

## Analytical Study for Stagnation-Point Flow and Heat Transfer of MHD Nanofluid Over a Stretching Sheet in Porous Medium via Modified Adomian Decomposition Method

D. M. Mostafa<sup>1,8</sup>, A. A. Gaber<sup>2,\*</sup>, G. Zaman<sup>3</sup>, Z. Ullah<sup>3</sup>, H. Ahmed<sup>4,5,6</sup>, Tawfik M. Younis<sup>7</sup>

<sup>1</sup>*Department of Mathematics, College of Science, Qassim University, P. O. Box 6644, Buraidah 51452, Saudi Arabia*

<sup>2</sup>*Department of Mathematics, College of Science Al-Zulfi, Majmaah University, Majmaah 11952, Saudi Arabia*

<sup>3</sup>*Department of Mathematics, University of Malakand Chakdara, Pakistan*

<sup>4</sup>*Operational Research Center in Healthcare, Near East University, Nicosia/TRNC, 99138 Mersin 10, Turkey*

<sup>5</sup>*Sustainability Competence Centre, Széchenyi István University, Egyetem tér 1, H-9026 Győr, Hungary*

<sup>6</sup>*Department of Mathematics, College of Science, Korea University, 145 Anam-ro, Seongbuk-gu, Seoul 02841, South Korea*

<sup>7</sup>*Department of Business Administration, College of Business Administration, Majmaah University, Majmaah 11952, Saudi Arabia*

<sup>8</sup>*Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Hiliopolis, Cairo, Egypt*

\*Corresponding author: a.gaber@mu.edu.sa, aagaber6@gmail.com

**Abstract.** In the current study, we investigate the stagnation-point flow of a MHD nanofluid toward stretching sheet in porous media with suction or injection. Whereas, the contribution of the velocity, temperature, and nanoparticle distributions to identify the advantages or disadvantages that nanoparticles like bacteria, microbes and viruses, cause in the flow stretching sheet is what makes this work significant. A new procedure is suggested for the analytical treatment of the governing system of partial differential equations, where the boundary condition at infinity is converted from the unbounded domain to the bounded domain by using some transformations and then modified adomian decomposition method is utilized. The effects of parameters (porous medium, magnetic number, surface heat flux, suction or injection and Prandtl number) on velocity, temperature and concentration profiles are shown graphically and analyzed. Finally, we compared our obtained results with the other techniques used before in literature.

Received: Oct. 22, 2025.

2020 *Mathematics Subject Classification.* 76W05.

*Key words and phrases.* nanofluid; stagnation point; MHD; adomian decomposition method.

## 1. INTRODUCTION

The investigation of boundary layer slipt and melting heat transfer over stretching sheet has significant importance in Manufacturing and industrial procedures. Many researchers, such as Carragher [1], Abu-Sitta[2], Ishak [3], Bhattacharyya [4] and Zamman [5] have studied the difficulty of stretching sheet by employing varied effectance, such as heat flux, permeability and steadiness characteristics, etc.

Many important industrial applications can be found by studying heat transfer [6 – 20], like casting and welding. Melting permafrost and warming frozen ground are also important applications. There are a lot of things in engineering that use water, oil, and ethylene glycol to move heat, but they don't move heat very well. This can be a big problem if you want to improve the efficiency and compactness of things like heat exchangers or electronic machines. For that reason, researchers have looked into how to make fluids more conductive of heat by adding small solid particles to them. These types of fluids, which have small solid particles, should be able to transfer more heat than other fluids.

lately, Bhattacharyya and et. [21] studied heat transfer of a non-isothermal MHD stagnation-point flow of electrically conducting fluid and shrinking/stretching sheet in a permeable medium. That mean, he has not studied the mass transfer.

The ADM was built by George Adomian and has recently become quite well-known in related fields [22]. ADM has been used to solve differential and integral linear and non-linear equations in the fields of mathematics, physics, biochemistry, and molecular biology in a number of research that have been published to far to show its viability. New developments [23, 24] of the technique and its applications to many tayps of nonlinear ODEs, were given.

Therefore, this paper investigates the flow, heat and mass transfer in a conducting fluid in a porous medium. The Mathematical formulas were formed for this model. Then, the analytical solutions are obtained by modified Adomian decomposition method (MAD) and discussed.

