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Anti Synchronization of New Chaotic Systems via Adaptive Backstepping Control: An Application to Image Encryption

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Abstract. Based on an adaptive backstepping control strategy, the anti-synchronization phenomenon between two identical chaotic systems is proposed to achieve global and exponential anti-synchronization. The theoretical analysis is supported by Lyapunov-based stability proofs. Through numerical simulations, it is demonstrate that the synchronization errors vanish asymptotically, thus confirming the validity of the proposed scheme. Furthermore, the practical applicability of the methodology is illustrated through its application instance to image encryption, where the master system states are employed in an XOR based process to encrypt visual data. The obtained results both the theoretical of the methodology and its applicability in secure communications and related fields.

1. Introduction

Chaotic system synchronization has gradually acquired more attention as it may find its applications in secure communication, data encryption, or control areas [1]- [5]. Among various kinds of synchronization, anti-synchronization is a precious phenomenon whereby the slave system's state variables move in the opposite direction of the master system variables [6]- [12]

Backstepping design [13]- [20] has also extensively been utilized in the synchronization and control of nonlinear chaotic systems with parametric uncertainties and time varying dynamics. Tsinia [13] proposed a foundational framework for backstepping design of time varying nonlinear systems characterized by unknown parameters. Mazenc and Iggidr [16] generalized the method

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further by integrating bounded feedbacks within the backstepping design, providing higher stability margins as well as theoretical guarantees. Vaidyanathan [19] then suggested an adaptive backstepping synchronizer and controller of the Arneodo chaotic system with unknown parameters, yet another proof of the general applicability of the method for various chaotic models. Vaidyanathan et al. [20] also investigated anti-synchronization of WINDMI models via adaptive backstepping.

With the rapid growing trend in digital communication technologies, multimedia data transmission has become an essential aspect of modern human life, underpinning social interaction, education, information exchange, daily activities and global connectivity. Despite the fact that such spread in digital communication technologies has highlighted the necessity of establishing robust mechanisms for strong data security. This makes image data specialized encryption algorithm design an active current research area in the domain of information security [21]- [24].

Based on a new 3-D autonomous chaotic system presented in [25], we study in this work research, the anti-synchronization behavior between two identical systems of this sort using an adaptive backstepping control approach. The primary objective is to develop a control input that achieves global convergence of the synchronization error to zero. For better readability of the study, the reminder of this paper is arranged in the following sequence. Section 2 introduces the new novel chaotic system. The scheme of anti synchronization is investigated in Section 3. In Section 4, establishes the anti synchronization problem and derives an adaptive backstepping controller to guarantee global error convergence. Section 5 applies the proposed synchronization framework to provide secure image communication. Section 6 provides intensive simulation results. including histogram analysis, to validate the robustness and security of the proposed scheme. Finally, Section 7 summarizes the main finding of the work.

2. New Chaotic System

The proposed three-dimensional autonomous chaotic system [25] is described by the following set of differential equations:

$$\begin{aligned}
\dot{x} &= y \\
\dot{y} &= z \\
\dot{z} &= -x - y - z - 2.3z^2 + xy
\end{aligned}$$

where x, y and z represent the state variables of the system. The system exhibits complex nonlinear behavior and generates a chaotic attractor, as described in Figure 1.

