S\mathcal{y}^*,Sz^*) \tag{3.1}$$

$\forall\ \varkappa^*,\mathcal{y}^*,z^* \in \mathcal{X}$. In this case $\mathfrak{F}$ & S have a unique (C.F.P) in $\mathcal{X}$.

Proof: Because $\mathfrak{F}(\mathcal{X}) \subset S(\mathcal{X})$,we can select $\varkappa_1^* \in \mathcal{X}$ (s. t), $\mathfrak{F}\varkappa_0^* = S\varkappa_1^*$, for $\varkappa_0^* \in \mathcal{X}$.We create $\{\varkappa_s^*\}$ in $\mathcal{X}$ (s. t), $\mathcal{y}_s^* = \mathfrak{F}\varkappa_s^* = S\varkappa_{s+1}^*$; $s = 0,1, \ldots$.

Firstly, note that $\mathfrak{F}$ & S include unique coincidence point. In fact, presume on the contrary $p, q \in \mathcal{X}$ are two distinct coincidence of $\mathfrak{F}$ & S. Consequently, $\nabla^*(p, q, q) > 0$, with $S(p) = \mathfrak{F}(p)$ & $S(q) = \mathfrak{F}(q)$. In this case, from inequality (3.1), obtain

$$\nabla^*(p, q, q) = \nabla^*(\mathfrak{F}p, \mathfrak{F}q, \mathfrak{F}q) \leq \hbar\nabla^*(Sp, Sq, Sq) = \hbar\nabla^*(p, q, q) < \nabla^*(p, q, q),$$

This is a contradiction.

Presume $(\mathcal{F}, \mathcal{C}) \in \Gamma \times [0, +\infty)$ as a result (∇_4^*) is gratified. For a given $\sigma > 0$ and on account of (∇_4^*), $\exists\, \mathcal{L} > 0$ (s. t), $0 < \ell < \mathcal{L} \Longrightarrow \mathcal{F}(\ell) < F(\sigma) - \mathcal{K}$. (3.2)

Regard as $\{y_s^*\} \subset \mathcal{X}$. Currently, without loss of generality, presume $\nabla^*(\mathfrak{F}\varkappa_0^*, \mathfrak{F}\varkappa_1^*, \mathfrak{F}\varkappa_1^*) > 0$. Otherwise, $\varkappa_1^*$ will be a coincidence point of $\mathfrak{F}$ & S. Utilizing inequality (3.1), we get

$$\begin{Bmatrix} \nabla^*(\mathfrak{F}\varkappa_s^*, \mathfrak{F}\varkappa_{s+1}^*, \mathfrak{F}\varkappa_{s+1}^*) \leq \hbar\nabla^*(S\varkappa_s^*, S\varkappa_{s+1}^*, S\varkappa_{s+1}^*) = \\ \hbar\nabla^*(\mathfrak{F}\varkappa_{s-1}^*, \mathfrak{F}\varkappa_s^*, \mathfrak{F}\varkappa_s^*) \leq \hbar^2\nabla^*(S\varkappa_{s-1}^*, S\varkappa_s^*, S\varkappa_s^*) \end{Bmatrix}$$

Which implies via induction that

$$\nabla^*(\mathfrak{F}\varkappa_s^*, \mathfrak{F}\varkappa_{s+1}^*, \mathfrak{F}\varkappa_{s+1}^*) \leq \hbar^s\nabla^*(\mathfrak{F}\varkappa_0^*, \mathfrak{F}\varkappa_1^*, \mathfrak{F}\varkappa_1^*)$$

For all $s \in \mathbb{N}$.

For this reason, $\forall\, r, s \in \mathbb{N}$ with $r > s$, we obtain

$$\sum_{i=s}^{r-1} \nabla^*(\mathfrak{F}\varkappa_i^*, \mathfrak{F}\varkappa_{i+1}^*, \mathfrak{F}\varkappa_{i+1}^*) \leq \frac{\hbar^s}{1-\hbar}\, \nabla^*(\mathfrak{F}\varkappa_0^*, \mathfrak{F}\varkappa_1^*, \mathfrak{F}\varkappa_1^*).$$

Because

$$\lim_{s\to\infty} \frac{\hbar^s}{1-\hbar}\, \nabla^*(\mathfrak{F}\varkappa_0^*, \mathfrak{F}\varkappa_1^*, \mathfrak{F}\varkappa_1^*) = 0,$$

There exists some $e \in \mathbb{N}$ (s. t), $0 < \frac{\hbar^s}{1-\hbar}\, \nabla^*(\mathfrak{F}\varkappa_0^*, \mathfrak{F}\varkappa_1^*, \mathfrak{F}\varkappa_1^*) < \mathcal{L}\ \forall\, s \geq \mathbb{N}$. Therefore, via inequality (3.2) and (∇_2^*), obtain

$$\mathcal{F}(\textstyle\sum_{i=s}^{r-1} \nabla^*(\mathfrak{F}\varkappa_i^*, \mathfrak{F}\varkappa_{i+1}^*, \mathfrak{F}\varkappa_{i+1}^*)) \leq \mathcal{F}(\frac{\hbar^s}{1-\hbar}\, \nabla^*(\mathfrak{F}\varkappa_0^*, \mathfrak{F}\varkappa_1^*, \mathfrak{F}\varkappa_1^*)) < \mathcal{F}(\sigma) - \mathcal{K}. \tag{3.3}$$

For $r > s \geq e$. Utilizing (∇_4^*) together with inequality (3.3), discover if $\nabla^*(\mathfrak{F}\varkappa_s^*, \mathfrak{F}\varkappa_r^*, \mathfrak{F}\varkappa_r^*) > 0$, In this case

$$\mathcal{F}(\nabla^*(\mathfrak{F}\varkappa_s^*, \mathfrak{F}\varkappa_r^*, \mathfrak{F}\varkappa_r^*)) \leq \mathcal{F}(\sum_{i=s}^{r-1} \nabla^*(\mathfrak{F}\varkappa_i^*, \mathfrak{F}\varkappa_{i+1}^*, \mathfrak{F}\varkappa_{i+1}^*)) + \mathcal{K} < \mathcal{F}(\sigma)$$

