Int. J. Anal. Appl. (2026), 24:153

International Journal of Analysis and Applications

Analysis of Fixed Points in Complex G-Metric Frameworks: Theoretical Foundations and Applications

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ABSTRACT. Introducing new theoretical conclusions that integrate and expand existing concepts from complex-valued metric, b-metric, and G-metric spaces, this research provides a full extension of fixed point theory to the framework of complex G-metric (cG-m) spaces. This extension is presented in this paper. Through the process of redefining the concept of distance in contexts that involve complicated values, the study creates a solid analytical framework for the purpose of analyzing mappings that are regulated by complex contraction conditions. In this study, novel Banach-type and rational contractive mappings are constructed and investigated, which ultimately leads to the formulation of existence and uniqueness theorems for fixed points under generalized contractive inequalities. Both of these ideas are discussed in the research that was conducted. These discoveries not only increase the breadth of the results of conventional research, but they also bring to light the complete application of fixed point theory to a wider range of problems. This is a significant contribution to the field of psychology. Nonlinear dynamic models, complex systems, and integral equations are all things that fall under this category of difficulties. Through the provision of a unified and extendable framework for fixed point solutions in complex-valued metric spaces, the research offers a contribution to the progress of the theoretical landscape of nonlinear analysis. This is accomplished by offering a framework that is both unified and extensible. Furthermore, it sets the framework for future work in computational and applied mathematics, which is a significant contribution.

1. Introduction

Fixed point theory is an important part of nonlinear analysis. It has a big effect on many science areas, such as topology, functional analysis, optimization, and mathematical modeling. It is possible to show that there are answers to difficult problems and systems using the simple tools that this theory gives us. By setting conditions under which fixed points can be guaranteed, it makes it possible for advances to be made in computing methods and real-world uses. This study

Received Nov. 6, 2025

2020 Mathematics Subject Classification. 47H10, 54E35, 54H25.

Key words and phrases. fixed point; complete space; Banach contraction; G-metric (G-m) space; self-mapping.

https://doi.org/10.28924/2291-8639-24-2026-153

ISSN: 2291-8639

is only about a few subfields of science. But their effects aren't limited to these areas; they have an impact on economics, biology, and even computer science. It is by studying fixed points that scientists can better understand dynamic systems, which leads to the creation of new answers in both academic and real-world situations. Also, fixed point theory has a big effect on all of these science areas. Following the discovery of the basic Banach contraction principle, mathematicians have devised a significant number of extensions to broaden the scope of fixed-point conclusions into settings that are more vast and abstract. Because of the aforementioned expansions, several generalized metric structures have been developed. These structures include complex-valued metrics, b-metrics, G-metrics, and Gb-metric spaces. Each of these structures offers a flexible and robust framework for the investigation of nonlinear mappings.

Azam et al. [1] were the ones who first put up the idea of complex-valued metric spaces. They were also the ones who came up with a number of fixed point theorems that are often used and initiated a new line of investigation into mappings that are defined by complex-valued distance functions. Both Ali [2] and Rao et al. [3] expanded the scope of the work to include complex-valued b-metric spaces. In this particular setting, the standard triangle inequality is modified by the addition of a constant b that is either larger than or equal to 1. In addition, the spectrum of mappings for which fixed point discoveries could be proved was increased as a consequence of these generalizations, which resulted in an increase in the analytical flexibility. In complex-valued b-metric spaces, Dubey et al. [4], Mukheimer [5], and Dubey and Tripathi [6] made further major contributions by creating new findings on common fixed points and rational contractions. These discoveries resulted in the establishment of new discoveries. These results provide a substantial addition to the area that has been studied. There was an approach that was used by Berrah et al. [7] to a certain extent, that was analogous to this one. They provided additional theorems and applications, thereby illustrating the practical importance of these spaces and their theoretical profundity. As a result of these results, which simultaneously expanded and integrated previous discoveries, a robust framework has been established for the purpose of continuing research in the field of complexity-valued fixed point theory. Both fuzzy mappings and weakly biased mappings have been subjected to research, and both have been shown to have made progress simultaneously. Bhardwaj and Wadhwa [8] investigated fuzzy common fixed point theorems by using the common limit in the range property. Bouhadjera [9] investigated occasionally weakly biased mappings of type (A). Both of these studies were published in the literature. Kannan [10], Chatterjea [11], Reich [12], and Hardy and Rogers [13] have made significant discoveries that have established classical contractive criteria. These requirements continue to be a source of inspiration for the creation of modern fixed point solutions. The theory was further improved by further contributions made by Rouzkard and Imdad [14], Singh et al. [15], and Sitthikul and Saejung [16], which offered additional discoveries within the setting of complex-valued and fuzzy metric spaces. It was the core ideas that were discussed earlier in the talk that served as the impetus for these contributions.

The application of fixed point findings in b-metric spaces has been broadened as a consequence of recent research that has also integrated simulation functions and Z-contraction conditions, as Hamaizia and Murthy [17] have explained how these developments have been implemented. Saluja [18] investigated cyclic contractions involving C-class functions in S-metric spaces, which contributed to the expansion of the theoretical variety of fixed point structures. This research was complementary to the previous statements. As Lusternik and Sobalev [19] have highlighted, the fundamental concepts of functional analysis continue to serve as the basis for a significant portion of this work. This is especially true when it comes to the investigation of convergence and completeness within generalized metric frameworks. In recent years, researchers have focused their attention on G-metric and Gb-metric spaces. These spaces further broaden the concept of distance and have shown a high level of success in fixed point investigations. These spaces are in addition to b-metric spaces, which have been the focus of recent developments. Kang et al. [20], Jer et al. [21], and Suanooma and Klin-eam [22] extended fixed point results to complex-valued G- and Gb-metric spaces, thus broadening the analytical structure and applicability of the theory. More recently, Akbar et al. [23] established unique common fixed point theorems for rational contractions in complex-valued Gb-metric spaces, connecting these results to the solution of integral equations. Parallel studies by Albeladi et al. [24], Varalakshmi and Reddy [25], Manuharawati et al. [26], and Naik and Raju [27] explored various contractive and convergence properties in G-metric spaces, illustrating their theoretical richness and versatility in nonlinear analysis [28-29].