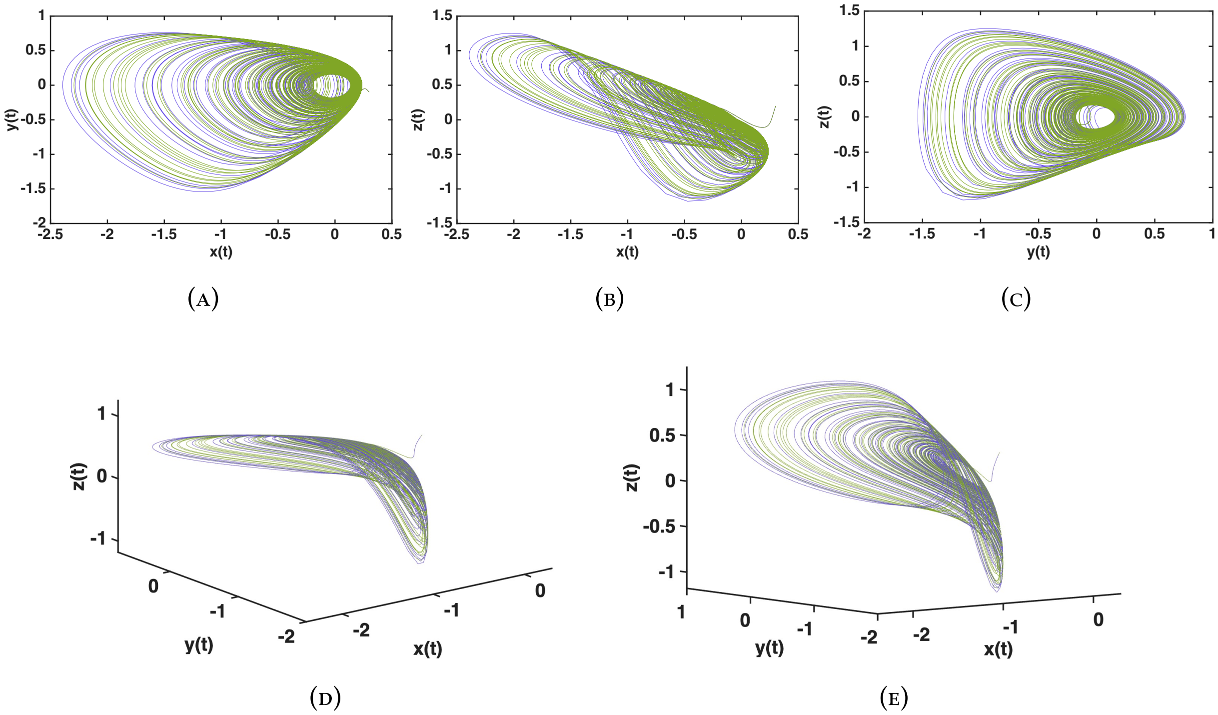


FIGURE 1. Chaotic system's phase space projections : (a) $x(t)$ vs. $y(t)$, (b) $x(t)$ vs. $z(t)$, (c) $y(t)$ vs. $z(t)$, (d) and (e) Plots of 3D chaotic attractors.

3. The Scheme of Anti Synchronization

Anti synchronization is achieved when the sum of the corresponding state variables of two coupled systems asymptotically converges to zero as time approaches infinity. The master and slave systems are defined as follows:

$$\dot{x} = g(x)$$
$$\dot{y} = h(y) + u(x, y, t)$$

where $x = (x_1, x_2, ..., x_n)^T$, $y = (y_1, y_2, ..., y_n)^T \in R^n$ denote the state vectors of the master and slave systems, respectively. The nonlinear functions $g, h : R^n \to R^n$ define the system dynamics, while $u(x, y, t)$ is the controller vector which will be designed to achieve anti-synchronization.

Definition 3.1. *It is said that the derive and response systems are achieved anti synchronization if the following equation hold:*

$$\lim_{t\to\infty} \|x(t) + y(t)\| = 0$$

4. Anti Synchronization Between Two Identical Systems

The chaotic system according to the master system can be demonstrated through the following equations:

$$\begin{aligned}\dot{x}_1 &= y_1\\ \dot{y}_1 &= z_1\\ \dot{z}_1 &= -x_1 - y_1 - z_1 - 2.3z_1^2 + x_1y_1\end{aligned} \tag{4.1}$$

where x_1, y_1 and z_1 are the state variables of the master system. Moreover, the corresponding slave system is given by:

$$\begin{aligned}\dot{x}_2 &= y_2\\ \dot{y}_2 &= z_2\\ \dot{z}_2 &= -x_2 - y_2 - z_2 - 2.3z_2^2 + x_2y_2 + U\end{aligned} \tag{4.2}$$

where x_2, y_2 and z_2 are the state variables of the slave system, and $u_i,\ (i = 1, 2, 3)$ are the controller to be designed such that the anti-synchronization is satisfied.

