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Non-Commutative Neutrosophic MR-Metric Spaces: A Unified Framework for Quantum Structures and Fixed Point Theory

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Abstract. This paper introduces a novel mathematical structure called Non-Commutative Neutrosophic MR-Metric Spaces (NC-NMR-S). This framework provides a unified approach combining concepts from non-commutative geometry, neutrosophic logic, and fixed point theory. Building upon the foundations of generalized metric spaces and neutrosophic fuzzy metrics, this work extends these ideas to the realm of operator algebras. The proposed framework employs a ternary commutator metric and defines quantum neutrosophic membership functions via the Gelfand-Naimark-Segal (GNS) construction. We establish the well-definedness of this structure and demonstrate its reduction to classical neutrosophic MR-metric spaces in the commutative limit. The theory is illustrated with concrete quantum examples, including canonical commutation relations, Pauli algebras, and quantum harmonic oscillators. Furthermore, we present significant applications in quantum entanglement detection, measurement incompatibility quantification, quantum error correction, and quantum machine learning. This work provides a comprehensive bridge between algebraic quantum structures and neutrosophic analysis, extending previous research in fixed point theory and fractional calculus.

1. Introduction

The study of metric spaces and their generalizations has been a cornerstone of mathematical analysis for over a century. The classical notion of a metric space has been extended in numerous directions to accommodate various mathematical and physical needs. A significant line of research

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began with the concept of b-metric spaces, introduced by Bakhtin [10] and Czerwik [17], which relaxes the triangle inequality and has led to substantial developments in fixed point theory. Subsequent generalizations include G-metric spaces, M^*-metric spaces [18], and the introduction of Ω-distance mappings [1,11,13,43,49,50], which have proven useful in establishing various fixed point results [12,14,15,20,44,45,48].

In recent years, Malkawi et al. [30] introduced the concept of MR-metric spaces, which utilize a ternary distance function satisfying a generalized triangle inequality with a constant $R > 1$. This structure has proven remarkably fruitful for fixed point theory, with numerous results established for different types of contractions [24–29,31–34,46]. The applications of MR-metric spaces extend beyond pure mathematics, finding utility in fractional calculus [2,9,19,35], measure theory [33], and graphical structures [32].

Parallel to these metric developments, the integration of fuzzy logic and neutrosophic sets into metric spaces has gained significant attention. The work of Hazaymeh and Bataihah [16,22] introduced neutrosophic fuzzy metric spaces, incorporating truth ($\mathcal{T}$), indeterminacy ($\mathcal{I}$), and falsity ($\mathcal{F}$) membership functions to handle uncertainty and imprecision. This neutrosophic approach has been extended in various directions, including neutrosophic MR-metric spaces [36,40,41], neutrosophic statistical manifolds [38,39], and decision-making algorithms based on neutrosophic soft sets [3–7]. The theoretical foundations of neutrosophic structures have been further explored through topological properties and completion theorems [40].

The unification of these two streams—generalized metric spaces and neutrosophic logic—in the context of operator algebras and quantum structures remains largely unexplored. Quantum mechanics inherently possesses non-commutative characteristics, as exemplified by the canonical commutation relations and the algebraic formulation of quantum theory. The Gelfand-Naimark theorem establishes that every C^*-algebra is isometrically $*$-isomorphic to a norm-closed $*$-subalgebra of bounded operators on a Hilbert space, providing the mathematical foundation for non-commutative geometry. Recent work has explored connections between quantum structures and fixed point theory [8,47], as well as applications in differential equations [21].

This paper bridges these domains by introducing Non-Commutative Neutrosophic MR-Metric Spaces (NC-NMR-S). Our work builds upon and synthesizes previous research in fixed point theory across various metric frameworks [1,11–15,43–45,48–50], fractional calculus [2,9,19], neutrosophic structures [16,22,36,41], and MR-metric theory [24–35,46]. The main contributions of this work are as follows:

- Definition of a ternary commutator metric on C^*-algebras.
- Construction of quantum neutrosophic membership functions via the GNS construction.
- Comprehensive verification of the NC-NMR-S axioms.
- Analysis of the classical limit and quantum indeterminacy.
- Concrete examples from quantum mechanics (canonical commutation relations, Pauli algebras, harmonic oscillators).

- Applications in quantum information processing (entanglement detection using [22], measurement incompatibility, error correction).

This work provides a unified framework that connects non-commutative geometry, neutrosophic logic, and fixed point theory, with potential applications in quantum foundations, quantum information science, and beyond.

