International Journal of Analysis and Applications

Fuzzifications of Almost Ideals in Ternary Semirings

Napaporn Sarasit[1], Ronnason Chinram[2], Amornrat Rattana[2,*]

[1]*Division of Mathematics, Faculty of Engineering, Rajamangala University of Technology Isan, Khon Kaen Campus, Khon Kaen 40000 Thailand*

[2]*Division of Computational Science, Faculty of Science, Prince of Songkla University, Hat Yai, Songkhla 90110 Thailand*

[*]*Corresponding author:* amornrat.r@psu.ac.th

Abstract. We introduce an almost ideal (shortly A-ideal) and a fuzzy A-ideal of a ternary semiring. We present that an A-ideal of a ternary semiring is formed by the union of A-ideals, but not by their intersection. We further study the defining properties of minimal fuzzy A-ideals in ternary semirings. Additionally, we relate the A-ideals of ternary semirings to their fuzzifications and find that an A-ideal is equivalent to its characteristic mapping, while a fuzzy A-ideal is equivalent to its support. Moreover, we demonstrate that its characteristic mapping is similar to that of a minimum A-ideal.

1. Introduction

A ring is a structure that uses addition and multiplication to generalize integer arithmetic, whereas a semiring is a more general concept that does not require additive inverses. A mathematical structure that extends the idea of a vector space is called a module over a ring, although the scalars originate from a ring rather than a field. A module over a ring is directly generalised to deal with the more flexible structure of semirings as a semimodule over a semiring. There are so many papers related to semirings and semimodules (see, for example [1]- [3]).

A ternary semiring is a semiring structure reformulated using a single ternary operation. We now explore this concept. A particular ternary algebraic system called triplexes was originally investigated in 1932 by Lehmer [4]. Then Lister [6] defined ternary rings as abelian groups that are closed and associative under the ternary multiplication which is distributive. Later, the concept of ternary semirings as commutative semigroups that are closed and associative under the ternary multiplication which is distributive, were introduced in [5]. This algebraic structure generalizes

Received: Oct. 29, 2025.

2020 *Mathematics Subject Classification.* 16Y60, 16Y80, 20N10.

Key words and phrases. A-ideals; fuzzy sets; fuzzy A-ideals; minimal; ternary semirings.

https://doi.org/10.28924/2291-8639-23-2025-326
© 2025 the author(s).
ISSN: 2291-8639

the concepts of rings, semirings, and ternary semirings. Ideal theory in ternary semirings explores the properties of various kinds of ideals, which are special subsets, for example, quasi-ideals and bi-ideals by Kar [7]; generalized semi-ideals by Daddi and Pawar [8]; prime quasi-ideals by Dubey and Anuradha [9]; irreducible principal T-ideals of non-positive integers by Chaudhari and Ingale [10]; quasi primary ideals by Yiarayong [11], etc. Moreover, Dutta and Kar [12] looked into the notion of regular ternary semirings.

The function that maps each element in a set to membership value between 0 and 1 is called a fuzzy subset that introduced by Zadeh [13]. The concepts of fuzzy subsets were then extended to other ideas. Fuzzy sets theory were applied to study in ternary semirings, for example, we can see in [14]- [19], etc.

Grosek and Satko [20] introduced an almost ideal (shortly A-ideal) of semigroups in 1980. Moreover, they also examined minimum, maximal and smallest A-ideals [21]. The notions of A-ideals and their fuzzifications were extended to define in ternary semigroups and explored by Suebsung et al. [22] in 2019. They introduced the concept of minimality of FA-ideals and thoroughly investigated their characteristics. This work laid the groundwork for further exploration of these ideals in more complex algebraic structures. We now focus on ternary semirings as an extension of these concepts.

The article begins with an introduction highlighting the importance of semirings and the reasons for concentrating on these ideas. In Section 2, we review basic notations and definitions for semigroups that provides the foundation for understanding ternary semirings and their ideals. Section 3 includes the definitions about A-ideal of ternary semiring and the investigations of their properties, including closure properties and examples. In Section 4, we introduce fuzzy A-ideals (shortly FA-ideals) and minimal fuzzy A-ideals (shortly MFA-ideals) and investigate their properties. Moreover, we also study some relationships between A-ideals and their fuzzifications.