The error dynamical system can be expressed by:

$$\begin{aligned}e_1(t) &= x_1(t) + x_2(t) \Rightarrow \dot{e}_1 = e_2\\ e_2(t) &= y_1(t) + y_2(t) \Rightarrow \dot{e}_2 = e_3\\ e_3(t) &= z_1(t) + z_2(t) \Rightarrow \dot{e}_3 = -e_1 - e_2 - e_3 - 2.3(z_1^2 + z_2^2) + x_1y_1 + x_2y_2 + U\end{aligned}$$

Now, the primary objective is to design an adaptive backstepping controller U to achieve global and exponential anti synchronization.

Theorem 4.1. *The identical chaotic systems (1) and (2) achieve global and exponential anti synchronization under the adaptive backstepping control law defined as:*

$$U = -2(e_1 + 2e_2 + e_3) + 2.3(z_1^2 + z_2^2) - x_1y_1 - x_2y_2 \tag{4.3}$$

where $e_i,\ (i = 1, 2, 3)$ denote the synchronization errors.

Proof:

$$v_1(z) = \frac{1}{2}\omega_1^2, \quad \text{where } \omega_1 = e_1$$

Differentiate v_1:

$$\begin{aligned}\frac{dv_1}{dt} &= \omega_1\dot{\omega}_1 = e_1\dot{e}_1 = e_1e_2\\ \Rightarrow \frac{dv_1}{dt} &= -e_1^2 + e_1^2 + e_1e_2 = -\omega_1^2 + \omega_1(e_1 + e_2),\\ \Rightarrow \frac{dv_1}{dt} &= -\omega_1^2 + \omega_1\omega_2, \quad \text{where } \omega_2 = (e_1 + e_2)\end{aligned}$$

Next: define a quadratic Lyapunov function

$$v_2(\omega_1, \omega_2) = v(\omega_1) + \frac{1}{2}\omega_2^2 = \frac{1}{2}\omega_1^2 + \frac{1}{2}\omega_2^2$$

Differentiate v_2 we get:

$$\Rightarrow \frac{dv_2}{dt} = \omega_1\dot{\omega}_1 + \omega_2\dot{\omega}_2$$
$$= \omega_1 e_2 + \omega_2(\dot{e_1} + \dot{e_2})$$
$$= -\omega_1^2 - \omega_2^2 + (e_1 + e_2)\left[e_3 + 2e_1 + 2e_2\right],$$

$$\text{So;} \quad \frac{dv_2}{dt} = -\omega_1^2 - \omega_2^2 + \omega_2\omega_3 \quad \text{where } \omega_3 = e_3 + 2e_1 + 2e_2$$

Finally: define a quadratic function

$$V = v_2 + \frac{1}{2}\omega_3^2 \quad \text{where} \quad \omega_3 = 2e_1 + 2e_2 + e_3$$

Clearly: V is positive definite function on $\mathbb{R}^3$.

The time derivative of V is obtained as

$$\frac{dV}{dt} = \dot{v}_2 + \omega_3\dot{\omega}_3$$
$$= -\omega_1^2 - \omega_2^2 + \omega_2\omega_3 + \omega_3\dot{\omega}_3$$
$$= -\omega_1^2 - \omega_2^2 - \omega_3^2 + \omega_3\left[2e_1 + 4e_2 + 2e_3 - 2.3(z_1^2 + z_2^2) + x_1y_1 + x_2y_2 + U\right]$$

$$\text{let } U = -2(e_1 + 2e_2 + e_3) + 2.3(z_1^2 + z_2^2) - x_1y_1 - x_2y_2$$

substituting the value of (3) into $\dot{V}$

$$\Rightarrow \dot{V} = -\omega_1^2 - \omega_2^2 - \omega_3^2$$

which is a negative definite function on $\mathbb{R}^3$. Thus, the drive and the response systems are achieving the anti synchronization.