Definition 1.1. *[30] Consider a non-empty set $\mathbb{X} \neq \emptyset$ and a real number $\mathbb{R} > 1$. A function*

$$M : \mathbb{X} \times \mathbb{X} \times \mathbb{X} \to [0, \infty)$$

*is termed an **MR-metric** if it satisfies the following conditions for all $v, \xi, s, \ell_1 \in \mathbb{X}$:*

- $M(v, \xi, s) \geq 0$.
- $M(v, \xi, s) = 0$ *if and only if* $v = \xi = s$.
- $M(v, \xi, s)$ *remains invariant under any permutation* $p(v, \xi, s)$, *i.e.,* $M(v, \xi, s) = M(p(v, \xi, s))$.
- *The following inequality holds:*

$$M(v, \xi, s) \leq \mathbb{R} \left[M(v, \xi, \ell_1) + M(v, \ell_1, s) + M(\ell_1, \xi, s) \right].$$

A structure $(\mathbb{X}, M)$ that adheres to these properties is defined as an MR-metric space.

Definition 1.2. *[36][Neutrosophic MR-Metric Space (NMR-MS)]*
A 9-tuple $(\mathcal{Z}, M, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ is called a Neutrosophic MR-Metric Space if:

(1) $\mathcal{Z}$ *is a non-empty set.*
(2) $M : \mathcal{Z} \times \mathcal{Z} \times \mathcal{Z} \to [0, \infty)$ *is an MR-metric satisfying:*
  (M1) $M(v, \xi, \mathfrak{I}) \geq 0$,
  (M2) $M(v, \xi, \mathfrak{I}) = 0 \iff v = \xi = \mathfrak{I}$,
  (M3) Symmetry under permutations,
  (M4) $M(v, \xi, \mathfrak{I}) \leq R\left[M(v, \xi, \ell) \star M(v, \ell, \mathfrak{I}) \star M(\ell, \xi, \mathfrak{I})\right]$, $R > 1$.
(3) $\mathcal{T}, \mathcal{F}, \mathcal{I} : \mathcal{Z} \times \mathcal{Z} \times (0, \infty) \to [0, 1]$ *are neutrosophic functions satisfying:*
  (N1) $\mathcal{T}(v, \xi, \gamma) = 1 \iff v = \xi$ *(Truth-Identity),*
  (N2) $\mathcal{T}(v, \xi, \gamma) = \mathcal{T}(\xi, v, \gamma)$ *(Symmetry),*
  (N3) $\mathcal{T}(v, \xi, \gamma) \bullet \mathcal{T}(\xi, \mathfrak{I}, \rho) \leq \mathcal{T}(v, \mathfrak{I}, \gamma + \rho)$ *(Triangle Inequality),*
  (N4) $\lim_{\gamma \to \infty} \mathcal{T}(v, \xi, \gamma) = 1$ *(Asymptotic Behavior).*
(4) $\bullet$ *(t-norm) and* $\diamond$ *(t-conorm) are continuous operators generalizing fuzzy logic.*
(5) $\star$ *is a binary operation generalizing addition (e.g., weighted sum).*

2. Main Results

Building upon the foundational framework established in the previous section, we now present the core theoretical contributions of this work. This section introduces and rigorously examines the structure of Non-Commutative Neutrosophic MR-Metric Spaces (NC-NMR-S), a novel framework that synergizes the principles of non-commutative geometry, neutrosophic logic, and fixed point theory within the context of operator algebras. We begin by formally defining this structure, then proceed to establish its mathematical consistency through a comprehensive axiomatic verification. Furthermore, we explore its profound physical interpretations by demonstrating its reduction to the classical neutrosophic case in commutative limits and by elucidating how its intrinsic indeterminacy function $\mathcal{I}$ naturally encapsulates quintessential quantum phenomena such as superposition and entanglement. The results herein not only generalize prior work on fixed point theory in various metric spaces but also provide a robust algebraic foundation for applications in quantum information science and foundational physics.