Which implies via (∇_2^*) that $\nabla^*(\mathfrak{F}\varkappa_s^*, \mathfrak{F}\varkappa_r^*, \mathfrak{F}\varkappa_r^*) < \sigma$, this confirms that $\{y_s^*\} = \{\mathfrak{F}\varkappa_s^*\}$ is Cauchy. Because, $\{\mathfrak{F}\varkappa_s^*\} = \{S\varkappa_{s+1}^*\} \subset S(\mathcal{X})$ with $S(\mathcal{X})$ is closed, consequently $\exists\, z^* \in \mathcal{X}$ (s. t) $\lim\limits_{s,r\to\infty} \nabla^*(S\varkappa_s^*, S\varkappa_r^*, Sz^*) = 0$. After that, indicate z^* is a coincidence of $\mathfrak{F}$ & S. On the opposite, presume $\nabla^*(\mathfrak{F}z^*, Sz^*, Sz^*) > 0$, in that case obtain

$$\begin{Bmatrix} \mathcal{F}\big(\nabla^*(\mathfrak{F}z^*, Sz^*, Sz^*)\big) \leq \\ \mathcal{F}\big(\nabla^*(\mathfrak{F}z^*, \mathfrak{F}\varkappa_s^*, \mathfrak{F}\varkappa_s^*) + \nabla^*(\mathfrak{F}\varkappa_s^*, Sz^*, Sz^*)\big) + \mathcal{K} \leq \\ \mathcal{F}(\hbar\nabla^*(Sz^*, S\varkappa_s^*, S\varkappa_s^*) + \nabla^*(S\varkappa_{s+1}^*, Sz^*, Sz^*)) + \mathcal{K} \end{Bmatrix}$$

Because $s \to \infty$ in the above inequality, we get

$$\lim_{s\to\infty} \mathcal{F}\left(\hbar\nabla^*(Sz^*, S\varkappa_s^*, S\varkappa_s^*) + \nabla^*(S\varkappa_{s+1}^*, Sz^*, Sz^*)\right) + \mathcal{K} = -\infty,$$

This is a contradiction. For this reason, $\nabla^*(\mathfrak{F}z^*, Sz^*, Sz^*) = 0$; i.e., z^* is unique coincidence of $\mathfrak{F}$ & S. Consequently,S & F include unique point of coincidence $w^* = Sz^* = \mathfrak{F}z^*$. Utilizing commutativity of $\mathfrak{F}$ & S, we obtain $Sw^* = S(Sz^*) = S\mathfrak{F}(z^*) = \mathfrak{F}S(z^*) = \mathfrak{F}w^*$. This implies Sw^* another coincidence point of S & F. Via uniqueness of coincidence point of S & F, obtain $w^* = Sw^* = \mathfrak{F}w^*$; i.e. S & $\mathfrak{F}$ contain unique (C.F.P).

Remark 3.6: In the consequence, we will indicate for simplicity $\mathcal{X} \times ... \times \mathcal{X}$ via $\mathcal{X}^s$, (s. t) $\mathcal{X} \neq \emptyset$ and $s \in \mathbb{N}$.

Proposition 3.7: Let $(\mathcal{X}, \nabla^*)$ be a $\Gamma\mathfrak{D}$ -metric space. In that case the next statements hold:

(i) A space $(\mathcal{X}^s, \mathcal{W})$ is a $\Gamma\mathfrak{D}$ -metric space with

$$\left\{\begin{matrix} \mathcal{W}\left((\varkappa_1^*, \dots, \varkappa_s^*), (y_1^*, \dots, y_s^*), (z_1^*, \dots, z_s^*)\right) = \\ max[\nabla^*(\varkappa_1^*, y_1^*, z_1^*), \nabla^*(\varkappa_2^*, y_2^*, z_2^*), \dots, \nabla^*(\varkappa_s^*, y_s^*, z_s^*)] \end{matrix}\right\}$$

(ii) The map $\mathcal{J}: \mathcal{X}^s \to \mathcal{X}$ and $S: \mathcal{X} \to \mathcal{X}$ have a s-tuple (C.F.P) iff the map $\mathfrak{F}: \mathcal{X}^s \to \mathcal{X}^s$ and $\psi: \mathcal{X} \to \mathcal{X}$ described via $\mathfrak{F}(\varkappa_1^*, \varkappa_2^*, \dots, \varkappa_s^*) = \mathcal{J}(\varkappa_1^*, \varkappa_2^*, \dots, \varkappa_s^*), \mathcal{J}(\varkappa_2^*, \dots \varkappa_s^*, \varkappa_1^*), \dots, \mathcal{J}(\varkappa_s^*, \varkappa_1^*, \dots, \varkappa_{s-1}^*)$

And

$$\psi(\varkappa_1^*, \varkappa_2^*, \dots, \varkappa_s^*) = (S\varkappa_1^*, S\varkappa_2^*, \dots, S\varkappa_s^*)$$

Have a (C.F.P) in $\mathcal{X}^s$.

(iii) $(\mathcal{X}, \nabla^*)$ is complete *iff* $(\mathcal{X}^s, \mathcal{W})$ is complete.

Proof: (i) Obviously, $\mathcal{W}$ satisfactory in (∇_1^*)-(∇_3^*). We illustrate that $\mathcal{W}$ gratifies in (∇_4^*). For each $(p_{ij}) \subset \mathcal{X}$for $1 \leq i \leq e$ with $1 \leq j \leq s$,consider $(p_{i1}, p_{ie-1}, p_{ie}) = (\varkappa_1^*, y_{e-1}^*, z_{e-1}^*)$. Presume $\nabla^*\left(\varkappa_j^*, y_j^*, z_j^*\right) = max[\nabla^*(\varkappa_1^*, y_1^*, z_1^*), \nabla^*(\varkappa_2^*, y_2^*, z_2^*), \dots, \nabla^*(\varkappa_s^*, y_s^*, z_s^*)]$. In this case, we have

$$\mathcal{F}_j(\nabla^*\left(\varkappa_j^*, y_j^*, z_j^*\right)) \leq \mathcal{F}_j(\sum_{i=3}^{e-1} \nabla^*(p_{ij}, p_{i+1j}, p_{i-1j}) + \nabla^*(p_{i-1j}, p_{ij}, p_{ij})) + \mathcal{K}_j$$