5. Numerical Simulations

To verify the theoretical findings and the synchronization controller 4.3 , numerical experiments are conducted using the proposed chaotic system for both the master and the slave configurations. The control law is implemented based on adaptive backstepping, the synchronization error signals $e_1(t), e_2(t), e_3(t)$ are defined as follows:

$$e_1 = x_1 + x_2, \quad e_2 = y_1 + y_2, \quad e_3 = z_1 + z_2.$$

The initial conditions are assumed to be:

$$[x_1(0), y_1(0), z_1(0)]^T = [0.1, 0.2, -0.2]^T \quad [x_2(0), y_2(0), z_2(0)]^T = [-0.5, 0.3, 0.4]^T$$

Figure 2 show the time evolution of these error signals under the designed control scheme. All error components converge asymptotically to zero, confirming that global and exponential anti-synchronization is successfully achieved between the two systems. The effectiveness of the control strategy is also reflected in the combined error plot in Figure 2, which demonstrates stable error suppression over time. Figure 3 depict the time responses of the time responses of the state variables $x(t), y(t), z(t)$ for both the master and slave systems governed by the proposed control law. It is clearly observed that the trajectories of the slave system evolve in opposite phase with respect to those of the master system which confirms the achievement of anti-synchronization.

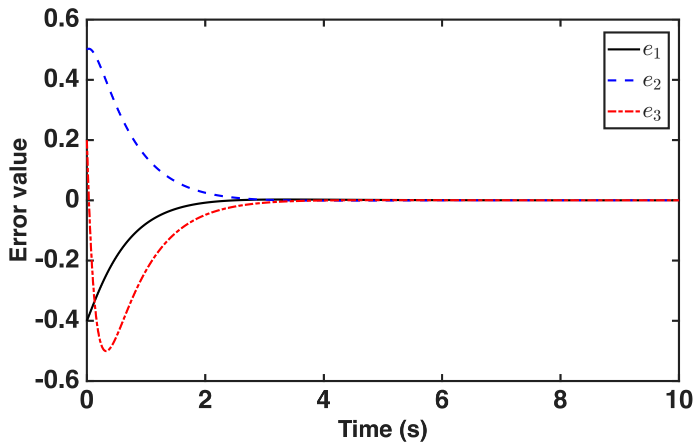


FIGURE 2. Time evolution of the synchronization error signals $e_1(t), e_2(t), e_3(t)$ under the adaptive backstepping control scheme.

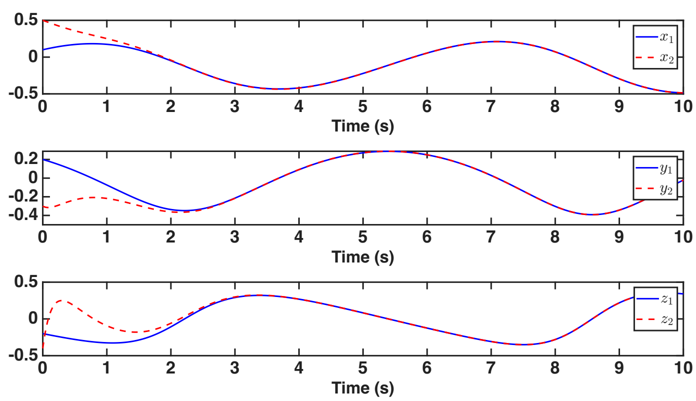


FIGURE 3. Time evolution of the master and slave systems showing the trajectories of state variables $x(t), y(t), z(t)$.

6. Image Encryption

The proposed image encryption scheme is founded exclusively on the chaotic *drive system*, which serves as the primary source of complexity in the design. By exploiting the inherent nonlinear characteristics of the drive system, highly complex and unpredictable sequences are generated and then employed as encryption key-streams. Due to the extreme sensitivity of the drive system to its initial conditions and system parameters, the generated sequences exhibit strong randomness and robustness.

(1) Image Preprocessing
- The original image I is first converted into an RGB matrix of 8-bit depth.
- If the input is grayscale, it is expanded into three channels to maintain consistency.

(2) Keystream Generation
- After discarding the transient behavior (burn-in), the trajectories of the master system are sampled at fixed intervals.
- A nonlinear mapping is applied to the master system to produce raw keystream values.
- The resulting K is matched to the number of pixel in I.