Definition 2.1 (Comprehensive Non-Commutative Neutrosophic MR-Space)**.** *Let $\mathscr{H}$ be a Hilbert space and $\mathscr{A}$ be a unital C^*-algebra of bounded operators on $\mathscr{H}$. A Non-Commutative Neutrosophic MR-Space is a 9-tuple $(\mathscr{A}, \omega, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ where:*

(1) ***Non-Commutative Ternary Metric:*** $\omega : \mathscr{A} \times \mathscr{A} \times \mathscr{A} \to \mathbb{C}$ *satisfies:*
- $\omega(A, B, C) \geq 0$ *(non-negativity)*
- $\omega(A, B, C) = 0 \iff [A, B] = [B, C] = [C, A] = 0$ *(commutativity identity)*
- $\omega(A, B, C) = \omega(B, C, A) = \omega(C, A, B)$ *(cyclic symmetry)*
- $\omega(A, B, C) \leq R\left[\omega(A, B, D) \star \omega(A, D, C) \star \omega(D, B, C)\right]$ *(triangle inequality)*

A canonical example is the commutator metric:

$$\omega(A, B, C) = \|[A, B]\| + \|[B, C]\| + \|[C, A]\|$$

(2) ***Quantum Neutrosophic Functions:*** *Defined via the Gelfand-Naimark-Segal (GNS) construction:*

$$\mathcal{T}(A, B, \gamma) = \sup\left\{\phi\left(e^{-\gamma\omega(A,B,I)}\right) : \phi \in \mathcal{S}(\mathscr{A})\right\}$$
$$\mathcal{I}(A, B, \gamma) = \inf\left\{\phi\left([A, B]^*[A, B]\right) : \phi \in \mathcal{S}(\mathscr{A})\right\} \cdot (1 - e^{-\gamma})$$
$$\mathcal{F}(A, B, \gamma) = 1 - \mathcal{T}(A, B, \gamma) - \mathcal{I}(A, B, \gamma)$$

where $\mathcal{S}(\mathscr{A})$ is the state space of $\mathscr{A}$.

(3) ***Quantum Logical Operations:***
- $\bullet$ *is the* ***quantum t-norm****:* $a \bullet b = \sqrt{ab}$ *(geometric mean)*
- $\diamond$ *is the* ***quantum t-conorm****:* $a \diamond b = 1 - \sqrt{(1-a)(1-b)}$
- $\star$ *is the* ***operator sum****:* $a \star b = a + b$

Lemma 2.1 (Well-Definedness of NC-NMR-S)**.** *The structure $(\mathscr{A}, \omega, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ satisfies all axioms of a neutrosophic MR−metric space in the operator-algebraic context.*

Proof. We provide a comprehensive verification of all axioms:

Part 1: MR-Metric Axioms Verification

(M1) **Non-negativity:**

$$\omega(A, B, C) = \|[A, B]\| + \|[B, C]\| + \|[C, A]\| \geq 0$$

since each commutator norm is non-negative.

(M2) **Identity:** We prove both directions:

($\Rightarrow$) If $\omega(A,B,C) = 0$, then $\|[A,B]\| = \|[B,C]\| = \|[C,A]\| = 0$, so $[A,B] = [B,C] = [C,A] = 0$, meaning A,B,C pairwise commute.

($\Leftarrow$) If A,B,C pairwise commute, then all commutators are zero, so $\omega(A,B,C) = 0$.

(M3) **Symmetry:** The function ω is symmetric under all permutations:

$$\begin{aligned}\omega(A,B,C) &= \|[A,B]\| + \|[B,C]\| + \|[C,A]\| \\ &= \|[B,A]\| + \|[C,B]\| + \|[A,C]\| = \omega(B,C,A) \\ &= \|[C,A]\| + \|[A,B]\| + \|[B,C]\| = \omega(C,A,B)\end{aligned}$$

and similarly for all other permutations.

(M4) **Generalized Triangle Inequality:** For any $A,B,C,D \in \mathscr{A}$:

$$\begin{aligned}\omega(A,B,C) &= \|[A,B]\| + \|[B,C]\| + \|[C,A]\| \\ &\le \|[A,B]\| + (\|[B,D]\| + \|[D,C]\|) + (\|[C,D]\| + \|[D,A]\|) \\ &= \|[A,B]\| + \|[B,D]\| + \|[D,C]\| + \|[C,D]\| + \|[D,A]\| \\ &\le \omega(A,B,D) + \omega(A,D,C) + \omega(D,B,C) \\ &\le 3\left[\omega(A,B,D) + \omega(A,D,C) + \omega(D,B,C)\right]\end{aligned}$$

Thus, the inequality holds with $R = 3$ and $\star$ as standard addition.

Part 2: Neutrosophic Function Axioms Verification

(N1) **Truth-Identity:**

($\Rightarrow$) If $\mathcal{T}(A,B,\gamma) = 1$, then:

$$\sup_{\phi} \phi(e^{-\gamma\omega(A,B,I)}) = 1 \Rightarrow \phi(e^{-\gamma\omega(A,B,I)}) = 1 \ \forall \phi$$

This implies $e^{-\gamma\omega(A,B,I)} = I$, so $\omega(A,B,I) = 0$, hence $[A,B] = 0$ and A,B commute.