## 2. EXPLAINING THE PROBLEM

Consider the magnetohydrodynamics (MHD) flow of viscous incompressible nanofluid near of the stagnation point on a sheet at a porous medium and the flow is assumed to be confined to  $y > 0$ . A transverse magnetic field  $B_0$  is applied in the y-direction so that the wall is stretched preserving the origin fixed. The regular temperature and nanoparticle concentration nearby the sheet is supposed that  $T_w$  and  $C_w$ , are the temperature and nanoparticle concentration afar from the sheet is supposed that  $T_\infty$  and  $C_\infty$  respectively. The governing equations of the current flow are stated as follows under the following assumptions:

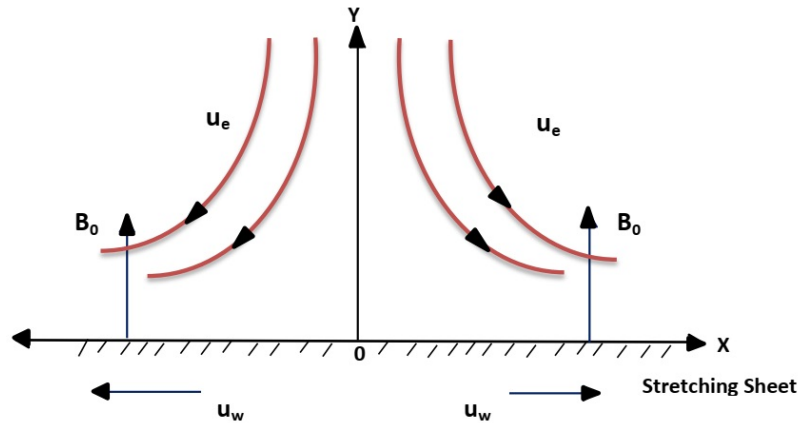


FIGURE 1. Physical model flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + u_e \frac{\partial u_e}{\partial x} + \frac{\sigma B_0^2(x)}{\rho} (u_e - u) + \frac{v}{k} (u_e - u), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} + \tau \left( D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right), \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

where  $T$  is the temperature and  $u$  and  $v$  are the components of velocity in the  $x$  and  $y$  directions,  $C$  is nanoparticle concentration,  $u_e(x)$  is the stagnation-point velocity in the open stream,  $k$ ,  $\mu$ ,  $\rho$ ,  $\nu (= \frac{\mu}{\rho})$ ,  $B_0(x)$ ,  $\sigma$ ,  $\kappa$ ,  $\tau$ ,  $D_B$  and  $D_T$  are the permeability of the porous material, coefficient of fluid viscosity, fluid density, is the kinematic-viscosity, magnetic field of uniform strength, electrical conductivity, coefficient of thermal diffusivity, the ratio effectiveness of heat capacity of the nanoparticle, Brownian motion coefficient and thermophoretic diffusion coefficient. The boundary conditions (B.C.) are given as following

$$\begin{aligned} u &= u_w = ax, \quad v = -v_w, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0, \\ u &\rightarrow u_e = bx, \quad T = T_\infty, \quad C = C_\infty \quad \text{at } y = \infty. \end{aligned} \quad (5)$$

Here  $a (>0)$  and  $b (>0)$  are constants,  $v_w$  is the wall velocity,  $T_w$  is the uniform wall temperature,  $T_\infty$  and  $C_\infty$  are the free stream temperature and nanoparticles.

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad \text{and} \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad (6)$$

where  $\psi$  is the stream function. The system of PDE are reduced to ODE by the following similarity transformations

$$\eta = \sqrt{\frac{a}{v}}y, \psi = \sqrt{av}xf(\eta). \quad (7)$$

Utilizing the boundary layer (1-4) relations (6,7). we get the following equations

$$\begin{aligned} f'''(\eta) + f(\eta)f''(\eta) - f^2(\eta) + (K + M^2)(\lambda - f'(\eta)) + \lambda^2 &= 0, \\ \theta''(\eta) + \text{Pr}[Nb\phi'(\eta)\theta'(\eta) + Nt\theta'^2(\eta) + f(\eta)\theta'(\eta)] &= 0, \\ \phi''(\eta) + \frac{Nt}{Nb}\theta''(\eta) + Le\text{Pr}f(\eta)\phi'(\eta) &= 0, \end{aligned} \quad (8)$$

B.C. convert to

$$\begin{aligned} f' &= 1, \quad f = S, \quad \theta = 1, \quad \phi = 1 \text{ at } \eta = 0 \\ f' &= A, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty, \end{aligned} \quad (9)$$

where  $A = \frac{b}{a}$  (ratio of rates of velocities),  $M^2 = \frac{\sigma B_0^2(x)}{\rho a}$  (parameter of the magnetic),  $K = \frac{v}{ka}$  (permeability parameter of the porous medium),  $\text{Pr} = \frac{v}{\kappa}$  (Prandtl number),  $Nb = \frac{\tau D_B}{v}(T_w - T_\infty)$  (Brownian motion),  $Nt = \frac{\tau D_T}{v T_\infty}(T_w - T_\infty)$  (thermophoresis parameter),  $S = \frac{v_w}{\sqrt{av}}$ ,  $S > 0$  (refer to suction) and  $S < 0$  refer to injection,  $Le = \frac{\kappa}{D_B}$  (Lewis number).