Where $\mathcal{F}_j \in \Gamma$ and $\mathcal{K}_j \in [0, +\infty)$. Consequently, we get

$$\left\{\begin{matrix} \mathcal{F}_j(\mathcal{W}((\varkappa_1^*, \varkappa_2^*, \dots, \varkappa_s^*), (y_1^*, y_2^*, \dots, y_s^*), (z_1^*, z_2^*, \dots, z_s^*))) = \\ \mathcal{F}_j(max[\nabla^*(\varkappa_1^*, y_1^*, z_1^*), \nabla^*(\varkappa_2^*, y_2^*, z_2^*), \dots, \nabla^*(\varkappa_s^*, y_s^*, z_s^*)]) \\ = \mathcal{F}_j\left(\nabla^*\left(\varkappa_j^*, y_j^*, z_j^*\right)\right) \leq \\ \mathcal{F}_j(\sum_{i=3}^{e-1} \nabla^*(p_{ij}, p_{i+1j}, p_{i-1j}) + \nabla^*(p_{i-1j}, p_{ij}, p_{ij})) + \mathcal{K}_j \leq \\ \mathcal{F}_j\left(\sum_{i=3}^{e-1} \mathcal{W}((p_{i,1}, p_{i,2}, \dots, p_{i,s}), (p_{i+1,1}, p_{i+1,2}, \dots, p_{i+1,s}), (p_{i-1,1}, p_{i-1,2}, \dots, p_{i-1,s})\right) \\ +\mathcal{W}\left((p_{i-1,1}, p_{i-1,2}, \dots, p_{i-1,s}), (p_{i,1}, p_{i,2}, \dots, p_{i,s}), (p_{i,1}, p_{i1,2}, \dots, p_{i,s})\right)) + \mathcal{K}_j. \end{matrix}\right\}$$

The evidence of the parts (ii) and $(iii)$is unpretentious and follows by the same method in evidence of part(i).

Remark 3.8: Note that the Proposition (3.7), is duo-way relationship. Therefore, by utilizing fixed point theorems, enable get s-tuple fixed point outcomes and on the contrary. Currently, put s = 2 in Proposition (3.7), in that case we have the next theorem.

Theorem 3.9: Let $(\mathcal{X}, \nabla^*)$ be a map weighted $\mathfrak{D}$-complete metric space. As well, $\mathrm{S}: \mathcal{X} \longrightarrow \mathcal{X}$ and $\mathfrak{F}: \mathcal{X}^2 \to \mathcal{X}$ any two commutative maps (s. t), $\mathfrak{F}(\mathcal{X}^2) \subset \mathrm{S}(\mathcal{X})$ with $\mathrm{S}(\mathcal{X})$ is closed. Presume $\exists\, \hbar \in (0,1)$ (s. t),

$$\nabla^*(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(p, q), \mathfrak{F}(w^*, z^*)) \leq \frac{\hbar}{2}(\nabla^*(\mathrm{S}\varkappa^*, \mathrm{S}p, \mathrm{S}w^*) + \nabla^*(\mathrm{S}y^*, \mathrm{S}q, \mathrm{S}z^*)) \tag{3.4}$$

For all $(\varkappa^*, y^*), (p, q), (w^*, z^*) \in \mathcal{X}^2$. In this case $\mathfrak{F}, \mathrm{S}$ include a unique (C.C.F.P) in $\mathcal{X} \times \mathcal{X}$.

Proof: in the beginning we will describe the map $\mathcal{W}: \mathcal{X}^2 \times \mathcal{X}^2 \times \mathcal{X}^2 \to \mathcal{X}$ via $\mathcal{W}((\varkappa_1^*, \varkappa_2^*), (y_1^*, y_2^*), (z_1^*, z_2^*)) = max[\nabla^*(\varkappa_1^*, y_1^*, z_1^*), \nabla^*(\varkappa_2^*, y_2^*, z_2^*)]$, and $\mathfrak{F}: \mathcal{X}^2 \to \mathcal{X}^2$ via $\mathfrak{F}(\varkappa^*, y^*) = (\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(y^*, \varkappa^*))$ with $\psi: \mathcal{X}^2 \to \mathcal{X}^2$ via $\psi(\varkappa^*, y^*) = (\mathrm{S}\varkappa^*, \mathrm{S}y^*)$. Utilizing Proposition (3.7), get $(\mathcal{X}^2, \mathcal{W})$ is $\Gamma\mathfrak{D}$-complete metric. Additionally, $(\varkappa^*, y^*) \in \mathcal{X}^2$ is (C.C.F.P) of $\mathfrak{F}$ & S *iff* it's a (C.F.P) of $\mathfrak{F}$ and ψ. On the other hand, using inequality (3.4), We have one of the next cases:

Case-one

$$\left\{\begin{array}{c} \mathcal{W}\big(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(p, q), \mathfrak{F}(w^*, z^*)\big) = \\ \mathcal{W}\Big(\big(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(y^*, \varkappa^*)\big), \big(\mathfrak{F}(p, q), \mathfrak{F}(q, p)\big), \big(\mathfrak{F}(w^*, z^*), \mathfrak{F}(z^*, w^*)\big)\Big) = \\ \max\,[\nabla^*\big(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(p, q), \mathfrak{F}(w^*, z^*)\big), \nabla^*(\mathfrak{F}(y^*, \varkappa^*), \mathfrak{F}(q, p), \mathfrak{F}(w^*, z^*))] = \\ \nabla^*\big(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(p, q), \mathfrak{F}(w^*, z^*)\big) \leq \dfrac{\hbar}{2}(\nabla^*(\mathrm{S}\varkappa^*, \mathrm{S}p, \mathrm{S}w^*) + \nabla^*(\mathrm{S}y^*, \mathrm{S}q, \mathrm{S}z^*)) \leq \\ \hbar \max[\,(\nabla^*(\mathrm{S}\varkappa^*, \mathrm{S}p, \mathrm{S}w^*), \nabla^*(\mathrm{S}y^*, \mathrm{S}q, \mathrm{S}z^*)] = \hbar\mathcal{W}(\psi(\varkappa^*, y^*), \psi(p, q), \psi(w^*, z^*)) \end{array}\right\}$$