2. Preliminaries

We will review some basic notations and definitions needed throughout this paper (cf. [14], [16]-[18]).

Firstly, we recall a ternary semiring. A *ternary semiring* is a nonempty set T with binary addition and ternary multiplication that satisfies the following conditions:

(1) $(rst)uv = r(stu)v = rs(tuv)$ (Associative Law)
(2) $(r+s)uv = ruv + suv$ (Right Distributive Law)
(3) $r(u+v)s = rus + rvs$ (Lateral Distributive Law)
(4) $rs(u+v) = rsu + rsv$ (Left Distributive Law)

for all $r, s, t, u, v \in T$.

From this point forward, a ternary semiring is always denoted by T unless otherwise stated.

For nonempty subsets U, V, W of T, define a product UVW by

$$UVW = \{uvw \mid u \in U, v \in V, w \in W\}.$$

Moreover, $U^3 := UUU$. The nonempty subset U of T is defined as an *almost ternary subsemiring* (shortly A-ternary subsemiring) of T if $(U+U)\cap U \neq \emptyset$ and $U^3 \cap U \neq \emptyset$.

Let U be an additive closed nonempty subset of T . Then U is referred to as a *ternary subsemiring* of T if $U^3 \subseteq U$. Furthermore, U is a *left ideal* of T if $TTU \subseteq U$; *a middle ideal* of T if $TUT \subseteq U$; and a *right ideal* of T if $UTT \subseteq U$. If U satisfies all three conditions, then U is called an *ideal* of T.

For fuzzy subsets σ and v of T, define $\sigma \cap v$ and $\sigma \cup v$ by fuzzy subsets of T as follows:

$$(\sigma \cap v)(t) = \min\{\sigma(t), v(t)\}$$

and

$$(\sigma \cup v)(t) = \max\{\sigma(t), v(t)\}$$

for all $t \in T$. If $\sigma(t) \leq v(t)$ for all $t \in T$, we say that $\sigma \subseteq v$.

Next, the support $\mathcal{SP}(\sigma)$ of σ is defined by

$$\mathcal{SP}(\sigma) = \{t \in T \mid \sigma(t) \neq 0\}.$$

Define the characteristic mappings λ_U and λ_t of $U \subseteq T$ and $t \in T$, respectively, as follows:

$$\lambda_U(t) = \begin{cases} 1 & \text{if } t \in U, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } \lambda_t(x) = \begin{cases} 1 & \text{if } x = t, \\ 0 & \text{otherwise.} \end{cases}$$

We notice that we write $T := \lambda_T$ that is $T(t) = 1$ for all $t \in T$.

Let σ, v and ω be any three fuzzy subsets of T. Fuzzy subsets $\sigma + v$ and $\sigma \circ v \circ \omega$ are defined as follows:
For any $t \in T$,

$$(\sigma + v)(t) = \begin{cases} \sup\limits_{t=a+b} \{\min\{\sigma(a), v(b)\}\} & \text{if } t = a+b \text{ for some } a, b \in T, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$(\sigma \circ v \circ \omega)(x) = \begin{cases} \sup\limits_{t=abc} \{\min\{\sigma(a), v(b), \omega(c)\}\} & \text{if } t = abc \text{ for some } a, b, c \in T, \\ 0 & \text{otherwise.} \end{cases}$$

For a fuzzy subset σ of T under the condition $\sigma + \sigma \subseteq \sigma$, σ is a *fuzzy left ideal* (shortly FL-ideal) of T if $\sigma(rst) \geq \sigma(t)$ for all $r, s, t \in T$. Similarly, for all $r, s, t \in T$, σ is a *fuzzy middle ideal* (shortly FM-ideal) and a *fuzzy right ideal* (shortly FR-ideal) if $\sigma(rst) \geq \sigma(s)$ and $\sigma(rst) \geq \sigma(r)$, respectively. A fuzzy subset σ is a *fuzzy ideal* (shortly F-ideal) of T if it is simultaneously a fuzzy left, fuzzy middle and fuzzy right ideal of T.