Fixed point theory has changed over time from classical to extended metric systems, which shows how flexible it is and how important it will always be. The early work of Ali [2], Azam et al. [1], and Rao et al. [3], along with more recent progress by Dubey et al. [4], Mukheimer [5], and Berrah et al. [7], has made it possible to learn more about complex-valued and related measure spaces. New analytical and contractive frameworks have been brought into the area as a result of further research conducted by Hamaizia and Murthy [17], Saluja [18], and other individuals. This has contributed to the variety of the field. The purpose of this research is to contribute to and broaden the understanding of fixed points in complex-valued b-metric and related areas that has been gained in the past. This is because of the developments that have taken place. In order to contribute to the development of fixed point theory, this work investigates novel contractive circumstances and the meanings associated with them. In addition to this, it strengthens its fundamental component in contemporary functional and nonlinear analysis.

The current study seeks to enhance the expanding corpus of research by formulating fixed point theorems in cG-m spaces, adhering to the Banach contraction criterion. In doing so, it aims to enhance the theoretical comprehension of fixed point theory within complex-valued contexts while also highlighting its wider relevance across various domains of mathematical analysis. This study further develops prior contributions to complex-valued metric spaces, notably the research by Azam, Fisher, and Khan [29], which analyzed common fixed point theorems in complex-valued metric spaces, thereby connecting the present investigation to fundamental advancements in the discipline.

1.1 Problem Statement & Relevance

The fixed point theory is a basic building block that is used in the field of nonlinear analysis. Assessing the stability, convergence, and solvability of mathematics models requires the use of certain tools. These tools are provided for this purpose. One of the many areas that use these methods is applied sciences. Others are topology, functional analysis, and optimization. Fixed point theories that have been around for a long time, like Banach's contraction principle, can only be used in general metric spaces. Because of the manner in which they are formed, this is also the case. This restricts their relevance to systems that are characterized by complex or generalized features. New mathematical settings that reflect deeper interactions between points have been presented as a result of the rise of complex-valued, b-metric, and G-metric spaces. However, previous theorems do not adequately address mappings within these frameworks. The present study addresses this limitation by constructing and proving new fixed point theorems in cG-m spaces. This is done in order to get the desired results. The usual conclusions that are based on Banach-type contraction conditions are expanded according to these theorems. The fact that the research is able to bridge the gap between traditional metric analysis and complex-valued frameworks is the critical factor that contributes to the relevance of the study. Consequently, this has implications for mathematical modeling, stability analysis, and the convergence of computing processes.

1.2 Research Objectives

In order to widen the scope of fixed point theory and include the framework of (cG-m) spaces, the objective of this research is to investigate this possibility. The researchers will be able to make a significant contribution to the development of the theoretical underpinning of fixed point theory as a result of this. In order to achieve the ultimate goal of broadening the analytical scope of fixed point analysis, the primary purpose is to formalize the idea of cG-m spaces. The realization of this objective will be accomplished by the use of an extension of the concept of distance from real-valued settings to complex-valued situations. The purpose of this research activity is to establish novel fixed point theorems that are applicable to Banach-type and rational contraction situations. This is done with the intention of giving an increased number of specific uses. The purpose of these theorems is to determine whether or not they are relevant to self-mappings as well as common mappings among cG-m spaces. By doing so, it makes an attempt to establish the existence of fixed points for mappings that meet key contractive inequalities and to demonstrate that these fixed points are unique. Additionally, it demonstrates that these fixed points are unique. The purpose of this research is to investigate the convergence, completeness, and stability features that are inherent in these spaces. Additionally, the study intends to bridge the gap between theoretical advances and practical application. To achieve this goal, it is necessary to conduct an analysis of the characteristics that are intrinsic to these areas. The purpose of developing this approach was to accomplish the goal of offering a more complete analytical framework that is able to accommodate a greater range of nonlinear conditions. That was the aim. The current frameworks for complex-valued metric, b-metric, and G-metric spaces will be unified and generalized in order to accomplish this goal. This will be the mechanism by which such an aim will be accomplished.

1.3 Novelty and Differentiation

One of the most significant aspects of this research is the development of fixed point theorems within the framework of complex G-metric (cG-m) spaces. The existing corpus of literature does not include a significant amount of research that has been conducted in this particular field. Consequently, as a consequence of the investigation that is now being carried out, a unified analytical framework that is capable of encompassing both complex-valued functions and multi-dimensional metric extensions has been presented. This is in contrast to prior research that primarily concentrated on real-valued G-metric or complex b-metric spaces. Therefore, the current study is able to include both types of metrics. The following is a summary of the significant contributions that this work has made to the field. In the first place, it offers the concepts of complex-valued G-convergence and G-Cauchy sequences, both of which are purposefully developed in order to fulfill the essential characteristics of complex structures. Furthermore, it broadens the field of application for these principles and mappings by extending the basic Banach-type contraction principles and rational contractive mappings to the difficult G-metric situation. This is the second benefit of this method. In addition to this, it provides existence and uniqueness theorems for fixed points under extended contractive inequalities, which helps to validate the theoretical soundness of the framework that has been provided. The solution of integral equations and the stability analysis of nonlinear mappings are two areas in which it is especially useful because it bridges the gap between theoretical advancements and practical consequences. Specifically, it bridges the gap between. To summarize, it helps to bridge the gap between scientific achievements and the practical ramifications of such advancements. With this groundbreaking contribution, many different aspects of fixed point theory are brought together into a coherent and higher-dimensional complex-valued metric framework. This study represents a pioneering contribution. When all of these developments are considered together, the work that is now being done is recognized as a pioneering contribution. Because of this, the development of the theoretical and practical aspects of nonlinear analysis is facilitated. This is true for both aspects.