### 3. A TRANSFORMATION OF GOVERN EQUATIONS

In order to solve (8) with the boundary condition (9), we should be proceed on the following steps:

we first transform the system differential equations (9) into the next system of differential equations:

$$\begin{aligned} f'(\eta) &= u(\eta), \\ u''(\eta) &= f(\eta)u'(\eta) - u'(\eta)^2 + A^2 + (K + M^2)(A - u(\eta)), \\ \theta''(\eta) &= -\text{Pr}(f(\eta)\theta'(\eta) + Nb\phi'(\eta)\theta'(\eta) + Nt\theta'^2(\eta)), \\ \phi''(\eta) &= -Le f(\eta)\phi'(\eta) - \frac{Nt}{Nb}\theta''(\eta), \end{aligned} \quad (10)$$

the B.C. are given as following

$$u(0) = 1, \quad f(0) = S, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad u(\infty) = A, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \quad (11)$$

We can be converted the B.C. from unbounded domain,  $\eta \in [0, \infty)$ , to bounded domain,  $\zeta(\text{say}) \in [0, 1)$ , the relation between the independent variable  $\eta$  and independent variable  $\zeta$  is the transformation  $\zeta = 1 - e^{-\eta}$ . As a result, the system of (10) should be dependent on the new variable

$\zeta$ , hence, we show the relations between the derivatives for  $\eta$  and the derivatives for  $\zeta$  [25 – 27]

$$\begin{aligned}\frac{d}{d\eta}() &= (1 - \zeta) \frac{d}{d\zeta}(), \\ \frac{d^2}{d\eta^2}() &= (1 - \zeta)^2 \frac{d^2}{d\zeta^2}() - (1 - \zeta) \frac{d}{d\zeta}().\end{aligned}\quad (12)$$

Substituting (12) into the system (10), we arrive to the following system of ordinary equations

$$f'(\zeta) = \frac{u(\zeta)}{1 - \zeta}, \quad (13)$$

$$u''(\zeta) = \frac{u'(\zeta)}{1 - \zeta} - \frac{f(\zeta)u'(\zeta)}{1 - \zeta} + \frac{u^2(\zeta)}{(1 - \zeta)^2} + \frac{(K + M^2)u(\zeta)}{(1 - \zeta)^2} + \frac{A(M + A)}{(1 - \zeta)^2}, \quad (14)$$

$$\theta''(\zeta) = \frac{\theta'(\zeta)}{1 - \zeta} - \text{Pr}(Nb\phi'(\zeta)\theta'(\eta) + Nt\theta'^2(\zeta) + \frac{f(\zeta)\theta'(\zeta)}{1 - \zeta}). \quad (15)$$

$$\phi''(\eta) = \frac{\phi'(\zeta)}{1 - \zeta} - \text{LePr} \frac{f(\eta)\phi'(\zeta)}{1 - \zeta} - \frac{Nt}{Nb}(\theta''(\zeta) + \frac{\theta'(\zeta)}{1 - \zeta}), \quad (16)$$

B.C. become

$$u(0) = 1, \quad f(0) = S, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad u(1) = A, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \quad (17)$$

Integrate (13) with respect to  $\zeta$  from 0 to  $\zeta$

$$f(\zeta) = f(0) + \int_0^\zeta \frac{u(\zeta)}{1 - \zeta} d\zeta \quad (18)$$

#### 4. APPLICATION OF MAD METHOD

In order to apply the modified adomian decomposition method, the solutions of  $f(\zeta)$ ,  $u(\zeta)$ ,  $\theta(\zeta)$  and  $\phi(\zeta)$  are expressed in a series form given by

$$f(\zeta) = \sum_{n=0}^{\infty} f_n(\zeta), \quad u(\zeta) = \sum_{n=0}^{\infty} u_n(\zeta), \quad \theta(\zeta) = \sum_{n=0}^{\infty} \theta_n(\zeta), \quad \phi(\zeta) = \sum_{n=0}^{\infty} \phi_n(\zeta). \quad (19)$$

Now, we define the inverse operator to solve (14-16) with boundary condition (17)

$$L^{-1}() = \int_0^\zeta \int_c^\zeta () d\zeta d\zeta - \zeta \int_0^\zeta \int_c^\zeta () d\zeta d\zeta, \quad c \neq 1. \quad (20)$$

On using the operator (20) for both side of (14-16), we have

$$\begin{aligned}u(\zeta) &= 1 + (A - 1)t + L^{-1}\left(\frac{u'(\zeta)}{1 - \zeta} - \frac{f(\zeta)u'(\zeta)}{1 - \zeta} + \frac{u^2(\zeta)}{(1 - \zeta)^2} + \frac{(K + M^2)u(\zeta)}{(1 - \zeta)^2} + \frac{A(M^2 + A)}{(1 - \zeta)^2}\right), \\ \theta(\zeta) &= -t + L^{-1}\left(\frac{\theta'(\zeta)}{1 - \zeta} - \text{Pr}\left(\frac{f(\zeta)\theta'(\zeta)}{1 - \zeta} + Nb\omega'(\zeta)\theta'(\eta) + Nt\theta'^2(\zeta)\right)\right), \\ \phi(\zeta) &= -t + L^{-1}\left(\frac{\phi'(\zeta)}{1 - \zeta} - \text{LePr} \frac{f(\eta)\phi'(\zeta)}{1 - \zeta} - \frac{Nt}{Nb}(\theta''(\zeta) + \frac{\theta'(\zeta)}{1 - \zeta})\right).\end{aligned}\quad (21)$$