Theorem 2.1. Let σ be a fuzzy subset of T such that $\sigma + \sigma \subseteq \sigma$. Then

(1) σ is an FL-ideal (FM-ideal, FR-ideal, respectively) of T if and only if $T \circ T \circ \sigma \subseteq \sigma$ ($T \circ \sigma \circ T \subseteq \sigma$, $\sigma \circ T \circ T \subseteq \sigma$, respectively).

(2) σ is an F-ideal of T if and only if $T \circ T \circ \sigma \subseteq \sigma$, $T \circ \sigma \circ T \subseteq \sigma$ and $\sigma \circ T \circ T \subseteq \sigma$.

3. A-ideals

In this section, we first introduce A-ideal, left A-ideal, middle A-ideal and right A-ideal of ternary semirings.

Definition 3.1. *A nonempty subset U of T such that $(U+U) \cap U \neq \emptyset$ is called a* left A-ideal *(shortly LA-ideal) [*a middle A-ideal *(shortly MA-ideal) and* a right A-ideal *(shortly RA-ideal), respectively] of T if for all $t \in T$, $ttU \cap U \neq \emptyset$ [$tUt \cap U \neq \emptyset$ and $Utt \cap U \neq \emptyset$, respectively].*

If U is a LA-ideal, MA-ideal and RA-ideal of T, then U is called an A-ideal of T.

Example 3.1. Let $\mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ be a ternary semiring under usual addition and ternary multiplication modulo 6. We can see that $U = \{\bar{0}, \bar{1}, \bar{3}\}$ is an A-ternary subsemiring of T, but it is not a ternary subsemiring, i.e. $(U+U) \cap U \neq \emptyset$ and $U^3 \cap U \neq \emptyset$ but $(U+U) \nsubseteq U$.

Since $(\bar{0} \cdot \bar{0} \cdot U) \cap U = \{\bar{0}\}$, $(\bar{1} \cdot \bar{1} \cdot U) \cap U = \{\bar{0}, \bar{1}, \bar{3}\}$, $(\bar{2} \cdot \bar{2} \cdot U) \cap U = \{\bar{0}\}$, $(\bar{3} \cdot \bar{3} \cdot U) \cap U = \{\bar{0}, \bar{3}\}$, $(\bar{4} \cdot \bar{4} \cdot U) \cap U = \{\bar{0}\}$ and $(\bar{5} \cdot \bar{5} \cdot U) \cap U = \{\bar{0}, \bar{1}, \bar{3}\}$. Thus U is an LA-ideal of $\mathbb{Z}_6$. Since $\bar{2} \cdot \bar{2} \cdot \bar{1} = \bar{4} \notin U$, U is not a left ideal of $\mathbb{Z}_6$.

LA-ideals need not be left ideals as demonstrated by the previous example.

Theorem 3.1. *Let U and V be a subset of T such that $U \subseteq V$. If U is an LA-ideal (MA-ideal, RA-ideal, A-ideal, respectively) of T, then V is also.*

Proof. Assume that U is an LA-ideal of T and let $t \in T$. Let V be a subset of T where $U \subseteq V$. Then $(U+U) \cap U \neq \emptyset$. So $(V+V) \cap V \neq \emptyset$. Since U is an LA-ideal of T, $tta \in ttU \cap U$ for some $a \in U$. The assumption yields $tta \in ttV \cap V$ which means $ttV \cap V \neq \emptyset$. Thus V is an LA-ideal of T.

We can prove in the same manner for the others. □

Corollary 3.1. *Let U and V be nonempty subsets of T. If U is an LA-ideal (MA-ideal, RA-ideal, A-ideal, respectively) of T, then $U \cup V$ is also.*

Proof. We assume that U is an LA-ideal of T. Since $U \subseteq U \cup V$, by Theorem 3.1, $U \cup V$ is an LA-ideal of T. This argument can be applied to prove others. □

Corollary 3.2. *Let U and V be nonempty subsets of T. If U, V are LA-ideals (MA-ideals, RA-ideals, A-ideals, respectively) of T, then $U \cup V$ is also.*