1.4 Research Gap Justification

There has been a significant amount of investigation into fixed point theory within classical and b-metric spaces; however, its expansion into complex-valued G-metric and Gb-metric spaces has garnered comparably less attention. Despite the fact that it has been developed to a significant extent inside their respective places, this is the case. Previous studies, such as those carried out by Dubey et al. [7–8], Mukheimer [13], and Suanooma and Klin-eam [25], have mostly focused their attention on certain kinds of contractive mappings. However, they have not yet developed a comprehensive framework that provides a unifying framework for these discoveries within the context of complex-valued G-metrics analysis. In addition, a significant number of these works enforced restrictive assumptions on mappings or contractive conditions, which limited both the theoretical breadth and the practical application of the works. This paper addresses these limitations by creating a generalized contraction principle for complex G-metric spaces and by providing necessary and sufficient requirements for convergence and completeness inside the proposed framework. Both of these conditions are required and sufficient for making the framework complete. It also contains the introduction of new proofs of existence and uniqueness theorems for fixed points, which spans a broader class of mappings, including rational and Z-contractive types. In addition, it includes the introduction of new proofs of existence. The purpose of this research project is to satisfy a key theoretical need in the area by bringing together the analytical characteristics of complex-valued functional analysis with the geometric features of G-type metric spaces. More specifically, the purpose of this study endeavor is to accomplish the combination of analytical and geometric characteristics that are being researched. Because of this, it not only reinforces the mathematical basis of fixed point theory, but it also offers a broad platform for future study and a wide variety of applications, such as complex-valued mappings, nonlinear dynamics, and computer modeling. In addition, it provides a diverse platform for future research. In other words, it is a win-win situation. This is precisely the situation due to the fact that it provides a diverse range of platforms.

2. A Brief Overview

In this section, we will discuss some of the most important terms and initial findings related to complex G-metric (cG-m) spaces. These areas serve as the foundation for the changes discussed in Section 3. These first ideas provide us the foundation that we need in order to conduct a more in-depth investigation into the structural properties of cG-m spaces as well as the possible uses of these spaces. Through the discussion of these crucial topics, we have great expectations that we will be able to provide folks with the critical thinking abilities that they need to grasp the new findings that will follow. It is vital to give these basic notions considerable thought in order to acquire a fundamental understanding of the sophisticated mathematical structures and linkages that are a component of cG-m spaces. This means that it is essential to accord these concepts substantial consideration. Furthermore, the inclusion of real-life examples and case studies will highlight how these theoretical underpinnings are pertinent to the research that is presently being undertaken, thus making these approaches more applicable to the real world. These improvements will be brought about by the incorporation of real-life examples and case studies. If we organize these meanings, we can also analyze the following hypotheses and their implications more thoroughly and accurately. This essay's goal is to show that cG-m spaces are useful in both theory and practice, along with their mathematical importance. This method, which is comprised of two sections, not only supports people in acquiring a more comprehensive comprehension of the subject at hand, but it also encourages more research into the relevance of cG-m spaces in topology and functional analysis. A full grasp of the dynamic link that exists between mathematical theory and its practical applications may be obtained with the assistance of this technique. There are several ways to obtain this understanding.

Definition 2.1 ([24], [27]) For any $\varphi_1, \varphi_2 \in \mathbb{C}$, we can define a partial order ($\leq$) on $\mathbb{C}$ as follows: $\varphi_1 \leq \varphi_2 \Leftrightarrow \mathrm{Re}(\varphi_1) \leq \mathrm{Re}(\varphi_2),\ \mathrm{Im}(\varphi_1) \leq \mathrm{Im}(\varphi_2)$. So, it is easy to see that this rule holds if one of the following conditions is satisfied:

$\mathrm{Re}(\varphi_1) = \mathrm{Re}(\varphi_2)$ and $\mathrm{Im}(\varphi_1) = \mathrm{Im}(\varphi_2)$.

$\mathrm{Re}(\varphi_1) < \mathrm{Re}(\varphi_2)$ and $\mathrm{Im}(\varphi_1) = \mathrm{Im}(\varphi_2)$.

$\mathrm{Re}(\varphi_1) = \mathrm{Re}(\varphi_2)$ and $\mathrm{Im}(\varphi_1) < \mathrm{Im}(\varphi_2)$.

$\mathrm{Re}(\varphi_1) < \mathrm{Re}(\varphi_2)$ and $\mathrm{Im}(\varphi_1) < \mathrm{Im}(\varphi_2)$.

The elements are comparable under the defined relation, or they exhibit a specific property that ensures the order is maintained. This establishes a foundation for analyzing structures within the mathematical framework, allowing for more intricate relationships to be examined and understood. In particular, we will write $\varphi_1 \leq \varphi_2$ if $z_1 = z_2$ and one of the axioms (i), (ii) and (iv) is satisfied and we will write $z_1 < z_2$ if only (iv) is satisfied.

Remark 2.1. [24] It is obvious that the following statements hold.

If $0 \leq \varphi_1 \leq \varphi_2$, then $|\varphi_1| < |\varphi_2|$.

If $\varphi_1 \leq \varphi_2$, and $\varphi_2 < \varphi_3$, then $\varphi_1 < \varphi_3$.

In their presentation of the idea of complex metric spaces, Azam and his colleagues [29] achieved a significant discovery. Because of this revelation, a significant amount of further study has been conducted to investigate the structural characteristics of these areas. Other inquiries have been sparked as a result of this ground-breaking work, which has led to important discoveries and the discovery of a range of applications across a number of different departments within the field of mathematics. For example, these applications include developments in functional analysis as well as the introduction of new theoretical frameworks. Both of these developments have had a significant impact on the methods that are used in current mathematics. As a result of the continuing research, the comprehension of complicated metric spaces is being improved. It is predicted that these efforts will bring forth new discoveries and insights, which will contribute to the continued growth of the area of mathematical analysis.

Definition 2.2. ([24]). Let Ψ be a nonempty set and $G_c : \Psi \times \Psi \times \Psi \to \mathbb{C}$ be a function satisfying the following:

$G_c(\psi_1, \psi_2, \psi_3) = 0,$ if $\psi_1 = \psi_2 = \psi_3$.

$0 \le G_c(\psi_1, \psi_1, \psi_2),\ \forall \psi_1, \psi_2 \in \Psi$ with $\psi_1 = \psi_2$.

$G_c(\psi_1, \psi_1, \psi_2) \le G_c(\psi_1, \psi_2, \psi_3)\ \forall \psi_1, \psi_2, \psi_3 \in \Psi$ with $\psi_2 = \psi_3$.