Substituting (19) into (21), we obtain

$$\begin{aligned}
 \sum_{n=0}^{\infty} u_n(\zeta) &= 1 + (A-1)t + L^{-1} \left( \sum_{n=0}^{\infty} \sum_{i=0}^n \zeta^{n-i} u'_i - \sum_{n=0}^{\infty} \sum_{j=0}^n \sum_{i=0}^j \zeta^{n-i} f_i u'_{j-i} \right. \\
 &\quad + \sum_{n=0}^{\infty} \sum_{j=0}^n \sum_{i=0}^j (n-j-1) \zeta^{n-i} u_i u_{j-i} + (K+M^2) \sum_{n=0}^{\infty} \sum_{i=0}^n (n-j-1) \zeta^{n-i} u_j \\
 &\quad \left. - A(M+A)(n+1)\zeta^n \right), \\
 \sum_{n=0}^{\infty} \theta_n(\zeta) &= 1 - t + L^{-1} \left( \sum_{n=0}^{\infty} \sum_{i=0}^n \zeta^{n-i} \theta'_i - \Pr \left( \sum_{n=0}^{\infty} \sum_{j=0}^n \sum_{i=0}^j \zeta^{n-i} f_i \theta'_{j-i} \right. \right. \\
 &\quad \left. \left. + Nb \sum_{n=0}^{\infty} \sum_{i=0}^n \phi'_{n-i} \theta'_i + Nt \sum_{n=0}^{\infty} \sum_{i=0}^n \theta'_{n-i} \theta'_i \right) \right), \\
 \sum_{n=0}^{\infty} \phi(\zeta) &= -t + L^{-1} \left( \sum_{n=0}^{\infty} \sum_{i=0}^n \zeta^{n-i} \phi'_i - Le \Pr \sum_{n=0}^{\infty} \sum_{j=0}^n \sum_{i=0}^j \zeta^{n-i} f_i \phi'_{j-i} \right. \\
 &\quad \left. - \frac{Nt}{Nb} \left( \sum_{n=0}^{\infty} \phi''_n(\zeta) + \sum_{n=0}^{\infty} \sum_{i=0}^n \zeta^{n-i} \theta'_i \right) \right). \tag{22}
 \end{aligned}$$

Accordingly, we obtain the recurrence relation

$$\begin{aligned}
 u_0(\zeta) &= 1, \quad f_1(\zeta) = S, \quad v_1(\zeta) = 1, \\
 u_1(\zeta) &= (A-1)t + L^{-1}(u'_0 - f_0 u'_0 + u_0^2 + (K+M^2)u_0 - (M^2\lambda + \lambda^2)), \\
 \theta_1(\zeta) &= -t + L^{-1}(\theta' - \Pr(f_0 \theta'_0 + Nb \theta'_0 \phi'_0 + Nt \theta_0'^2)), \\
 \phi_1(\zeta) &= t + L^{-1}(\phi'_0 - Le \Pr f'_0 \phi'_0 - \frac{Nt}{Nb}(\theta_0'' + \theta'_0)), \tag{23}
 \end{aligned}$$

$$\begin{aligned}
 f_{n+1}(\zeta) &= \int_0^{\zeta} \sum_{i=0}^n \zeta^{n-i} u_i d\zeta, \\
 u_{n+1}(\zeta) &= L^{-1} \left( \sum_{i=0}^n \zeta^{n-i} u'_i - \sum_{j=0}^n \sum_{i=0}^j \zeta^{n-i} f_i u'_{j-i} + \sum_{j=0}^n \sum_{i=0}^j (n-j-1) \zeta^{n-i} u_i u_{j-i} \right. \\
 &\quad \left. + (K+M) \sum_{i=0}^n (n-j-1) \zeta^{n-i} u_j - A(M+A)(n+1)\zeta^n \right), \\
 \theta_{n+1}(\zeta) &= L^{-1} \left( \sum_{i=0}^n \zeta^{n-i} \theta'_i - \Pr \left( \sum_{j=0}^n \sum_{i=0}^j \zeta^{n-i} f_i \theta'_{j-i} + Nb \sum_{i=0}^n \phi'_{n-i} \theta'_i + Nt \sum_{i=0}^n \theta'_{n-i} \theta'_i \right) \right), \\
 \phi_{n+1}(\zeta) &= -L^{-1} \left( \sum_{i=0}^n \zeta^{n-i} \phi'_i - Le \sum_{j=0}^n \sum_{i=0}^j \zeta^{n-i} f_i \phi'_{j-i} - \frac{Nt}{Nb} (\theta_n''(\zeta) + \sum_{i=0}^n \zeta^{n-i} \theta'_i) \right). \tag{24}
 \end{aligned}$$