Proof. The proof is completed by Theorem 3.1. □

Lemma 3.1. *Assume that T is not a singleton.*

1. *The set T has no proper LA-ideal (MA-ideal, RA-ideal, respectively) if and only if for each $a \in T$, we have $t_a t_a (T - \{a\}) = \{a\}$ ($t_a (T - \{a\}) t_a = \{a\}$, $(T - \{a\}) t_a t_a = \{a\}$, respectively) for some $t_a \in T$.*
2. *The set T has no proper A-ideal if and only if for each $a \in T$, we have $t_a t_a (T - \{a\}) = \{a\}$ or $t_a (T - \{a\}) t_a = \{a\}$ or $(T - \{a\}) t_a t_a = \{a\}$ for some $t_a \in T$.*

Proof. (1) Assume that T has no proper LA-ideal. We let an element $a \in T$. It obtains from the assumption that $T - \{a\}$ is not an LA-ideal of T and $T - \{a\} \neq \emptyset$. Therefore $t_a t_a (T - \{a\}) \cap (T - \{a\}) = \emptyset$ for some $t_a \in T$. Hence $t_a t_a (T - \{a\}) = \{a\}$. Let us now prove the converse. Suppose that for all $a \in T$, we obtain $t_a t_a (T - \{a\}) = \{a\}$ for some $t_a \in T$. Then $t_a t_a (T - \{a\}) \cap (T - \{a\}) = \{a\} \cap (T - \{a\}) = \emptyset$. This means that $T - \{a\}$ is not an LA-ideal of T. We can conclude that T has no proper LA-ideals. Others can be proved similarly.

(2) Assume that T has no proper A-ideal. It implies that T has no proper LA-ideal or has no proper MA-ideal or has no proper RA-ideal. It follows from (1) that for any element a of T, there is an element $t_a \in T$ that satisfies the conditions $t_a t_a (T - \{a\}) = \{a\}$ or $(T - \{a\}) t_a t_a = \{a\}$ or $t_a (T - \{a\}) t_a = \{a\}$. Conversely, It obtains from (1) that T has no proper LA-ideal or has no proper MA-ideal or has no proper RA-ideal. Therefore T has no A-ideal. □

Definition 3.2. *An element $a \in T$ is called a ternary idempotent (shortly t-idempotent) if $a^3 = a$.*

Theorem 3.2. *Let T be not a singleton and let $a \in T$.*

(1) *If T has no proper LA-ideal (MA-ideal, RA-ideal, respectively), then a is a t-idempotent or $a^3 = a^7$.*
(2) *If T has no proper A-ideal, then either a is a t-idempotent or $a^3 = a^7$.*

Proof. (1) It follows by Lemma 3.1(1) that $t_a t_a (T - \{a\}) = \{a\}$ for some $t_a \in T$. Assume that $a^3 \neq a$. Then $a^3 \in T - \{a\}$ which means $t_a t_a a^3 = a$.

Case $t_a = a$: We have $a = a^5$. This implies that $a^3 = a^7$.

Case $t_a \neq a$: Thus $t_a \in T - \{a\}$. We now have $t_a t_a t_a = a$. Suppose that $t_a t_a a = a$, so $a^3 = t_a t_a a^3 = a$ that is a contradiction. Hence $t_a t_a a \neq a$. It follows that $a = t_a t_a (t_a t_a a) = t_a (t_a t_a t_a) a = t_a a a$. Thus $a^3 = (t_a a a) a a = (t_a (t_a a a) a) a a = (t_a (t_a (t_a a a) a) a) a a = (t_a t_a t_a) a a^5 = a a a^5 = a^7$. Others can be proved as in the same manner.

(2) can be proved by (1). □

Corollary 3.3. *Let T be a ternary semiring and T be not a singleton set. Let $a \in T$. If a is not a t-idempotent and $a^3 \neq a^7$, then $T - \{a\}$ is an A-ideal of T.*

Proof. The proof is completed by Theorem 3.2. □

4. Fuzzy A-ideals

This section includes the definitions of fuzzy almost ideals (shortly FA-ideals) of ternary semirings and properties of them.