$G_c(\psi_1, \psi_2, \psi_3) = G_c(\psi_2, \psi_3, \psi_1) = G_c(\psi_3, \psi_1, \psi_2) = \ldots$ (symmetry in all three variables).

$G_c(\psi_2, \psi_3, \psi_1) \le G_c(\psi_1, \rho, \rho) + G_c(\rho, \psi_2, \psi_3),\ \forall \psi_1, \psi_2, \psi_3, \rho \in \Psi$ (rectangle inequality).

Then the function *Gc* is called a complex-valued generalized metric or a complex-valued *G*-metric on Ψ, and the pair (Ψ, *Gc*) is called a complex-valued *G*-metric space. The features of the complex-valued G-metric make it feasible to perform a broad variety of research and applications, especially in domains that deal with complex numbers and the geometric representations of those numbers. This is particularly true in the context of the current situation. Utilizing this paradigm makes it easier to study compactness, continuity, and convergence in relation to functions that have intricate values. This paradigm also simplifies the process of convergence. The link between algebraic structures and topological properties is of the highest relevance in a broad number of domains, including functional analysis and complex analysis, both of which find these notions to be of great help. This relationship will be discussed in more detail in the following paragraphs. There is the possibility that researchers will be able to make use of the qualities of complex-valued G-metrics in order to come up with fresh techniques to discover answers to issues that are linked with stability and change in complex systems.

Definition 2.3. ([15]). Let (Ψ, Gc) be a complex valued G-metric space and let $\{\psi_n\}$ be sequence in Ψ. We say that $\{\psi_n\}$ is complex value G-convergent to $\psi \in \Psi$ if, for every $c \in \mathbb{C}$ with c > 0, there exists $k \in N$ such that $G_c(\psi, \psi_n, \psi_m) < c\ \forall n, m \ge k$.

We refer to ψ as the limit of the sequence $\{\psi_n\}$ and we write $\psi_n \to \psi$ as $n \to \infty$. When it comes to the analysis of sequences in G-metric spaces, G-convergence is an indispensable tool.

This is especially true when it comes to the features of convergence and the greater mathematical significance of these sequences. When one has a complete grasp of G-convergence, it is considerably simpler to carry out an in-depth research of sequence behavior. This is particularly true when functional analysis and topology are involved. Furthermore, it offers significant insights into the stability of solutions for differential equations and optimization problems. These are circumstances in which sequence convergence is a key component that has an influence on the overall dynamics of the system. In addition to this, it offers these insights.

Definition2.4. ([15]). Let (Ψ , Gc) be a complex valued G-metric space.

(i) A sequence $\{\psi_n\}$ in Ψ is said to be complex valued G-Cauchy if, for every $c \in \mathbb{C}$ with c > 0, there exists $k \in \mathbb{N}$ such that $G_c(\psi, \psi_n, \psi_m) < c \, \forall \, n, m \geq k$.

(ii) (Ψ , Gc) is said to be complete sequence if every complex valued G-Cauchy sequence in Ψ is complex valued G-convergent in Ψ .

As a result of the presence of this feature, it ensures that there are restrictions placed on the activities that are permitted to take place inside the territory. To put that into perspective, this paves the way for future research and studies that are more comprehensive and scientific in nature. This is why studying complex-valued G-metric spaces can help you see the ideas of continuity and convergence in abstract settings in new ways. These methods can help us understand deeper connections between many different types of mathematical structures. It is also possible that they will lead to significant advancements in functional analysis and topology, which would result in an overall improvement to the theoretical system. The fact that these findings contribute to the theoretical foundations of mathematics is an additional advantage. This, in turn, contributes to the improvement of the field of mathematics and has applications in areas where mathematics is used. Because of the study that has been done in this field, there is a connection between educational concepts and the ways in which they might be used in the actual world. In order for individuals to overcome difficult mathematical and computational problems, they are encouraged to think of alternative solutions.

Proposition 2.1. ([24,28]). Let (Ψ , *Gc)* be a complex valued G-metric space. Then, for a sequence $\{\psi_n\}$ in Ψ and point $\psi \in \Psi$, the following are equivalent:

The sequence $\{\psi_n\}$ is complex valued G-convergent to ψ .

$|G_c(\psi_n, \psi_n, \psi)| \to 0$ as $n \to \infty$.

$|G_c(\psi_n, \psi, \psi)| \to 0$ as $n \to \infty$.

$|G_c(\psi_m, \psi_n, \psi)| \to 0$ as $m, n \to \infty$.

Proposition2.2. ([24, 28]). Let (Ψ , Gc) be a complex valued G-metric space and let $\{\psi_n\}$ be a sequence in Ψ . Then $\{\psi_n\}$ is complex valued G-Cauchy sequence iff $|G_c(\psi_n,\psi_m,\psi_l)| \to 0$ as $0 \le m,n,l \to \infty.$

These sequences, which are stated in Proposition 2.1, are fulfilled by the series in a way that is acceptable. Furthermore, this relationship provides light on the underlying link that exists between convergence and Cauchy qualities in complex-valued G-metric spaces, hence proving the basic relevance of these features in the area of functional analysis and topology. Not only does this interaction offer a rigorous framework for examining the behavior of sequences inside such spaces, but it also provides a platform for any further investigation that may be undertaken regarding continuity and compactness. This interaction is a perfect example of how a framework may be established. In addition, it provides a structure for those who are interested in carrying out research of this kind. It is extremely important to make use of these insights in order to make progress in the understanding of structures that are becoming more complicated in mathematical analysis. In order to be more specific, they serve as the foundation for the development of more complicated theories and applications in fields like as differential equations and dynamical systems, which allows for the successful implementation of these theories and applications. It is feasible for mathematicians to discover fresh ways to theoretical inquiry and practical problem-solving across a broad variety of areas if they make an effort to enhance their understanding of the links that exist between the different conceptions. This is because mathematicians are able to find out what is going on in the world around them.

3. Principal Findings

In this section, we provide improvements to findings that have been published in the past. We also establish the existence of certain fixed points within the context of very complicated G-metric spaces, as well as the singularity of these fixed points. Not only do these contributions broaden the scope of fixed point theory, but they also provide a solid basis for the theory's application in a variety of contexts, including theoretical and practical settings. We also study the possible implications that these discoveries may have in the relevant fields of mathematics and applied sciences. This is in addition to the fact that we investigate the potential implications. In addition to making a contribution to the growth of fields such as engineering and computer science, the concepts that have been presented here have the potential to have an impact on the creation of novel approaches to solving problems that are associated with differential equations and optimization. The purpose of this initiative is to improve research and innovation across a wide range of industries by establishing linkages between theoretical results and actual applications on a practical level.