Calculating a few terms of the recurrence expression (24), we obtain

$$\begin{aligned} u_1 &= \frac{1}{2}(A-1)(A+M^2+K+3)\zeta + \frac{1}{2}(A-1)(M^2+K-A-1)\zeta^2, \\ f_1 &= \zeta, \theta_1 = -\zeta, \phi_1 = -\zeta, \end{aligned} \quad (25)$$

$$\begin{aligned} u_2 &= \frac{1}{24}(16-20A+A^2(K+M^2-2(S+2)+A(M^2-K(M^2+4)+2M^2S+12S) \\ &\quad + (K+M^2)(K+M^2-2S)-13S+K)\zeta - 6(M^2-K-3+A(M^2+A+2)(S-1))\zeta^2 \\ &\quad + 2(K+M^2-2S+A^2(M^2+K+2S-4)-(K+M^2)(K+M^2+2S)+A(4+ \\ &\quad (M^2+2)+M^2(M^2+2S-2)))\zeta^3 + (M^2+K+2)(1+K+M^2-A(A+M^2))\zeta^4), \\ f_2 &= \frac{1}{12}(A+3A^2+3AM^2-3(1+K+M^2))\zeta^2 + -2(A^2+M^2A-(1+K+M^2))\zeta^3, \\ \theta_2 &= \frac{1}{2}(\text{Pr}S-1)(\zeta^2-\zeta), \\ \phi_2 &= \frac{1}{Nb}((\zeta^2-\zeta)(Nb(LeS-1)-Nt)). \end{aligned} \quad (26)$$

Calculating  $u_3, u_4, \dots, f_3, f_4, \dots, \theta_3, \theta_4, \dots$  and  $\phi_3, \phi_4, \dots$  Substituting terms of  $f_i, \theta_i$  and  $\phi_i$  into (19) with  $\zeta = 1 - e^\eta$ , then, the analytical solutions of (10) can be written in the form

$$\begin{aligned} f &= 1 - e^\eta + \frac{1}{12}(A+3A^2+3AM^2-3(1+K+M^2))(1-e^\eta)^2 \\ &\quad - 2(A^2+M^2A-(1+K+M^2))(1-e^\eta)^3 + \dots, \\ \theta &= -(1-e^\eta) + \frac{1}{2}(\text{Pr}S-1)((1-e^\eta)^2 - (1-e^\eta)) + \dots, \\ \phi &= -(1-e^\eta) + \frac{1}{Nb}(((1-e^\eta)^2 - (1-e^\eta))(Nb(LeS-1)-Nt) + \dots \end{aligned} \quad (27)$$

## 5. DISCUSSION

The current work illustrates incompressible nanofluid near of the stagnation point on a sheet at a porous medium. A number of graphs is shown and analyzed to acquire a physical interpretation of the relations between these limitations and the target distributions of the study. A number of graphs is produced and analyzed to acquire a physical interpretation of the interactions between these limitations and the goal distributions of the study. In what follows, selected values of the pertinent parameters are considered for drawing the profiles, which differ according to the studied parameter in each figure. 2-D drawings from 2 to 13 were used to depict the earlier effects.