(3) Encryption Process
- The encrypted process employs a bitwise XOR operation between the original image I and the generated keystream K, producing the cipher image C

$$C = I \oplus K,$$

Producing the encrypted image C

(4) Decryption Process
- The decryption process is symmetric, given by

$$R = C \oplus K,$$

which restores the original image $R = I$.
- Both encrypted and decrypted outputs are stored in PNG format to avoid lossy compression.

(5) Performance Verification
To evaluate the security strength and robustness of the proposed encryption framework, histogram analysis is utilized as statistical tool.
- **Histogram Analysis:** A uniformly distributed histogram of the encrypted image indicates that pixel values are well-randomized, thus preventing adversaries from extracting meaningful statistical information. In our results, the histogram of the cipher image exhibited an almost uniform distribution, thereby confirming the effectiveness of the proposed method in resisting statistical attacks.
- **Decryption Accuracy:** The decrypted image is expected to be perfectly identical to the original, i.e.,

$$R = I,$$

confirming the correctness and reversibility of the scheme.

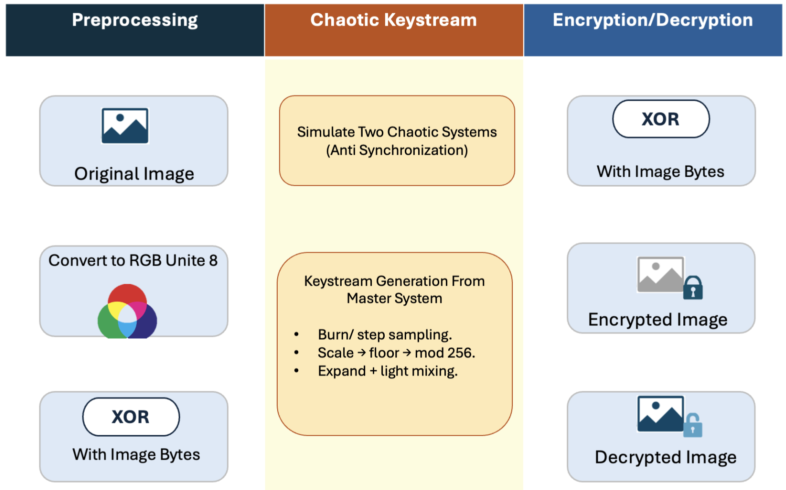


FIGURE 4. Schematic representation of the proposed chaotic image encryption framework

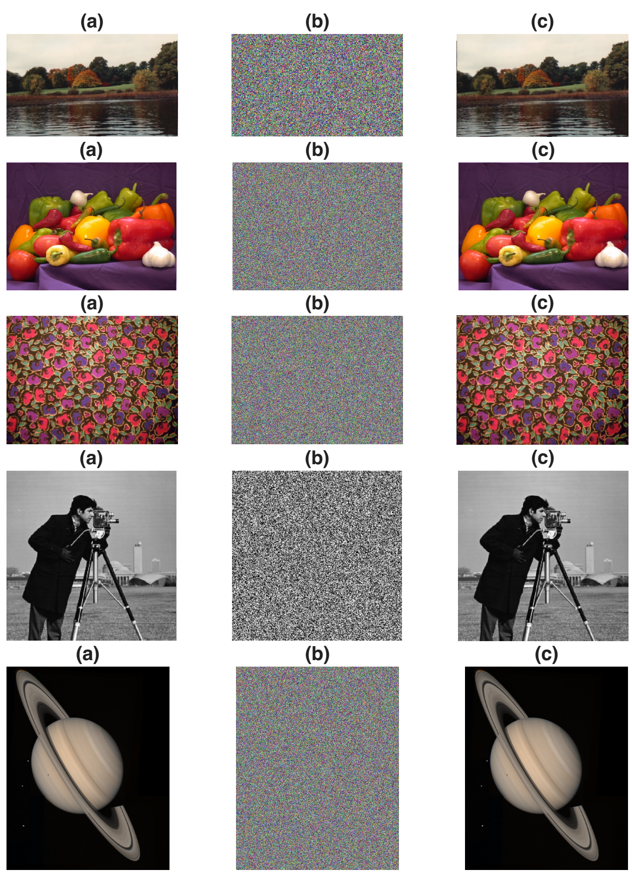
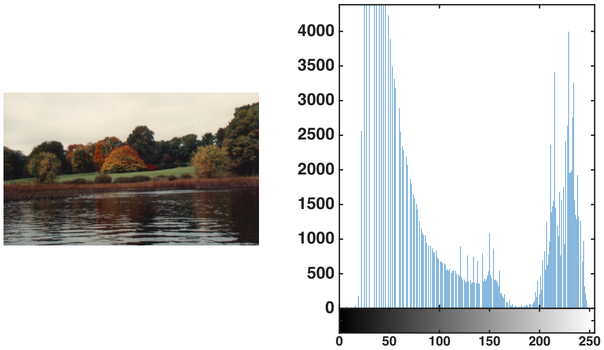
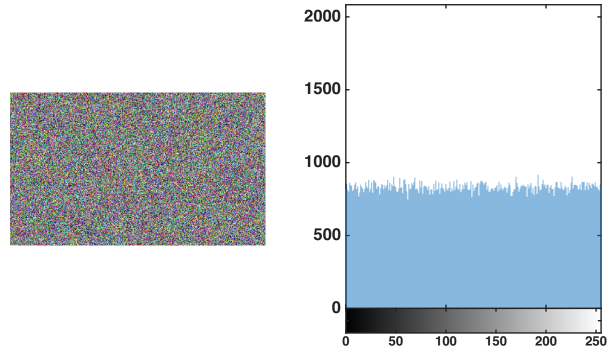