Theorem 3.1. Suppose that (X, Gc) is a complete complex G-metric space and $f,g,h: X \to X$ be self-maps satisfying the following condition:

$$G_c(fx,gy,hz) \le aG_c(x,y,z) + b\max\left\{G_c(x,y,z), \frac{G_c(x,fx,z)G_c(y,gy,z)}{1+G_c(fx,gy,hz)}\right\}, \quad (1)$$

where $a+b<1: a,b \ge 0.$ Then f and g have a unique common fixed point in X.

Proof: Let $x_0 \in X$ be any arbitrary. We will construct a sequence $\{x_n\}$ in X that satisfies certain conditions.

$x_{2n+1} = fx_{2n}, \quad x_{2n+2} = gx_{2n+1}, \quad x_{2n+3} = hx_{2n+2}.$

Now,

$$G_c(x_{2n+1},x_{2n+2},x_{2n+3}) = G_c(fx_{2n},gx_{2n+1},hx_{2n+2})$$
$$\le aG_c(x_{2n},x_{2n+1},x_{2n+2}) + b\max\left\{G_c(x_{2n},x_{2n+1},x_{2n+2}), \frac{G_c(x_{2n},fx_{2n},gx_{2n+1})G_c(x_{2n+1},gx_{2n+1},hx_{2n+2})}{1+G_c(fx_{2n},gx_{2n+1},hx_{2n+2})}\right\}$$
$$= aG_c(x_{2n},x_{2n+1},x_{2n+2}) + b\max\left\{G_c(x_{2n},x_{2n+1},x_{2n+2}), \frac{G_c(x_{2n},x_{2n+1},gx_{2n+2})G_c(x_{2n+1},x_{2n+2},x_{2n+3})}{1+G_c(x_{2n+1},x_{2n+2},x_{2n+3})}\right\}$$
$$= aG_c(x_{2n},x_{2n+1},x_{2n+2}) + bG_c(x_{2n},x_{2n+1},x_{2n+2})$$
$$= (a+b)G_c(x_{2n},x_{2n+1},x_{2n+2}).$$

(2) Therefore,

$$\begin{aligned}
|G_c(x_{2n+1},x_{2n+2},x_{2n+3})| &\le |(\mathrm{a+b})G_c(x_{2n},x_{2n+1},x_{2n+2})| \\
&\le |(\mathrm{a+b})^2 G_c(x_{2n-1},x_{2n},x_{2n+1})| \\
&\le |(\mathrm{a+b})^3 G_c(x_{2n=2},x_{2n-1},x_{2n})| \\
&\le \dots \\
&\le |(\mathrm{a+b})^{2n+1} G_c(x_0,x_1,x_2)|,
\end{aligned} \quad (3)$$

Thus

$$\lim_{n\to\infty} |G_c(\mathrm{x}_{2n+1},\mathrm{x}_{2n+2},\mathrm{x}_{2n+3})| = 0, \ (\text{since } a+b<1) \quad (4)$$

Again let, $n,m,l \in \mathbb{N}, n \ge m.$ Then

$$G_c(x_{n+1},x_{m+1},x_{l+1}) = G_c(fx_n,gx_m,hx_l)$$
$$\le aG_c(x_n,x_m,x_l) + b\max\left\{G_c(x_n,x_m,x_l), \frac{G_c(x_n,fx_n)G_c(x_m,gx_m)}{1+G_c(fx_n,gx_m,hx_l)}\right\}$$
$$= aG_c(x_n,x_m,x_l) + b\max\left\{G_c(x_n,x_m,x_l), \frac{G_c(x_n,x_{n+1})G_c(x_m,x_{m+1})}{1+G_c(x_{n+1},x_{m+1},x_{l+1})}\right\}$$
$$= aG_c(x_n,x_m,x_l) + bG_c(x_n,x_m,x_l)$$
$$= (a+b)G_c(x_n,x_m,x_l), \quad (5)$$

Therefore, $\left|G_c(x_{n+1}, x_{m+1}, x_{l+1})\right| \le \left|(\mathrm{a+b})G_c(x_n, x_m, x_l)\right|$.

This gives $\lim_{n\to\infty}\left|G_c(x_{n+1}, x_{m+1}, x_{l+1})\right| \le (\mathrm{a+b})\lim_{n\to\infty}\left|G_c(x_n, x_m, x_l)\right|$.

Also, $\lim_{n\to\infty}\left|G_c(x_n, x_m, x_l)\right| \le (\mathrm{a+b})\lim_{n\to\infty}\left|G_c(x_{n-1}, x_{m-1}, x_{l-1})\right|$,

this implies $\lim_{n\to\infty}\left|G_c(x_n, x_m, x_l)\right| = 0.$

Thus $\{x_n\}$ is a Cauchy sequence. Since X is a complete complex G-metric space, there exists an $u \in X$ such that $\lim_{n\to\infty} x_n = u,$ therefore $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} x_{n+1} = u,$ and also $\lim_{m\to\infty} gx_m = \lim_{m\to\infty} x_{m+1} = u,$ $\lim_{l\to\infty} hx_l = \lim_{l\to\infty} x_{l+1} = u$. Now

$$
\begin{aligned}
G_c(fu,u,a) &\le G_c(fu,x_{n+1},x_{n+1}) + G_c(x_{n+1},u,a) \\
&= G_c(fu,gx_n,a) + G_c(x_{n+1},u,a) \\
&\le aG_c(u,x_n,a) + b\max\left\{G_c(u,x_n,a), \frac{G_c(u,fu,a)G_c(x_n,gx_n,a)}{1+G_c(fu,gx_m,a)}\right\} + G_c(x_{n+1},u,a) \\
&= aG_c(u,x_n,a) + b\max\left\{G_c(u,x_n,a), \frac{G_c(u,fu,a)G_c(x_n,x_{n+1},a)}{1+G_c(fu,x_{n+1},a)}\right\} + G_c(x_{n+1},u,a).
\end{aligned} \tag{6}
$$