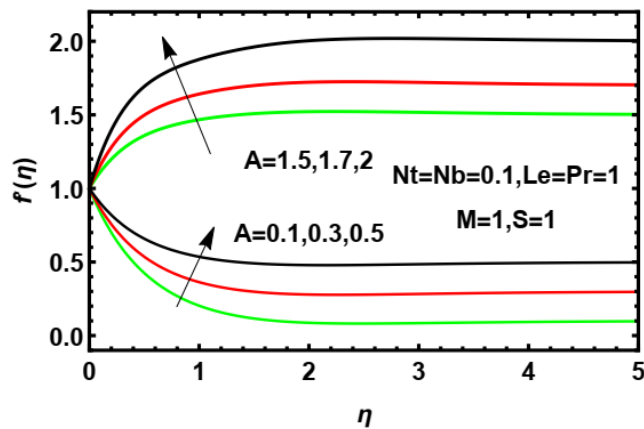


FIGURE 2. Impact of A on Velocity profile

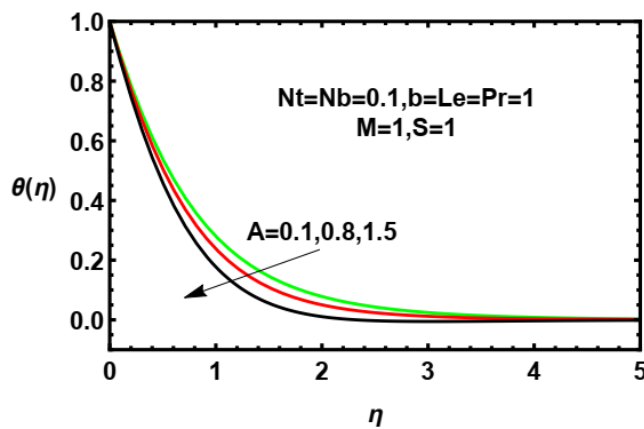


FIGURE 3. Impact of A on Temperature profile

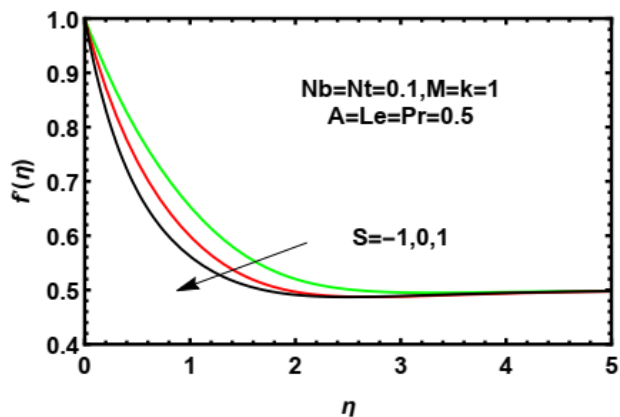


FIGURE 4. Impact of S in Velocity profile



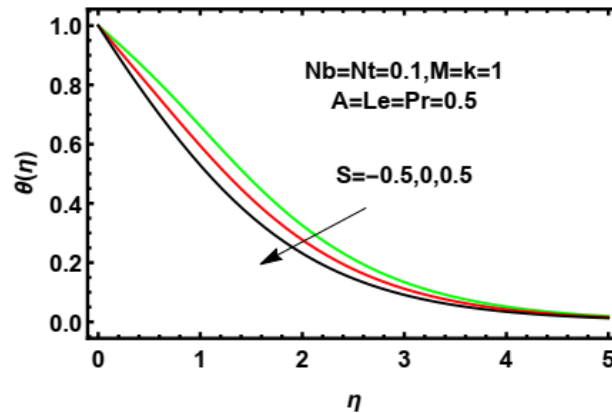
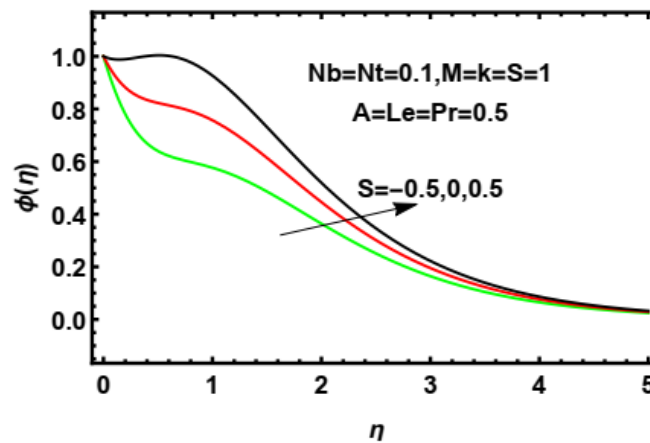
FIGURE 5. Impact of  $S$  on Temperature profile

Fig. 2. Shows that at  $A > 1$  the stream velocity is more than the stretching velocity, this means straining motion is progress near the stagnation region, It causes thickness of the boundary layer to decrease. The fluid flow changes to reverse When the stretching velocity stretching velocity is more than the free stream velocity at  $A < 1$

Fig. 3. For deferent values of  $A$ , the temperature profiles decreasing with increasing  $A$ .

Fig. 4. Show that for  $A < 1$  as  $S$  increasesm the velocity decreases. This means that the free stream velocity is less than the stretching velocity, which lead to stabilize the growth of the boundary layer.