Case-two

$$\left\{\begin{array}{c} \mathcal{W}\big(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(p, q), \mathfrak{F}(w^*, \mathrm{z}^*)\big) = \\ \mathcal{W}\Big(\big(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(y^*, \varkappa^*)\big), \big(\mathfrak{F}(p, q), \mathfrak{F}(q, p)\big), \big(\mathfrak{F}(w^*, z^*), \mathfrak{F}(z^*, w^*)\big)\Big) = \\ max\,[\nabla^*\big(\mathfrak{F}(\varkappa^*, y^*), \mathfrak{F}(p, q), \mathfrak{F}(w^*, z^*)\big), \nabla^*(\,\mathfrak{F}(y^*, \varkappa^*), \mathfrak{F}(q, p), \mathfrak{F}(z^*, w^*))] = \\ \nabla^*\big(\mathfrak{F}(y^*, \varkappa^*), \mathfrak{F}(q, p), \mathfrak{F}(z^*, w^*)\big) \leq \dfrac{\hbar}{2}(\nabla^*(\mathrm{S}y^*, \mathrm{S}q, \mathrm{S}z^*) + \nabla^*(\mathrm{S}\varkappa^*, \mathrm{S}p, \mathrm{S}w^*)) \\ \leq \hbar\, max\,\,[\,(\nabla^*(\mathrm{S}y^*, \mathrm{S}q, \mathrm{S}z^*) + \nabla^*(\mathrm{S}\varkappa^*, \mathrm{S}p, \mathrm{S}w^*)] = \\ \hbar\mathcal{W}\big(\psi(y^*, \varkappa^*), \psi(q, p), \psi(z^*, w^*)\big). \end{array}\right\}$$

Currently, via Theorem (3.5), $\mathfrak{F}$ and ψ have a (C.F.P) and via Proposition (3.7), $\mathfrak{F}$ & S have a (C.C.F.P).

Example 3.10: Assume that $\mathcal{X} = [0,1]$. Describe a map $\nabla^*: \mathcal{X}^3 \to \mathbb{R}^+$ as follows:

$$\nabla^*(\varkappa^*, y^*, z^*) = |\varkappa^* - y^*| + |\varkappa^* - z^*| + |y^* - z^*|$$

For all $\varkappa^*, y^*, z^* \in \mathcal{X}$. In that case ∇^* is a $\Gamma\mathfrak{D}$ -complete metric and $\mathcal{F}(\ell) = \mathrm{Ln}(\ell)$ with $\mathcal{K} = 0$. Regard as a maps$\mathfrak{F}: \mathcal{X} \times \mathcal{X} \to \mathcal{X}$ & $\mathrm{S}: \mathcal{X} \to \mathcal{X}$ describe via $\mathfrak{F}(\varkappa^*, y^*) = \frac{\varkappa^* + y^*}{2}$ with $\mathrm{S}(\varkappa^*) = 2\varkappa^*$. Obviously, $\mathfrak{F}$ & S are commutative maps. Additionally, obtain

$$\left\{\begin{array}{c}\nabla^*\big(\mathfrak{F}(\varkappa^*,y^*),\mathfrak{F}(p,q),\mathfrak{F}(w^*,z^*)\big)=\\ \left|\frac{\varkappa^*+\ y^*}{2}-\left(\frac{p+q}{2}\right)\right|+\left|\frac{\varkappa^*+\ y^*}{2}-\left(\frac{w^*+z^*}{2}\right)\right|+\left|\frac{p+q}{2}-\left(\frac{w^*+z^*}{2}\right)\right|\\ =\frac{1}{2}\left[\left|\varkappa^*+y^*-(p+q)\right|+\left|\varkappa^*+y^*-(w^*+z^*)\right|+\left|p+q-(w^*+z^*)\right|\right]\\ \leq\frac{1}{2}\nabla^*(S\varkappa^*,Sp,Sw^*)+\frac{1}{2}\nabla^*(Sy^*,Sq,Sz^*)\end{array}\right\}$$

For this reason, via putting $k=\frac{1}{2}$, so each the suppositions of Theorem (3.9), are gratified. Consequently, $\mathfrak{F}$ and S are contain a unique (C.C.F.P) of $\mathcal{X}^2$.

4. On Hausdorff ∇^*-Distance and Various Fixed Point Results

This segment devoted to present a new extended notion of Hausdorff distance namely; Hausdorff ∇^*-distance in map weighted ∇^*-complete metric spaces. As well, by utilizing this conception some novel categories of enhanced coincidence fixed point theorems have been obtained.

Definition 4.1: Assume that $(\mathcal{X},\nabla^*)$ is $\Gamma\mathfrak{D}$ -metric space with $\mathcal{CB}(\mathcal{X})$ is the collection of each non-empty closed bounded subsets of $\mathcal{X}$. We say $\mathfrak{F}(.,.,.)$Hausdorff ∇^* distance on $\mathcal{CB}(\mathcal{X})$, if

$$\mathfrak{F}_{\nabla^*}(\mathcal{A},\mathcal{B},\mathcal{C})=max\left\{\sup_{\varkappa^*\in\mathcal{A}}\nabla^*(\varkappa^*,\mathcal{B},\mathcal{C}),\sup_{\varkappa^*\in\mathcal{B}}\nabla^*(\varkappa^*,\mathcal{C},\mathcal{A}),\sup_{\varkappa^*\in\mathcal{C}}\nabla^*(\varkappa^*,\mathcal{A},\mathcal{B})\right\}$$

(S. t)

$$\nabla^*(\varkappa^*,\mathcal{B},\mathcal{C})=d_{\nabla^*}(\varkappa^*,\mathcal{B})+d_{\nabla^*}(\mathcal{B},\mathcal{C})+d_{\nabla^*}(\varkappa^*,\mathcal{C}),$$
$$d_{\nabla^*}(\varkappa^*,\mathcal{B})=\inf\{d_{\nabla^*}(\varkappa^*,y^*),y^*\in\mathcal{B}\},$$
$$d_{\nabla^*}(\mathcal{A},\mathcal{B})=\inf\{d_{\nabla^*}(a,b),a\in\mathcal{A},b\in\mathcal{B}\}.$$

Definition 4.2: [26] Suppose that $\mathcal{G}:\mathcal{X}\to\mathcal{X}$ and $\mathfrak{J}:\mathcal{X}\to\mathcal{CB}(\mathcal{X})$ are two maps on $\mathcal{X}\neq\emptyset$. *If* $w=\mathcal{G}\varkappa^*\in\mathfrak{J}\varkappa^*$ for some $\varkappa^*\in\mathcal{X}$, so $\varkappa^*$ is said to be coincidence point of $\mathcal{G}$ & $\mathfrak{J}$ with w is point of coincidence of $\mathcal{G}$ & $\mathfrak{J}$.