Definition 4.1. *Let σ be a fuzzy subset of T such that $(\sigma + \sigma) \cap \sigma \neq 0$. σ is called a* fuzzy left A-ideal *(simply as FLA-ideal) [*fuzzy middle A-ideal *(simply as FMA-ideal) and* fuzzy right A-ideal *(simply as FRA-ideal), respectively] of T if $(\lambda_t \circ \lambda_t \circ \sigma) \cap \sigma \neq 0$ [$(\lambda_t \circ \sigma \circ \lambda_t) \cap \sigma \neq 0$ and $(\sigma \circ \lambda_t \circ \lambda_t) \cap \sigma \neq 0$, respectively] for all $t \in T$.*

If σ is an FLA-ideal, FMA-ideal and FRA-ideal of T, then σ is called a FA-ideal of T.

Example 4.1. Let us examine the ternary semiring $\mathbb{Z}_6 = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5}\}$ under usual addition and usual ternary multiplication modulo 6. Let $\sigma : \mathbb{Z}_6 \to [0,1]$ defined by $\sigma(\bar{0}) = 1, \sigma(\bar{1}) = 1, \sigma(\bar{2}) = 0.6, \sigma(\bar{3}) = 0.3, \sigma(\bar{4}) = 0.5, \sigma(\bar{5}) = 0.8$. It is obvious that $(\sigma+\sigma)\cap\sigma \neq 0$. We now consider each t.

If $t = \bar{0}$, then $\bar{0} = \bar{0}\cdot\bar{0}\cdot\bar{0}$. Thus $[(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma](\bar{0}) = 1$. So $(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma \neq 0$.

If $t = \bar{1}$, then $\bar{1} = \bar{1}\cdot\bar{1}\cdot\bar{1}$. Thus $[(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma](\bar{1}) = 1$. So $(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma \neq 0$.

If $t = \bar{2}$, then $\bar{4} = \bar{2}\cdot\bar{2}\cdot\bar{1}$. Thus $[(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma](\bar{4}) = 0.5$. So$(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma \neq 0$.

If $t = \bar{3}$, then $\bar{3} = \bar{3}\cdot\bar{3}\cdot\bar{1}$. Thus $[(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma](\bar{3}) = 0.3$. So $(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma \neq 0$.

If $t = \bar{4}$, then $\bar{4} = \bar{4}\cdot\bar{4}\cdot\bar{1}$. Thus $[(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma](\bar{4}) = 0.5$. So $(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma \neq 0$.

If $t = \bar{5}$, then $\bar{5} = \bar{5}\cdot\bar{5}\cdot\bar{1}$. Thus $[(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma](\bar{5}) = 0.8$. So $(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma \neq 0$.

We now obtain that $(\lambda_t \circ \lambda_t \circ \sigma)\cap\sigma \neq 0$ for all $t \in \mathbb{Z}_6$. Thus σ is an FLA-ideal of $\mathbb{Z}_6$.

Consider $t = \bar{3} = \bar{3}\cdot\bar{3}\cdot\bar{1}$. Then $(\lambda_t \circ \lambda_t \circ \sigma)(\bar{3}) = 1$, but $f(\bar{3}) = 0.3$. This implies that $(\lambda_t \circ \lambda_t \circ \sigma) \nsubseteq \sigma$. Thus σ is not an FL-ideal of $\mathbb{Z}_6$. This demonstrates that, in general, an FLA-ideal is not necessarily an FL-ideal.

Theorem 4.1. *A nonempty subset S of T is an LA-ideal (MA-ideal, RA-ideal, A-ideal, respectively) of T if and only if λ_S is an FLA-ideal (FMA-ideal, FRA-ideal, FA-ideal, respectively) of T.*

Proof. Assume S is an LA-ideal of T. So $(S+S)\cap S \neq \emptyset$. Let $x \in (S+S)\cap S$. Then $x = y+z$ for some $y,z \in S$. This implies that $\lambda_S(x) = 1$ and $(\lambda_S+\lambda_S)(x) = (\lambda_S+\lambda_S)(y+z) = \min\{\lambda_S(y), \lambda_S(z)\} = 1$. Hence $[(\lambda_S+\lambda_S)\cap\lambda_S](x) \neq 0$ which means $(\lambda_S+\lambda_S)\cap\lambda_S \neq 0$. In addition, there exists $x \in ttS\cap S$ for all $t \in T$. We obtain $x = tta$ for some $a \in S$. Thus $(\lambda_t \circ \lambda_t \circ \lambda_S)(x) \neq 0$ and $\lambda_S(x) \neq 0$. This implies that $[(\lambda_t \circ \lambda_t \circ \lambda_S)\cap\lambda_S](x) = \min\{(\lambda_t \circ \lambda_t \circ \lambda_S)(x), \lambda_s(x)\} \neq 0$. Hence λ_S is an FLA-ideal of T.