FIGURE 6. Impact of  $S$  on nanoparticles concentration

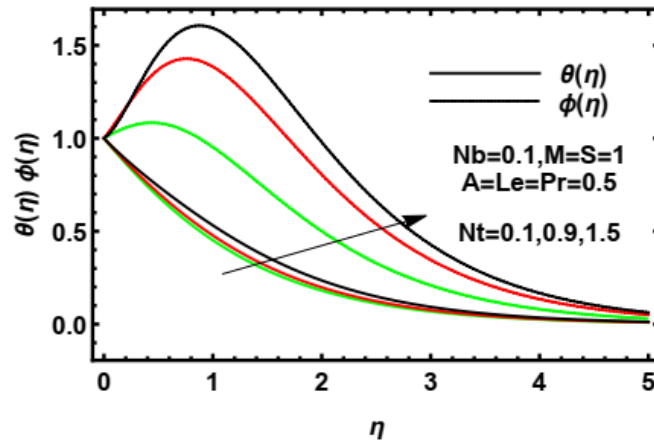


FIGURE 7. Impact of  $Nt$  on temperature and nanoparticles concentration

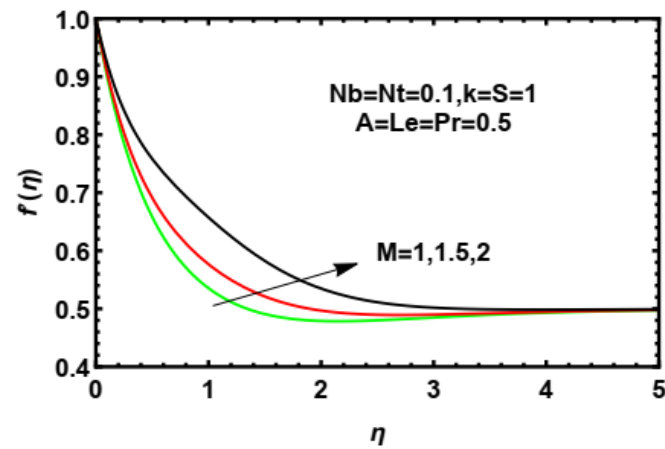


FIGURE 8. Impact of  $M$  on velocity profile

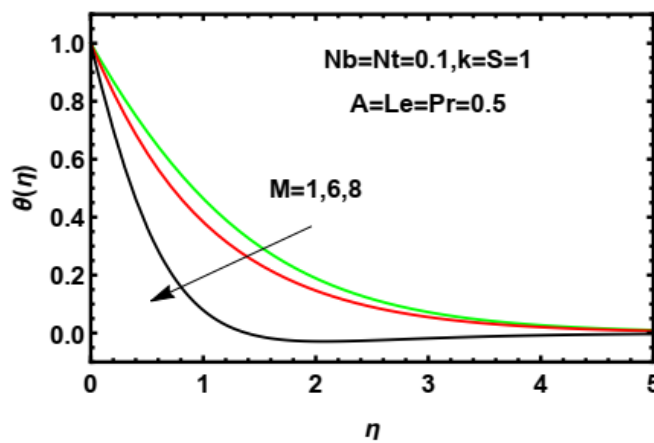


FIGURE 9. Impact of  $M$  on temperature profile

Fig. 5. and Fig. 6 It shows that the influence of suction ( $S > 0$ ) and injection ( $S < 0$ ) on the temperature and concentration, the injection lead to progress the temperature which causes a decrease in the rate of heat transfer and reverses with suction ( $S > 0$ ), the concentration changes to reverse when ( $S > 0$ ) and ( $S < 0$ ).

Fig. 7. It has been demonstrated that rising  $Nt$  values cause rising temperatures and concentrations of nanoparticles. The impact of increasing the value of  $Nt$  on the concentration volume profiles is to increase significantly in the boundary layer but it shows significant decreases in the surface boundary.

The magnetic field parameter effect is displayed in Fig. 8. Because  $M$  functions as a resistive force, fluid movement slows down in comparison, increasing the thickness of the boundary layer. As a result, the fluid's temperature rises as the magnetic field increases, as Fig. 9 illustrates.

Fig. 10. Demonstrates how the permeability parameter  $K$  affects horizontal velocity, with an increase in  $K$  causing a decrease in horizontal velocity. This indicates that an increase in permeability parameter  $K$  causes an increase in fluid resistance along the surface, which increases the thickness of the boundary layer.