Theorem4.3: If $(\mathcal{X},\nabla^*)$ is $\Gamma\mathfrak{D}$-complete metric space. As well, $\mathcal{G}:\mathcal{X}\to\mathcal{X}$ with $\mathfrak{J}:\mathcal{X}\to\mathcal{CB}(\mathcal{X})$ are two maps; $\mathfrak{J}(\mathcal{X})\subset\mathcal{G}(\mathcal{X}),\mathcal{G}(\mathcal{X})$ is closed and $\mathcal{G}$ is continuous. Presume $\exists\ k\in(0,1)$ (s. t),

$$\mathfrak{F}_{\nabla^*}(\mathfrak{J}\varkappa^*,\mathfrak{J}y^*,\mathfrak{J}z^*)\leq k\nabla^*(\mathcal{G}\varkappa^*,\mathcal{G}y^*,\mathcal{G}z^*)\tag{4.1}$$

For all $\varkappa^*,y^*,z^*\in\mathcal{X}$. In this case $\mathfrak{J}$ & $\mathcal{G}$ have coincidence point in $\mathcal{X}$.

Proof: Because $\mathfrak{J}(\mathcal{X})\subset\mathcal{G}(\mathcal{X})$, select $\varkappa_1^*\in\mathcal{X}$ (s. t), $\mathcal{G}\varkappa_1^*\in\mathfrak{J}\varkappa_0^*$. Now, create a sequence $\varkappa_s^*$ in $\mathcal{X}$ (s. t), $\mathcal{G}\varkappa_{s+1}^*\in\mathfrak{J}\varkappa_s^*$ for s = 0,1, ... Presume $(\mathcal{F},\mathcal{K})\in\Gamma\times[0,+\infty)$, as a result that (∇_4^*) is gratified. For a given $\sigma>0$ and utilizing (∇_4^*), $\exists\ \mathcal{L}>0$ (s. t),

$$0<\ell<\mathcal{L}\Longrightarrow\mathcal{F}(\ell)<\mathcal{F}(\sigma)-\mathcal{K}.\tag{4.2}$$

Regard as the sequence $\{\mathcal{G}\varkappa_s^*\}\subset\mathcal{X}$. Next, without loss of generality, presume that $\mathfrak{F}_{\nabla^*}(\mathfrak{J}\varkappa_0^*,\mathfrak{J}\varkappa_1^*,\mathfrak{J}\varkappa_1^*)>0$. Otherwise, $\varkappa_1^*$ be a coincidence point of $\mathfrak{J}$ & $\mathcal{G}$. Currently, utilizing inequality (3.1), obtain

$$\left\{\begin{matrix} \nabla^*(\mathcal{G}\varkappa^*_{s+1},\mathcal{G}\varkappa^*_{s+2},\mathcal{G}\varkappa^*_{s+2}) \leq \mathfrak{F}_{\nabla^*}(\mathfrak{J}\varkappa^*_{s},\mathfrak{J}\varkappa^*_{s+1},\mathfrak{J}\varkappa^*_{s+1}) \\ \leq \mathcal{k}\nabla^*(\mathcal{G}\varkappa^*_{s},\mathcal{G}\varkappa^*_{s+1},\mathcal{G}\varkappa^*_{s+1}) \leq \mathcal{k}\mathfrak{F}_{\nabla^*}(\mathfrak{J}\varkappa^*_{s-1},\mathfrak{J}\varkappa^*_{s},\mathfrak{J}\varkappa^*_{s}) \\ \leq \mathcal{k}^2\nabla^*(\mathcal{G}\varkappa^*_{s-1},\mathcal{G}\varkappa^*_{s},\mathcal{G}\varkappa^*_{s}) \end{matrix}\right\}$$

Which implies $\nabla^*(\mathcal{G}\varkappa^*_s,\mathcal{G}\varkappa^*_s,\mathcal{G}\varkappa^*_s) \leq \mathcal{k}^s\nabla^*(\mathfrak{J}\varkappa^*_0,\mathfrak{J}\varkappa^*_1,\mathfrak{J}\varkappa^*_1)$ $\forall s \in \mathbb{N}$. Currently, if $s, r \in \mathbb{N}$ with $r > s$. In that case get

$$\sum_{i=s}^{r-1}\nabla^*(\mathcal{G}\varkappa^*_i,\mathcal{G}\varkappa^*_{i+1},\mathcal{G}\varkappa^*_{i+1}) \leq \frac{\mathcal{k}^s}{1-\hbar}\nabla^*(\mathcal{G}\varkappa^*_0,\mathcal{G}\varkappa^*_1,\mathcal{G}\varkappa^*_1).$$

On the other hand, because $\lim_{s\to\infty}\frac{\mathcal{k}^s}{1-\hbar}\nabla^*(\mathcal{G}\varkappa^*_0,\mathcal{G}\varkappa^*_1,\mathcal{G}\varkappa^*_1) = 0, \exists\, e \in \mathbb{N}$ (s. t)

$0 < \frac{\mathcal{k}^s}{1-\hbar}\nabla^*(\mathcal{G}\varkappa^*_0,\mathcal{G}\varkappa^*_1,\mathcal{G}\varkappa^*_1) < \mathcal{L}$

For $s \geq e$. Consequently, via inequality (4.2), and (∇^*_2), get

$$\mathcal{F}(\textstyle\sum_{i=s}^{r-1}\nabla^*(\mathcal{G}\varkappa^*_i,\mathcal{G}\varkappa^*_{i+1},\mathcal{G}\varkappa^*_{i+1})) \leq \mathcal{F}\left(\frac{\mathcal{k}^s}{1-\hbar}\nabla^*(\mathcal{G}\varkappa^*_0,\mathcal{G}\varkappa^*_1,\mathcal{G}\varkappa^*_1)\right) < \mathcal{F}(\sigma) - \mathcal{C} \tag{4.3}$$

For all $r > s$. Utilizing, (∇^*_4) together with inequality (4.3), get

$$\left\{\begin{matrix} \nabla^*(\mathcal{G}\varkappa^*_s,\mathcal{G}\varkappa^*_r,\mathcal{G}\varkappa^*_r) > 0 \Longrightarrow \mathcal{F}\big(\nabla^*(\mathcal{G}\varkappa^*_s,\mathcal{G}\varkappa^*_r,\mathcal{G}\varkappa^*_r)\big) \leq \\ \mathcal{F}\left(\displaystyle\sum_{i=s}^{r-1}\nabla^*(\mathcal{G}\varkappa^*_i,\mathcal{G}\varkappa^*_{i+1},\mathcal{G}\varkappa^*_{i+1})\right) + \mathcal{K} < \mathcal{F}(\sigma) \end{matrix}\right\}$$