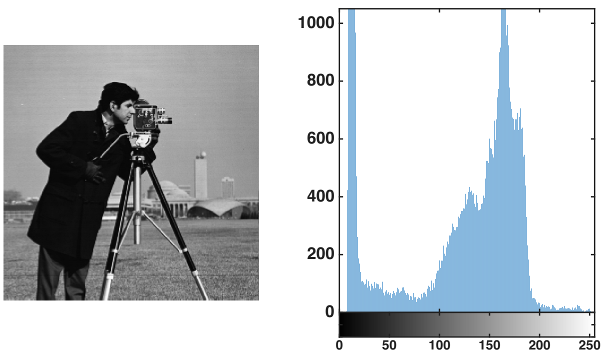
FIGURE 5. (a) Original image, (b) encrypted version, and (c) the perfectly recovered decrypted image using the proposed chaotic scheme.



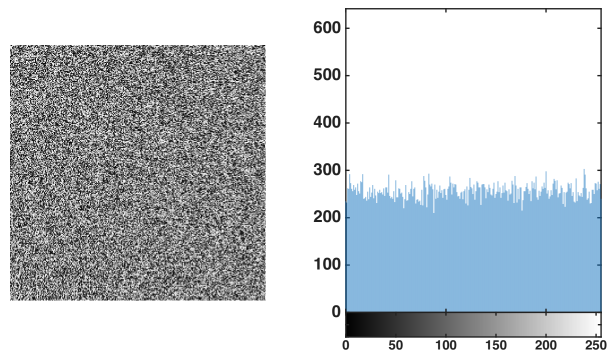
(A) Original image *autumn* and its histogram



(B) Encrypted image *autumn* and its histogram



(C) Original image *cameraman* and its histogram



(D) Encrypted image *cameraman* and its histogram

FIGURE 6. Comparison of original and encrypted images with their histograms. (a) Original *autumn*. (b) Encrypted *autumn*. (c) Original *cameraman*. (d) Encrypted *cameraman*.

7. CONCLUSION

In this paper, we addressed the anti-synchronization of a new chaotic system. With an adaptive backstepping control strategy, we designed a control input that derives the synchronization errors globally and exponentially to zero. Analytical derivations were confirmed by numerical simulations, which validated the system's controllability and the robustness of the synchronization method. Moreover, the scheme was also applied to image encryption by utilizing the master system states in an XOR based process. This technique can be extended to other chaotic systems and can find applications in secure data transmission and complex network control.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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