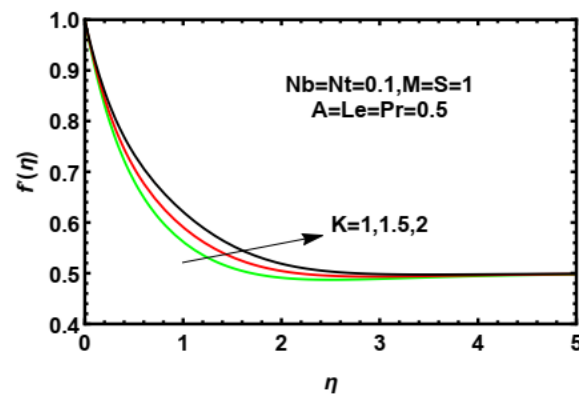


FIGURE 10. Effect of  $K$  on velocity profile

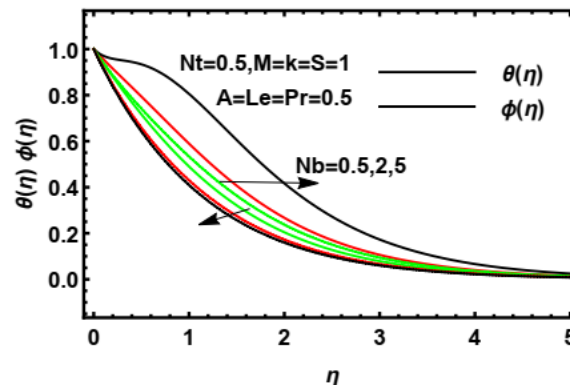


FIGURE 11. Effect of  $Nb$  on temperature and nanoparticles concentration

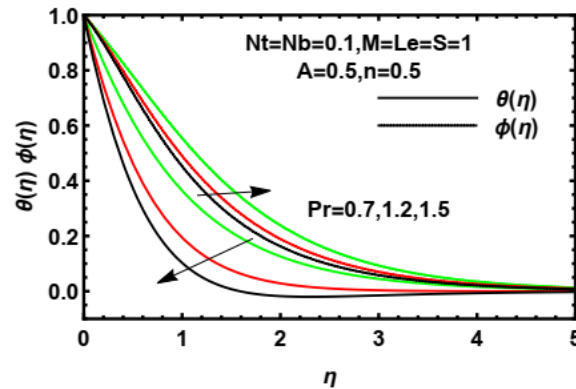


FIGURE 12. Effect of  $Pr$  on temperature and nanoparticles concentration

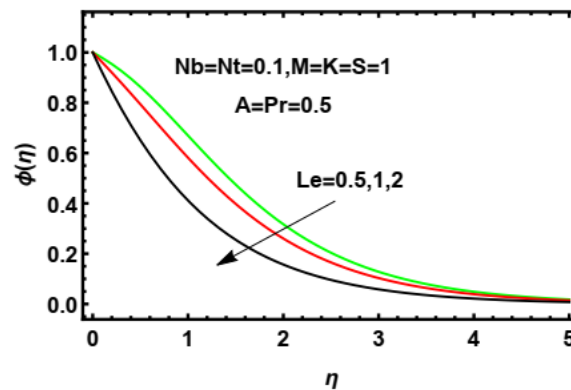


FIGURE 13. Effect of  $Le$  on nanoparticles concentration

From Fig. 11. indicates the effect of various values of  $Nb$  on the temperature  $\theta$  and nanoparticles concentration  $\phi$ . It is shown that the increase in the value of  $Nb$  is increasing  $\theta$  in the boundary layer, whereas increasing of  $Nb$  led to reducing concentration boundary layer which mean enhances the concentration at the sheet.

Fig. 12 Temperature profile and concentration increase with increase Prandtl number  $Pr$  were increasing of Prandtl number  $Pr$  causes increasing of thermal diffusivity, which led to upturn the temperature profile.

Fig. 13. It found that the variations in nanoparticles concentration  $\phi$  for various values of Lewis number  $Le$ , which mean increasing values of Lewis number causes decreasing of the concentration as a result the concentration boundary layer thickness decreases.

Table 2: Compares the outcomes for  $[f''(0)]$  at different values of velocity ratio  $A$ , when  $S = 0$  and  $Pr = 0.05$ .

$A$	Present study	Ibrahim et al [27]	Hayat et al [28]	Ramesh and Hayat [29]
0.1	-0.965	-0.9694	- 0.9695	-0.9696
0.5	-0.667	-0.6673	-0.6673	-0.6672
2.0	2.0175	2.0175	2.0176	2.0175
3.0	4.7293	4.7292	4.7296	4.7292

Table 3: Comparison of the results for  $[-\theta'(0)]$  at various values of velocity ratio  $A$ , when  $S = 0$ .

Pr	$A$	Present study	Ibrahim et al [27]	Hayat et al [28]	Ramesh and Hayat [29]
1	0.1	0.603	0.6022	0.6021	0.6048
	0.5	0.692	0.6924	0.6924	0.6925
1.5	0.1	0.777	0.7768	0.7768	0.7769
	0.5	0.864	0.8648	0.8647	0.8647

## 6. CONCLUSIONS

The results of this study of heat and mass transfer of MHD stagnation-point flow nanofluid past stretching sheet in the porous medium as follows:

- 1) The boundary layer solutions are obtained by modified adomian decomposition method and comparison of solutions with previous numerical solutions are shown in table 1 and 2
- 2) As the suction ( $S > 0$ ), permeability parameter ( $K$ ), magnetic parameter ( $M$ ), and Lewis number ( $Le$ ) increase, the boundary layer's thickness decreases
- 3) The temperature of the boundary layer decreases when the stream velocity is more than the stretching velocity (increasing  $A$ ) and same effect with the increase of Prandtl number, suction,

injection parameter and Lewis number. But the temperature increasing with increasing Brownian motion and thermophoresis parameter.

4) Nanoparticles concentration rises with the increase of suction, injection parameter and thermophoresis parameter but opposite with Prandtl number and Brownian motion.

**Conflicts of Interest:** The authors declare that there are no conflicts of interest regarding the publication of this paper.

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