Conversely, suppose that λ_S is an FLA-ideal of T. Since $(\lambda_S+\lambda_S)\cap\lambda_S \neq 0$, there exists $x \in S$ such that $(\lambda_S+\lambda_S)(x) \neq 0$ and $\lambda_S(x) \neq 0$. Hence there are $y,z \in S$ such that $x = y+z$ in which $\lambda_S(y) \neq 0$ and $\lambda_S(z) \neq 0$. Thus $\lambda_S(y) = 1$ and $\lambda_S(z) = 1$. This means $y,z \in s$. Then $x \in (S+S)\cap S$. Hence $(S+S)\cap S \neq \emptyset$. Moreover, for all $t \in T$, $[(\lambda_t \circ \lambda_t \circ \lambda_S)\cap\lambda_S](x) = \min\{(\lambda_t \circ \lambda_t \circ \lambda_S)(x), \lambda_S(x)\} \neq 0$ for some $x \in T$. Thus there exists $x \in ttS\cap S$ which implies that $ttS\cap S \neq \emptyset$. Therefore S is an LA-ideal of T.

Others can be proved in the same argument. □

Theorem 4.2. *A nonzero fuzzy subset σ of T is an FLA-ideal (FMA-ideal, FRA-ideal, FA-ideal, respectively) of T if and only if $\mathcal{SP}(\sigma)$ is an LA-ideal (MA-ideal, RA-ideal, A-ideal, respectively) of T.*

Proof. Suppose that σ is an FLA-ideal of T. Then $(\sigma+\sigma)\cap\sigma \neq 0$. This implies that $\sigma(x) \neq 0$ and $(\sigma+\sigma)(x) \neq 0$ for some $x \in S$. Then there are $y,z \in S$ such that $x = y+z$ with $\sigma(y) \neq 0$ and $\sigma(z) \neq 0$. So $y,z \in \mathcal{SP}(\sigma)$. Thus $x = y+z \in (\mathcal{SP}(\sigma)+\mathcal{SP}(\sigma))\cap\mathcal{SP}(\sigma)$. We can conclude that $(\mathcal{SP}(\sigma)+\mathcal{SP}(\sigma))\cap\mathcal{SP}(\sigma) \neq \emptyset$. Now let $t \in T$. By assumption, there are $x,a \in T$ such that $x = tta$, $(\lambda_t \circ \lambda_t \circ \sigma)(x) \neq 0$ and $\sigma(x) \neq 0$. This implies that $x,a \in \mathcal{SP}(\sigma)$, $(\lambda_t \circ \lambda_t \circ \lambda_{\mathcal{SP}(\sigma)})(x) \neq 0$ and $\lambda_{\mathcal{SP}(\sigma)}(x) \neq 0$. Therefore $[(\lambda_t \circ \lambda_t \circ \lambda_{\mathcal{SP}(\sigma)} \cap \lambda_{\mathcal{SP}(\sigma)}](x) \neq 0$. Thus $\lambda_{\mathcal{SP}(\sigma)}$ is an FLA-ideal of T. By Theorem 4.1(1), we can conclude that $\mathcal{SP}(\sigma)$ is an LA-ideal of T.

Conversely, assume that $\mathcal{SP}(\sigma)$ is an LA-ideal of T. Since $(\mathcal{SP}(\sigma) + \mathcal{SP}(\sigma)) \cap \mathcal{SP}(\sigma) \neq \emptyset$, $x \in (\mathcal{SP}(\sigma) + \mathcal{SP}(\sigma))$ and $x \in \mathcal{SP}(\sigma)$ for some $x \in S$. So $(\sigma + \sigma)(x) \neq 0$ and $\sigma(x) \neq 0$. Then $(\sigma + \sigma) \cap f \neq 0$. Moreover, we let $t \in T$. Then there exist $x, a \in \mathcal{SP}(\sigma)$ such that $x = tta$. This implies that $(\lambda_t \circ \lambda_t \circ \sigma)(x) \neq 0$ and $\sigma(x) \neq 0$. Thus $(\lambda_t \circ \lambda_t \circ \sigma) \cap \sigma \neq 0$. Hence σ is an FLA-ideal of T.