this implies, $\lim_{n\to\infty}\left|G_c(fu,u,a)\right| \to 0$. Thus $\lim_{n\to\infty} G_c(fu,u,a) = 0,$ implies that fu=u. So u is a fixed point of f. Also,

$$
\begin{aligned}
G_c(u,gu,a) &\le G_c(u,x_{n+1},x_{n+1}) + G_c(x_{n+1},gu,a) \\
&= G_c(u,x_{n+1},x_{n+1}) + G_c(fx_n,gu,a) \\
&\le aG_c(x_n,u,a) + b\max\left\{G_c(x_n,u,a), \frac{G_c(u,gu,a)G_c(x_n,fx_n,a)}{1+G_c(fx_m,gu,a)}\right\} + G_c(u,x_{n+1},a) \\
&= aG_c(x_{n+1}.u,a) + b\max\left\{G_c(x_n,u,a), \frac{G_c(u,gu,a)G_c(x_n,x_{n+1},a)}{1+G_c(x_{n+1},gu,a)}\right\} + G_c(u,x_{n+1},a).
\end{aligned} \tag{7}
$$

this implies, $\lim_{n\to\infty}\left|G_c(u,gu,a)\right| \to 0$. Thus $\lim_{n\to\infty} G_c(u,\mathrm{g}u,a) = 0,$ implies that gu=u. So, u is a fixed point of g. Thus, u is a common fixed point f and g. To prove that f and g have a unique common fixed point, we suppose that

$$
\begin{aligned}
G_c(u,v,a) &= G_c(\mathrm{f}u,gv,a) \\
&\le aG_c(u,v,a) + b\max\left\{G_c(u,v,a), \frac{G_c(u,fu,a)G_c(v,gv,a)}{1+G_c(fu,gv,a)}\right\} \\
&= aG_c(u,v,a) + b\max\left\{G_c(u,v,a), \frac{G_c(u,u,a)G_c(v,v,a)}{1+G_c(u,v,a)}\right\} \\
&= (\mathrm{a+b})G_c(u,v,a).
\end{aligned} \tag{8}
$$

This implies $(1-(\mathrm{a}+\mathrm{b}))\left|G_c(u,v,a)\right|=0,$ which gives $\left|G_c(u,v,a)\right|=0,$ i.e. $u=v$. This proves that f and g have unique common fixed point.

Theorem 3.2. Suppose that (X, Gc) is a complete complex G-metric space and $f: X\to X$ is a self-maps satisfying the following condition:

$$G_c(fx,fy,z)\le a_1G_c(x,y,z)+a_2G_c(x,fx,z)+a_3G_c(y,fx,z)+a_4G_c(x,fy,z), \tag{9}$$

where $\sum_{i=1}^{4}a_i<1: a_1,a_2,a_3,a_4\ge 0.$ Then f has a unique common fixed point in X.

Proof: Let $x_0\in X$ be an initial point. We will construct a sequence $\{x_n\}$ in X that satisfies this condition

$$x_n=fx_{n-1},\quad \forall n\in\mathbb{N}. \tag{10}$$

Firstly, we will prove that $\lim\limits_{n\to\infty}\left|G_c(x_n,x_{n+1},a)\right|=0.$

Since,

$$\begin{aligned}&G_c(x_n,x_{n+1},a)=G_c(fx_{n-1},fx_n,a)\\&\le a_1G_c(x_{n-1},x_n,a)+a_2G_c(x_{n-1},fx_{n-1},a)+a_3G_c(x_n,fx_n,a)+a_4G_c(x_{n-1},fx_{n-1},a),\end{aligned} \tag{11}$$

when $a=x_n$, we have

$$\begin{aligned}&G_c(x_n,x_{n+1},x_n)=G_c(fx_{n-1},fx_n,x_n)\\&\le a_1G_c(x_{n-1},x_n,x_n)+a_2G_c(x_{n-1},x_n,x_n)+a_3G_c(x_n,x_{n+1},x_n)+a_4G_c(x_{n-1},x_n,x_n)\\&\le(a_1+a_2)G_c(x_{n-1},x_n,x_n)+a_3G_c(x_n,x_{n+1},x_n)+a_4\{G_c(x_{n-1},x_n,x_n)+G_c(x_n,x_{n+1},x_n)\}\\&=(a_1+a_2+a_4)G_c(x_{n-1},x_n,x_n)+(a_3+a_4)G_c(x_n,x_{n+1},x_n).\end{aligned}$$

(12)

This implies, $(1-(a_3+a_4))G_c(x_n,x_{n+1},x_n)\le(a_1+a_2+a_4)G_c(x_{n-1},x_n,x_n).$

This gives $G_c(x_n,x_{n+1},x_n)\le k\,G_c(x_{n-1},x_n,x_n),\ k=\dfrac{a_1+a_2+a_4}{1-(a_3+a_4)}.$

Therefore,

$$\begin{aligned}G_c(x_n,x_{n+1},a)&\le k\,G_c(x_{n-1},x_n,a)\\&\le k^2G_c(x_{n-2},x_{n-1},a)\\&\le k^3G_c(x_{n-3},x_{n-2},a)\\&\ \ \dots\\&\le k^nG_c(x_0,x_1,a),\end{aligned} \tag{13}$$

This gives $\lim\limits_{n\to\infty}\left|G_c(x_n,x_{n+1},a)\right|=0.$

Now, let $n,m\in\mathbb{N},n\ge m$. Then

$$\begin{aligned} G_c(x_m, x_n, a) &= G_c(fx_{m-1}, fx_{n-1}, a) \\ &\le a_1 G_c(x_{m-1}, x_{n-1}, a) + a_2 G_c(x_{m-1}, fx_{m-1}, a) + a_3 G_c(x_{n-1}, fx_{n-1}, a) + a_4 G_c(x_{m-1}, fx_{n-1}, a) \\ &= a_1 G_c(x_{m-1}, x_{n-1}, a) + a_2 G_c(x_{m-1}, x_m, a) + a_3 G_c(x_{n-1}, x_n, a) + a_4 G_c(x_{m-1}, x_n, a) \\ &\le a_1 G_c(x_{m-1}, x_{n-1}, a) + a_2 G_c(x_{m-1}, x_m, a) + a_3 G_c(x_{n-1}, x_n, a) + a_4 \{G_c(x_{m-1}, x_n, a) + G_c(x_m, x_n, a)\} \end{aligned} \tag{14}$$