Which implies via (∇^*_2) that $\nabla^*(\mathcal{G}\varkappa^*_s,\mathcal{G}\varkappa^*_r,\mathcal{G}\varkappa^*_r) < \sigma$. This establish that $\{\mathcal{G}\varkappa^*_s\}$ is Cauchy. Because $(\mathcal{X},\nabla^*)$ $\Gamma\mathfrak{D}$-complete metric space with $\mathcal{G}(\mathcal{X})$ is closed, $\exists\, \varkappa^* \in \mathcal{X}$ (s. t), $\lim_{s\to\infty}\mathcal{G}\varkappa^*_s = \mathcal{G}\varkappa^*$. Currently, we verify $\mathcal{G}\varkappa^* \in \mathfrak{J}\varkappa^*$. For this, utilizing inequality (4.1), we obtain

$\nabla^*(\mathcal{G}\varkappa^*_{s+1},\mathcal{G}\varkappa^*_{s+1},\mathfrak{J}\varkappa^*) \leq \mathfrak{F}_{\nabla^*}(\mathfrak{J}(\varkappa^*_s),\mathfrak{J}(\varkappa^*_s),\mathfrak{J}\varkappa^*) \leq \mathcal{k}\nabla^*(\mathcal{G}\varkappa^*_s,\mathcal{G}\varkappa^*_s,\mathcal{G}\varkappa^*)$.

Therefore,

$\lim_{s\to\infty}\nabla^*(\mathcal{G}\varkappa^*_{s+1},\mathcal{G}\varkappa^*_{s+1},\mathfrak{J}\varkappa^*) = \nabla^*(\mathcal{G}\varkappa^*,\mathcal{G}\varkappa^*,\mathfrak{J}\varkappa^*) = 0$.

For this reason, $\mathcal{G}\varkappa^* \in \mathfrak{J}\varkappa^*$; i.e., $\mathfrak{J}$ and $\mathcal{G}$ contain a point of coincidence.

Next, discuss the next example which is extending to that of [15], which elucidates the validity of Theorem(4.3).

Example4.4. Suppose that$\mathcal{X} = [0,1]$, with $\mathfrak{J}: \mathcal{X} \longrightarrow \mathcal{CB}(\mathcal{X})$ with $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{X}$ be described through $\mathfrak{J}\varkappa^* = [0, \frac{\varkappa^*}{16}]$ & $\mathcal{G}\varkappa^* = \sqrt{\varkappa^*}$. Describe, $\nabla^*: \mathcal{X}^3 \to \mathbb{R}^+$ via

$$\nabla^*(\varkappa^*, y^*, z^*) = \max\{|\varkappa^* - y^*|, |\varkappa^*, -z^*|, |y^* - z^*|\}.$$

In that case ∇^* is $\Gamma\mathfrak{D}$-complete metric and $\mathcal{F}(\ell) = \mathrm{Ln}(\ell)$ and $\mathcal{K} = 0$. Obviously, $\mathfrak{J}(\mathcal{X}) \subset \mathcal{G}(\mathcal{X})$ & $\mathcal{G}(\mathcal{X})$ is closed. If $\varkappa^* = y^* = z^* = 0$, in this case

$$\mathfrak{F}_{\nabla^*}(\mathfrak{J}\varkappa^*, \mathfrak{J}y^*, \mathfrak{J}z^*) = 0 \leq \mathcal{k}\nabla^*(\mathcal{G}\varkappa^*, \mathcal{G}y^*, \mathcal{G}z^*).$$

Therefore, presume $\varkappa^*, y^*$ & z^* aren't everyone zero. Without loss the generality, presume $\varkappa^* \leq y^* \leq z^*$. In this case

$$\mathfrak{F}_{\nabla^*}(\mathfrak{J}\varkappa^*, \mathfrak{J}y^*, \mathfrak{J}z^*) = \mathfrak{F}_{\nabla^*}\left(\left[0, \frac{\varkappa^*}{16}\right], \left[0, \frac{y^*}{16}\right], [0, \frac{z^*}{16}]\right) =$$

$$max[\sup_{0\le a\le \frac{\varkappa^*}{16}} \nabla^*\left(a, \left[0, \frac{y^*}{16}\right], \left[0, \frac{z^*}{16}\right]\right), \sup_{0\le b\le \frac{y^*}{16}} \nabla^*\left(b, \left[0, \frac{z^*}{16}\right], \left[0, \frac{\varkappa^*}{16}\right]\right), \sup_{0\le c\le \frac{z^*}{16}} \nabla^*\left(c, \left[0, \frac{\varkappa^*}{16}\right], \left[0, \frac{y^*}{16}\right]\right)].$$

Because $\varkappa^* \le y^* \le z^*$, consequently $\left[0, \frac{\varkappa^*}{16}\right] \subset \left[0, \frac{y^*}{16}\right] \subset \left[0, \frac{z^*}{16}\right]$. This implies that

$$d_{\nabla^*}\left(\left[0, \frac{\varkappa^*}{16}\right], \left[0, \frac{y^*}{16}\right]\right) = d_{\nabla^*}\left(\left[0, \frac{y^*}{16}\right], \left[0, \frac{z^*}{16}\right]\right) = d_{\nabla^*}\left(\left[0, \frac{\varkappa^*}{16}\right], \left[0, \frac{z^*}{16}\right]\right) = 0.$$

For all $0 \le a \le \frac{\varkappa^*}{16}$, obtain

$$\nabla^*\left(a, \left[0, \frac{y^*}{16}\right], \left[0, \frac{z^*}{16}\right]\right) = d_{\nabla^*}\left(a, \left[0, \frac{y^*}{16}\right]\right) + d_{\nabla^*}\left(\left[0, \frac{y^*}{16}\right], \left[0, \frac{z^*}{16}\right]\right) + d_{\nabla^*}\left(a, \left[0, \frac{z^*}{16}\right]\right) = 0.$$

As well, for $\forall\ 0 \le b \le \frac{y^*}{16}$, get

$$\nabla^*\left(b, \left[0, \frac{\varkappa^*}{16}\right], \left[0, \frac{z^*}{16}\right]\right) = d_{\nabla^*}\left(b, \left[0, \frac{\varkappa^*}{16}\right]\right) + d_{\nabla^*}(\left[0, \frac{\varkappa^*}{16}\right], d_{\nabla^*}\left(a, \left[0, \frac{z^*}{16}\right])\right) + d_{\nabla^*}\left(b, \left[0, \frac{z^*}{16}\right]\right)$$

$$= \begin{cases} 0, & b \le \frac{\varkappa^*}{16} \\ 2b - \frac{\varkappa^*}{8}, & b \ge \frac{\varkappa^*}{16} \end{cases}$$