Others can be proved in the same argument. □

Definition 4.2. *An LA-ideal S of T is minimal if for every LA-ideal U of T, $U \subseteq S$ implies that $U = S$.*

We denote a minimal LA-ideal by M-LA-ideal. M-MA-ideal, M-RA-ideal, and M-A-ideal are defined in a similar manner.

Definition 4.3. *A FLA-ideal σ of T is minimal if for every FLA-ideal v of T, $v \subseteq \sigma$ implies that $\mathcal{SP}(v) = \mathcal{SP}(\sigma)$.*

We denote a minimal FLA-ideal by M-FLA-ideal. M-FMA-ideal, M-FRA-ideal, and M-FA-ideal are defined in a similar manner.

Theorem 4.3. *A nonempty subset S of T is an M-LA-ideal (M-MA-ideal, M-RA-ideal, M-A-ideal, respectively) of T if and only if λ_S is an M-FLA-ideal (M-FMA-ideal, M-FRA-ideal, M-FA-ideal, respectively) of T.*

Proof. Let S be an M-LA-ideal of T. It follows by Theorem 4.1 that S is an LA-ideal of T. Then $\mathcal{SP}(v) \subseteq \mathcal{SP}(\lambda_S) = S$. Since $v \subseteq \lambda_{\mathcal{SP}(v)}$,

$$(\lambda_S \circ \lambda_S \circ \lambda_S) \cap v \subseteq (\lambda_S \circ \lambda_S \circ \lambda_{\mathcal{SP}(v)}) \cap \lambda_{\mathcal{SP}(v)}.$$

Thus $\lambda_{\mathcal{SP}(v)}$ is an FLA-ideal of T. By Theorem 4.1, thus S is an LA-ideal of T. Since S is minimal, $\mathcal{SP}(v) = S = \mathcal{SP}(\lambda_S)$. Therefore λ_S is minimal.

Let us now prove the converse. Assume that λ_S is an M-FLA-ideal of T. It follows by Theorem 4.1 that S is an LA-ideal of T. To show that S is minimal, let $L \subseteq S$ be an LA-ideal of T. By Theorem 4.1, λ_L is an LA-ideal of T and $\lambda_L \subseteq \lambda_S$. Thus $L = \mathcal{SP}(\lambda_L) = \mathcal{SP}(\lambda_S) = S$. Hence S is minimal.

Others can be proved in the same argument. □

Conclusions

In this paper, we have first introduced the notion of A-ideals of ternary semirings. We also discussed the union and the intersection of two A-ideals of a ternary semiring. We found that the union of A-ideals is an A-ideal of a ternary semiring T, but the intersection of A-ideals is not an A-ideal of a ternary semiring T. This properties is also true for LA-ideals, MA-ideals and RA-ideals. Moreover, we presented FA-ideals and M-FA-ideals of ternary semirings. The relationships between A-ideal of ternary semirings and their fuzzifications were provided. We showed that an A-ideal of ternary semiring is equivalent to its characteristic mapping, a FA-ideal of ternary subsemiring is equivalent to its support, and an M-A-ideal of ternary semiring is equivalent to its characteristic mapping.

For future studies, the readers can extend the concept of A-ideal of ternary semirings to explore other types of ideals or to investigate A-ideals and FA-ideals of n-ary semirings.

Acknowledgment: Funding for this research was provided by Rajamangala University of Technology Isan. Contract No. ENG4/66.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] P. Jipsen, S. Vannucci, Injective and Projective Semimodules Over Involutive Semirings, J. Algebr. Appl. 21 (2021), 2250182. https://doi.org/10.1142/s0219498822501821.

[2] I. Chajda, H. Länger, Semimodules Over Commutative Semirings and Modules Over Unitary Commutative Rings, Linear Multilinear Algebr. 70 (2020), 1329–1344. https://doi.org/10.1080/03081087.2020.1760192.