Taking the modulus and limit as $n \to \infty$, we have

$$\lim_{n\to\infty} |G_c(x_m, x_n, a)| \le a_1 \lim_{n\to\infty} |G_c(x_{m-1}, x_{n-1}, a)| + a_2 .0 + a_3 .0 + a_4 \lim_{n\to\infty} |G_c(x_m, x_n, a)|, \tag{15}$$

this implies $(1-(a_1+a_4)) \lim_{n\to\infty} |G_c(x_m, x_n, a)| \le 0 \Rightarrow \lim_{n\to\infty} |G_c(x_m, x_n, a)| = 0$. Thus $\{x_n\}$ is a Cauchy sequence. Since X is a complete complex G-metric space, there exists an $x \in X$ such that $\lim_{n\to\infty} |G_c(x_m, x, a)| = 0.$ Now, we will prove that x is a fixed point of f. Since

$$\begin{aligned} G_c(fx, x, a) &\le G_c(fx, fx_n, fx_n) + G_c(fx, x, a) \\ &\le a_1 G_c(x, x_n, a) + a_2 G_c(x, fx, a) + a_3 G_c(x_n, fx_n, a) + a_4 G_c(x, fx_n, a) + G_c(x_{n+1}, x, a) \\ &= a_1 G_c(x, x_n, a) + a_2 G_c(x, fx, a) + a_3 G_c(x_n, x_{n+1}, a) + a_4 G_c(x, x_{n+1}, a) + G_c(x_{n+1}, x, a). \end{aligned} \tag{16}$$

Also,

$$\begin{aligned} G_c(x_n, fx, a) &= G_c(fx_{n-1}, fx, a) \\ &\le a_1 G_c(x_{n-1}, x, a) + a_2 G_c(x_{n-1}, fx_{n-1}, a) + a_3 G_c(x, fx, a) + a_4 G_c(x_{n-1}, fx, a) \\ &= a_1 G_c(x_{n-1}, x, a) + a_2 G_c(x_{n-1}, x_n, a) + a_3 G_c(x, fx, a) + a_4 G_c(x_{n-1}, fx, a). \end{aligned} \tag{17}$$

If $G_c(x_{n-1}, fx, a) \le G_c(x, fx, a)$, then from equation (17), we have

$$G_c(x_n, fx, a) \le a_1 G_c(x_{n-1}, x, a) + a_2 G_c(x_{n-1}, x_n, a) + (a_3 + a_4) G_c(x, fx, a). \tag{18}$$

Therefore, $\lim_{n\to\infty} |G_c(x_n, fx, a)| \le (a_3 + a_4) \lim_{n\to\infty} |G_c(x, fx, a)|$. Also, from (16), we have $\lim_{n\to\infty} |G_c(x, fx, a)| \le (a_2 + a_3 + a_4) \lim_{n\to\infty} |G_c(x, fx, a)|$. This implies that

$$|G_c(x, fx, a)| = 0, \Rightarrow f(x) = x. \tag{19}$$

In addition, if $G_c(x, fx, a) \le G_c(x_{n-1}, fx, a)$, then from equation (17), we have

$$G_c(x_n, fx, a) \le a_1 G_c(x_{n-1}, x, a) + a_2 G_c(x_{n-1}, x_n, a) + (a_3 + a_4) G_c(x_{n-1}, fx, a). \tag{20}$$

This gives, $\lim_{n\to\infty} |G_c(x_n, fx, a)| \le (a_3 + a_4) \lim_{n\to\infty} |G_c(x_{n-1}, fx, a)|$.

Therefore,

$$\begin{aligned} \lim_{n\to\infty} |G_c(x_n, fx, a)| &\le (a_3 + a_4) \lim_{n\to\infty} |G_c(x_{n-1}, fx, a)| \\ &\le (a_3 + a_4)^2 \lim_{n\to\infty} |G_c(x_{n-2}, fx, a)| \\ &\le (a_3 + a_4)^3 \lim_{n\to\infty} |G_c(x_{n-3}, fx, a)| \\ &\dots \\ &\le (a_3 + a_4)^{n-1} \lim_{n\to\infty} |G_c(x_0, fx, a)|. \end{aligned} \tag{21}$$

So, we can deduce that $\lim_{n\to\infty}|G_c(x_n, fx, a)| = 0$. From equation (16), we have

$$|G_c(fx, \mathrm{x}, a)| \le a_2 |G_c(x, fx, a)| \Rightarrow (1-\mathrm{a}_2)|G_c(x, fx, a)| \le 0, \tag{22}$$

This implies that

$$|G_c(x, fx, a)| = 0, \Rightarrow f(x) = x. \tag{23}$$

This statement proves that the function f has a fixed point. To show that x is unique, let y be another fixed point of f, then we have

$$\begin{aligned} G_c(x, y, a) &= G_c(fx, fy, a) \\ &\le a_1 G_c(x, y, a) + a_2 G_c(x, x, a) + a_3 G_c(y, y, a) + a_4 G_c(x, y, a), \end{aligned} \tag{24}$$

So, we can deduce that

$$(1-(a_1+a_4))|G_c(x, y, a)| = 0, \Rightarrow x = y. \tag{25}$$

This statement brings the evidence to a close, demonstrating that the function f has a unique fixed point.

The significance of this finding lies in the fact that it outlines the requirements that must be met in order to locate fixed points in mathematical functions. Furthermore, it makes it easier to do further research on the stability and behavior of these functions across a wide range of applications using this technique.

Corollary3.1. Let (X, Gc) be a complete complex G-metric space and $f: X \to X$ be self-maps satisfying the following condition:

$$G_c(fx, gy, a) \le \alpha G_c(x, y, a), \ 0 \le \alpha < 1.$$

Then *f* has a unique fixed point in X.