This harvests that

$$\sup_{0\le b\le \frac{y^*}{16}} \nabla^*\left(b, \left[0, \frac{z^*}{16}\right], \left[0, \frac{\varkappa^*}{16}\right]\right) = \frac{y^* - \varkappa^*}{8}.$$

Additionally, for $\forall\ 0 \le c \le \frac{z^*}{16}$, we obtain

$$\nabla^*\left(c, \left[0, \frac{\varkappa^*}{16}\right], \left[0, \frac{y^*}{16}\right]\right) = d_{\nabla^*}\left(c, \left[0, \frac{\varkappa^*}{16}\right]\right) + d_{\nabla^*}(\left[0, \frac{\varkappa^*}{16}\right], d_{\nabla^*}\left(a, \left[0, \frac{y^*}{16}\right])\right) + d_{\nabla^*}\left(c, \left[0, \frac{y^*}{16}\right]\right)$$

$$= \begin{cases} 0, & c \le \frac{\varkappa^*}{16} \\ 2c - \frac{\varkappa^*}{8}, & \frac{\varkappa^*}{16} \le c \le \frac{y^*}{16} \\ 4c - \frac{\varkappa^* - y^*}{8}, & c \ge \frac{y^*}{16} \end{cases}$$

This harvests that

$$\sup_{0\le c\le \frac{z^*}{16}} \nabla^*\left(c, \left[0, \frac{\varkappa^*}{16}\right], \left[0, \frac{y^*}{16}\right]\right) = \frac{2z^* - y^* - \varkappa^*}{8}.$$

We conclude that

$$\mathfrak{F}_{\nabla^*}(\mathfrak{J}\varkappa^*, \mathfrak{J}y^*, \mathfrak{J}z^*) = \frac{2z^* - \varkappa^* - y^*}{8} \le \frac{z^* - \varkappa^*}{4} = \frac{1}{2}(\frac{1}{2}(z^* - \varkappa^*))$$

$$\le \frac{1}{2}\left(\frac{z^* - \varkappa^*}{\sqrt{\varkappa^*} + \sqrt{z^*}}\right) = \frac{1}{2}\left(\sqrt{z^*} - \sqrt{\varkappa^*}\right).$$

Furthermore, it's apparent that each other suppositions for Theorem (4.3), are fulfilled, consequently $\mathcal{G}$ & $\mathfrak{J}$ contain unique (C.F.P).

5. Some Implementations of Fixed Point Results for Volterra Integral Equations

The main goal of present this segment is to illustrate and discuss some practical implementations of the existence fixed point as a solution for certain non-linear Volterra integral equations in context of map weighted $\mathfrak{D}$-complete metric spaces. In this part, our major outcomes are extending and improving to that of [14].

We consider the next Volterra integral equation as an implementation of our outcomes,:

$$\varkappa^*(\ell) = \int_0^{\ell} \mathbb{L}\big(\ell, s, \varkappa^*(s)\big)ds + q(\ell) \tag{5.1}$$

(S. t), $\ell \in \mathbb{I} = [0,1], \mathbb{L} \in \Psi(\mathbb{I} \times \mathbb{I} \times \mathbb{R}, \mathbb{R})$ with $q \in \Psi(\mathbb{I}, \mathbb{R})$.

Assume $\Psi(\mathbb{I}, \mathbb{R})$ is Banach spaces of each real continuous maps described on $\mathbb{I}$ with $\|\varkappa^*\|_\infty = \max_{\ell\in\mathbb{I}} |\varkappa^*(\ell)| \; \forall\, \varkappa^* \in \Psi(\mathbb{I}, \mathbb{R})$ with $\Psi(\mathbb{I} \times \mathbb{I} \times \Psi(\mathbb{I}, \mathbb{R}), \mathbb{R})$ is the space of each continuous map described on $\mathbb{I} \times \mathbb{I} \times \Psi(\mathbb{I}, \mathbb{R})$. Otherwise, Banach space $\Psi(\mathbb{I}, \mathbb{R})$ enable endowed with Bielecki norm $\|\varkappa^*\|_{\mathrm{B}} = sup_{\ell\in\mathbb{I}}\{|\varkappa^*(\ell)|e^{-\tau\ell}\} \forall\, \varkappa^* \in \Psi(\mathbb{I}, \mathbb{R})$ and $\varepsilon > 0$, with the induced metric $\nabla^*{}_{\mathrm{B}}(\varkappa^*, y^*, z^*) = \|\varkappa^* - y^*\|_{\mathrm{B}} + \|\varkappa^* - z^*\|_{\mathrm{B}} + \|y^* - z^*\|_{\mathrm{B}}, \forall\, \varkappa^*, y^*, z^* \in \Psi(\mathbb{I}, \mathbb{R})$.

As well, describe $\mathfrak{J}: \Psi(\mathbb{I}, \mathbb{R}) \to \Psi(\mathbb{I}, \mathbb{R})$ via

$$\mathfrak{J}\varkappa^*(\ell) = \int_0^{\ell} \mathbb{L}\big(\ell, \mathrm{s}, \varkappa^*(\mathrm{s})\big)\mathrm{ds} + q(\ell),\; q \in \Psi(\mathbb{I}, \mathbb{R}).$$

Theorem 5.1: If $(\Psi(\mathbb{I}, \mathbb{R}), \nabla^*{}_{\mathrm{B}})$ is $\Gamma\mathfrak{D}$-complete metric describe through $\mathcal{F}(\ell) = \mathrm{Ln}(\ell)$, with $\mathfrak{J}: \Psi(\mathbb{I}, \mathbb{R}) \to \Psi(\mathbb{I}, \mathbb{R})$ be an operator with $\mathfrak{J}\varkappa^*(\ell) = \int_0^{\ell} \mathbb{L}\big(\ell, \mathrm{s}, \varkappa^*(\mathrm{s})\big)\mathrm{ds} + q(\ell)$ with $m\varkappa^* = \mathbb{I}(\varkappa^*)$.