[3] A.J. Naeemah, S.A. Al-Saadi, Strongly Extending Semimodules over a Semiring, J. Discret. Math. Sci. Cryptogr. 27 (2024), 1677–1687. https://doi.org/10.47974/jdmsc-2009.

[4] D.H. Lehmer, A Ternary Analogue of Abelian Groups, Am. J. Math. 54 (1932), 329–338. https://doi.org/10.2307/2370997.

[5] T.K. Dutta, S. Kar, On Regular Ternary Semirings, in: Advances in Algebra, Proceedings of the ICM Satellite Conference in Algebra and Related Topics, World Scientific, pp. 343–355, (2003). https://doi.org/10.1142/9789812705808_0027.

[6] W.G. Lister, Ternary Rings, Trans. Am. Math. Soc. 154 (1971), 37–55. https://doi.org/10.2307/1995425.

[7] S. Kar, On Quasi-Ideals and Bi-Ideals in Ternary Semirings, Int. J. Math. Math. Sci. 2005 (2005), 3015–3023. https://doi.org/10.1155/ijmms.2005.3015.

[8] V.R. Daddi, Y.S. Pawar, Generalized Semi-Ideals in Ternary Semirings, Novi Sad J. Math. 41 (2011), 81–87.

[9] M.K. Dubey, Anuradha, A Note on Prime Quasi-Ideals in Ternary Semirings, Kragujevac J. Math. 37 (2013), 361–367.

[10] J.N. Chaudhari, K.J. Ingale, Ideals in the Ternary Semiring of Non-Positive Integers, Bull. Malays. Math. Sci. Soc. 37 (2014), 1149–1156.

[11] P. Yiarayong, On Weakly Completely Quasi Primary and Completely Quasi Primary Ideals in Ternary Semirings, Commun. Korean Math. Soc. 31 (2016), 657–665. https://doi.org/10.4134/ckms.c150164.

[12] T.K. Dutta, S. Kar, A Note on Regular Ternary Semirings, Kyungpook Math. J. 47 (2006), 357–365.

[13] L. Zadeh, Fuzzy Sets, Inf. Control. 8 (1965), 338–353. https://doi.org/10.1016/s0019-9958(65)90241-x.

[14] J. Kavikumar, A.B. Khamis, Y.B. Jun, Fuzzy Bi-Ideals in Ternary Semirings, Int. J. Comput. Math. Sci. 3 (2009), 164–168.

[15] D. Krishnaswamy, T. Anitha, Fuzzy Prime Ideals in Ternary Semiring, Ann. Fuzzy Math. Inform. 7 (2014), 755–763.

[16] S. Bashir, R. Mazhar, H. Abbas, M. Shabir, Regular Ternary Semirings in Terms of Bipolar Fuzzy Ideals, Comput. Appl. Math. 39 (2020), 319. https://doi.org/10.1007/s40314-020-01319-z.

[17] R. Chinram, S. Malee, L-Fuzzy Ternary Subsemirings and L-Fuzzy Ideals in Ternary Semirings, IAENG Int. J. Applied Math. 40 (2010), 3.

[18] S. Malee, R. Chinram, k-Fuzzy ideals of ternary semirings, Int. J. Comput. Math. Sci. 4 (2010), 206–210.

[19] V.R. Daddi, Almost Bi-Ideals and Fuzzy Almost Bi-Ideals of Ternary Semigroups, Ann. Commun. Math. 7 (2024), 100–107. https://doi.org/10.62072/acm.2024.070203.

[20] O. Grošek, L. Satko, A New Notion in the Theory of Semigroup, Semigroup Forum 20 (1980), 233–240. https://doi.org/10.1007/bf02572683.

[21] O. Grošek, L. Satko, Smallest A-Ideals in Semigroups, Semigroup Forum 23 (1981), 297–309. https://doi.org/10.1007/bf02676654.

[22] S. Suebsung, K. Wattanatripop, R. Chinram, A-Ideals and Fuzzy A-Ideals of Ternary Semigroups, Songklanakarin J. Sci. Technol. 41 (2019), 299–304.