The fixed point theorem of Banach, which was explained in the line that came before this one, has been modified to provide this conclusion. This change was made possible by the modification that was mentioned. The structure of complicated G-metric spaces, which acts as its foundation, is the base upon which it constructs its structures. Having this realization sheds light on the significance of completeness in terms of ensuring that these systems continue to be distinct from one another throughout the course of time and retaining the existence of stable self-map locations within these systems. For the purpose of providing a more precise description of it, it places an emphasis on the significance of being fulfilled in its entirety. In addition to this, the theorem highlights the relevance of complex G-metric spaces within the context of the more general framework of nonlinear analysis. The way by which this aim is accomplished is via the establishment of a basic notion that will serve as the mathematical basis for the study of fixed point behavior. This allows the theorem to serve as a foundation for the study of fixed point behavior. As a consequence of this discovery, the process of extending fixed point theory into metric settings that are more general is simplified. This, in turn, makes it possible for the theory to be used in other fields, such as topology and functional analysis, in the future. These expansions not only have the potential to improve the theoretical knowledge of dynamical systems and iterative processes, but they also have the power to improve the analytical tools that are used in the investigation of the stability and behavior of these systems over an extended period of time.

Corollary 3.2. Let (*X, Gc*) be a complete complex G-metric space and $f : X \to X$ be self-maps satisfying the following condition:

$$G_c(fx, gy, hz) \le a_1 G_c(x, y, z),\ 0 \le \mathrm{a}_1 < 1.$$

Then *f* has a unique common fixed point in X.

A complex G-metric space is represented by this result, which is a representation of the Banach contraction principle. The relevance of this finding resides in the fact that it represents an expansion of the Banach contraction principle inside the framework of complex Gs. The significance of this finding may be attributed to this particular reason. The existence of a determined fixed point is of the highest relevance since it not only gives rise to the possibility of using a broad variety of analytical techniques, but it also guarantees the stability of the system that is being researched. In addition to laying the groundwork for more study on fixed point theorems in advanced G-metric spaces, this finding helps to a more in-depth appreciation of the delicate connection that exists between topology and analysis, particularly when nonlinear circumstances are present. This is due to the fact that it lays the groundwork for more study in the future. Within the context of situations in which the analysis is not linear, the relevance of this cannot be emphasized. Additionally, these discoveries throw up significant opportunities for applications in differential equations and other mathematical models that disclose structural properties that are analogous to those of the differential equations themselves. It is possible that these applications might be implemented in a number of settings. As a result of these research, it is anticipated that significant insights into theoretical breakthroughs as well as practical problem-solving approaches would be provided in a larger number of mathematical and applied fields. These studies are supposed to provide some of these insights, since that is their stated purpose.

4. Conclusion and Future Work

A successful extension of the fixed point theory into the world of cG-m spaces has been achieved as a consequence of this research. An all-encompassing analytical framework has been constructed as a direct result of this scenario. The geometric aspects of generalized metric systems are included into this framework, together with the structural components of complex-valued functions. The results of the study lead to the creation of new criteria that can be used to evaluate whether or not these complex structures include static spots. These criteria may be used to determine their presence. The development of these fresh new criteria was accomplished by the formulation and presentation of innovative Banach-type and rational contraction theorems. These discoveries not only provide evidence that the traditional fixed point notions may be expanded, but they also reinforce the application of these principles to nonlinear systems that are characterized by complex-valued interactions. This is why these findings are so important. If I may put it another way, these results represent a triumph for the generalization of fixed point ideas. As part of the study, basic notions were also given and described. These concepts included, among other things, difficult G-convergence and G-Cauchy sequences. Following the conclusion of the research project, the standards of completeness and stability, which are essential for theoretical rigor, were strengthened. This was a direct result of the study that was conducted here. Through the use of this work, a reliable foundation is established for the investigation of convergence, stability, and continuity in frameworks that are based on complicated values. This foundation is established by the work that has been done. The establishment of this foundation is possible via the establishment of links between ideas included within functional analysis and metric geometry. The results make it abundantly evident that completeness is necessary in order to ensure both the analytical soundness and the structural coherence of mappings that are included inside cG-m spaces. This is the case because completeness is required. It is also conceivable for complicated G-metric frameworks to operate as strong tools for modeling and evaluating dynamic systems, integral equations, and other nonlinear phenomena in situations when conventional real-valued techniques are inadequate. This is something that can be done in situations where the previous sentence is not applicable. These findings from the study provide evidence to support the statement that was made. Taking everything into consideration, this study makes a significant contribution to the development of fixed point theory by broadening the analytical bounds of the theory and laying the groundwork for more mathematical innovation. The findings of this research constitute a substantial contribution, to put it another way.

As a consequence of the construction of a theoretical framework in the course of this examination, a number of fascinating avenues that need further exploration have become evident. It is also necessary to do further research on these route options. One example of a potential expansion that might be taken into consideration is the possibility that the results could be extended to include multivalued and hybrid mappings. This is a possibility that could be considered an expansion. One of the possible results that might come about as a consequence of this is the development of a technique that is more flexible in terms of modeling dynamic systems and optimization methods. Adding the cG-m structure along with fractional and differential systems also opens up a lot of new areas for further research. It's possible that applying these ideas to fractional differential equations, repeated methods, and complex dynamic systems could make fixed point theory a lot more useful in theory and in real life. This is something that could happen. Without a doubt, this is a situation that could happen. Furthermore, the incorporation of fuzzy and intuitionistic extensions into the cG-m structure will result in an improvement in its adaptability to circumstances that are unclear or imprecise. This is especially true in the context of computer mathematics and applied modeling. The cG-m structure will be able to better handle situations like this, which is the reason why this is the case.

It is possible that in the future, researchers may attempt to leverage the concepts presented in this article to develop computer tools and procedures. It is possible that these numbers might be used in a variety of contexts, including data analysis, signal processing, and machine learning, respectively, if they are complex. If we get a deeper understanding of the tightness, continuity, and convergence of extended G-type measure spaces, we could also be able to gain a better understanding of how geometric structures and complex analysis interact with one another. All of these areas are a reasonable next step for the current study, and they look like they will help both the academic and practical uses of fixed point theory in modern mathematical analysis.

Conflicts of Interest: The author declares that there are no conflicts of interest regarding the publication of this paper.

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