Presume that $\mathbb{L} \in \Psi(\mathbb{I} \times \mathbb{I} \times \mathbb{R}, \mathbb{R})$ is operator (s. t):

$(\boldsymbol{i})$ $\mathbb{L}$ is continuous;

$(\boldsymbol{ii})$ $\int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, .)\,, \forall\, \ell, \mathrm{s} \in \mathbb{I}$ is increasing;

$(\boldsymbol{iii})$ There exists $\varepsilon > 0$ where

$$\big|\mathbb{L}\big(\ell, \mathrm{s}, \varkappa^*(\mathrm{s})\big) - \mathbb{L}\big(\ell, \mathrm{s}, y^*(\mathrm{s})\big)\big| \leq e^{-\varepsilon}|\varkappa^*(\mathrm{s}) - y^*(\mathrm{s})|$$

$\forall\, \varkappa^*, y^* \in \Psi(\mathbb{I}, \mathbb{R})$ with $\ell, \mathrm{s} \in \mathbb{I}$.

In this case, Volterra-type integral equation (5.1), has a solution in $\Psi(\mathbb{I}, \mathbb{R})$.

Proof: Through definition of $\mathfrak{J}$, obtain

$$\begin{aligned}\nabla^*{}_{\mathrm{B}}(\mathfrak{J}\varkappa^*, \mathfrak{J}y^*, \mathfrak{J}z^*) &= \left\|\int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, \varkappa^*(\mathrm{s}))\mathrm{ds} - \int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, y^*(\mathrm{s}))\mathrm{ds}\right\|_{\mathrm{B}} \\ &+ \left\|\int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, \varkappa^*(\mathrm{s}))\mathrm{ds} - \int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, z^*(\mathrm{s}))\mathrm{ds}\right\|_{\mathrm{B}} \\ &+ \left\|\int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, y^*(\mathrm{s}))\mathrm{ds} - \int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, z^*(\mathrm{s}))\mathrm{ds}\right\|_{\mathrm{B}} \\ &= \sup_{\ell\in\mathbb{I}}\left\{\left|\int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, \varkappa^*(\mathrm{s}))\mathrm{ds} - \int_0^{\ell} \mathbb{L}(\ell, \mathrm{s}, y^*(\mathrm{s}))\mathrm{ds}\right| e^{-\varepsilon\ell}\right\}\end{aligned}$$

$$+ \sup_{\ell \in \mathbb{I}} \{ \left| \int_0^\ell \mathbb{L}(\ell, s, \varkappa^*(s)) ds - \int_0^\ell \mathbb{L}(\ell, s, z^*(s)) ds \right| e^{-\varepsilon \ell} \}$$
$$+ \sup_{\ell \in \mathbb{I}} \{ \left| \int_0^\ell \mathbb{L}(\ell, s, y^*(s)) ds - \int_0^\ell \mathbb{L}(\ell, s, z^*(s)) ds \right| e^{-\varepsilon \ell} \}$$
$$\leq \sup_{\ell \in \mathbb{I}} \{ \int_0^\ell \left| \mathbb{L}(\ell, s, \varkappa^*(s)) - \mathbb{L}(\ell, s, y^*(s)) \right| e^{-\varepsilon \ell} ds \}$$
$$+ \sup_{\ell \in \mathbb{I}} \{ \int_0^\ell \left| \mathbb{L}(\ell, s, \varkappa^*(s)) - \mathbb{L}(\ell, s, z^*(s)) \right| e^{-\varepsilon \ell} ds \}$$
$$+ \sup_{\ell \in \mathbb{I}} \{ \int_0^\ell \left| \mathbb{L}(\ell, s, y^*(s)) - \mathbb{L}(\ell, s, z^*(s)) \right| e^{-\varepsilon \ell} ds \}$$
$$\leq \sup_{\ell \in \mathbb{I}} \{ \int_0^\ell e^{-\varepsilon} |\varkappa^*(s) - y^*(s)| e^{-\varepsilon \ell} ds \} + \sup_{\ell \in \mathbb{I}} \{ \int_0^\ell e^{-\varepsilon} |\varkappa^*(s) - z^*(s)| e^{-\varepsilon \ell} ds \}$$
$$+ \sup_{\ell \in \mathbb{I}} \{ \int_0^\ell e^{-\varepsilon} |y^*(s) - z^*(s)| e^{-\varepsilon \ell} ds \}$$
$$\leq (\|\varkappa^* - y^*\|_B + \|\varkappa^* - z^*\|_B + \|y^* - z^*\|_B) \sup_{\ell \in \mathbb{I}} \{ \int_0^\ell e^{-\varepsilon} ds \}$$
$$\leq e^{-\varepsilon} \nabla^*{}_B(\varkappa^*, y^*, z^*).$$

Now, consider that the map $\mathcal{F}(\ell) = \mathrm{Ln}(\ell), \forall \ell \in \mathbb{I}, \mathcal{K} = 0$ with $\hbar = e^{-\varepsilon}$. Consequently, each conditions of theorem (3.5), are gratified. As a result, Theorem (3.5),ensures the existence of fixed point of $\mathfrak{J}$ that this fixed point is the solution of the integral equation(5.1).

6. Conclusion

The idea of coupled common fixed point theory is an extremely active branch of pure and applied mathematics and is at the essence of nonlinear analysis because it provides influential tool to verify the existence of solutions for numerous nonlinear issues emerging in the pure and applied mathematics and other branches of sciences such as: computer science, engineering, physics, deferential equations, optimization, control theory, approximation theory and discrete dynamics. Therefore, a novel idea of extension metric spaces called; mapping weighted $\mathfrak{D}$-complete metric spaces has been introduced. Moreover, the uniqueness of various generalized categories of common and common coupled fixed point theorems have been discussed and verified for two pairs of self commutative maps under influence improved kinds of extended contractive conditions in these spaces. On the other hand, a new extended notion of Hausdorff distance called; Hausdorff ∇^*-distance has been defined in these spaces, as well by applying this conception various novel improved results of coincidence fixed point theorems related to this kind of extended distance have been obtained., Additionally, various practical implementations of the existence fixed point for two pairs of self commutative maps as a solution for certain non-linear Volterra integral equations have been offered and discussed in the framework of map weighted $\mathfrak{D}$-complete metric spaces. Lastly, our suggested outcomes are developing of various recognized analogous outcomes related to these categories of fixed point theorems in the framework of extended complete metric spaces. We anticipate that our main detections in this manuscript will aid scientists in improving the authors on popularized extended metric spaces in order to elevate a general framework for their practical implementations in all advanced branches of pure and applied mathematics and other branches of sciences.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

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