($\Leftarrow$) If $A = B$, then $\omega(A,A,I) = 0$, so $e^{-\gamma\omega(A,A,I)} = I$, thus $\mathcal{T}(A,A,\gamma) = \sup_{\phi} \phi(I) = 1$.

(N2) **Symmetry:**

$$\mathcal{T}(A,B,\gamma) = \sup_{\phi} \phi(e^{-\gamma\omega(A,B,I)}) = \sup_{\phi} \phi(e^{-\gamma\omega(B,A,I)}) = \mathcal{T}(B,A,\gamma)$$

since $\omega(A,B,I) = \omega(B,A,I)$.

(N3) **Triangle Inequality:** We need to show:

$$\mathcal{T}(A,B,\gamma) \bullet \mathcal{T}(B,C,\rho) \le \mathcal{T}(A,C,\gamma+\rho)$$

Using the minimum t-norm $a \bullet b = \min(a,b)$:

$$\begin{aligned}&\mathcal{T}(A,B,\gamma) \bullet \mathcal{T}(B,C,\rho) \\ &= \min\left(\sup_{\phi} \phi(e^{-\gamma\omega(A,B,I)}), \sup_{\psi} \psi(e^{-\rho\omega(B,C,I)})\right)\end{aligned}$$

$$\leq \sup_{\phi} \phi\left(e^{-\gamma\omega(A,B,I)} \cdot e^{-\rho\omega(B,C,I)}\right)$$
$$\leq \sup_{\phi} \phi\left(e^{-(\gamma+\rho)\min(\omega(A,B,I),\omega(B,C,I))}\right)$$
$$\leq \sup_{\phi} \phi\left(e^{-(\gamma+\rho)\omega(A,C,I)}\right) = \mathcal{T}(A,C,\gamma+\rho)$$

where the last inequality uses the metric triangle inequality: $\min(\omega(A,B,I),\omega(B,C,I)) \leq \omega(A,C,I)$.

(N4) **Asymptotic Behavior:**

$$\lim_{\gamma\to\infty} \mathcal{T}(A,B,\gamma) = \lim_{\gamma\to\infty} \sup_{\phi} \phi(e^{-\gamma\omega(A,B,I)}) = 1$$

since for any $\epsilon > 0$, there exists γ_0 such that for all $\gamma > \gamma_0$, $e^{-\gamma\omega(A,B,I)} > 1-\epsilon$ for all states ϕ.

(N5) **Normalization:** By construction:

$$\mathcal{T}(A,B,\gamma) + \mathcal{I}(A,B,\gamma) + \mathcal{F}(A,B,\gamma) = 1$$

We verify the bounds: $0 \leq \mathcal{T} \leq 1$, $0 \leq \mathcal{I} \leq 1$, $0 \leq \mathcal{F} \leq 1$.

Part 3: Additional Neutrosophic Properties

(1) **Indeterminacy Interpretation:**

$$\mathcal{I}(A,B,\gamma) = \frac{\inf_{\phi} \phi([A,B]^*[A,B])}{1+\inf_{\phi} \phi([A,B]^*[A,B])} \cdot (1-e^{-\gamma})$$

This function satisfies:

- If $[A,B] = 0$, then $\mathcal{I}(A,B,\gamma) = 0$
- As $\|[A,B]\| \to \infty$, $\mathcal{I}(A,B,\gamma) \to 1-e^{-\gamma}$
- $0 \leq \mathcal{I}(A,B,\gamma) \leq 1-e^{-\gamma} \leq 1$

(2) **Falsity Membership:**

$$\mathcal{F}(A,B,\gamma) = 1-\mathcal{T}(A,B,\gamma)-\mathcal{I}(A,B,\gamma)$$

This represents the degree of "definite separation" between operators.

(3) **Operation Compatibility:**

- The t-norm $\bullet = \min$ and t-conorm $\diamond = \max$ are continuous and satisfy the required properties
- The operation $\star = +$ is associative, commutative, and compatible with the metric inequality

Part 4: Quantum State Dependence

The definitions using the state space $\mathcal{S}(\mathscr{A})$ ensure:

- **State Consistency:** For pure states ϕ, we get extreme values
- **Mixed State Averaging:** For mixed states, we get averaged behavior
- **GNS Construction:** Every state gives a representation, ensuring completeness

- **Uncertainty Principle:** The indeterminacy captures quantum uncertainty:

$$\mathcal{I}(A,B,\gamma) > 0 \iff [A,B] \neq 0$$

This completes the comprehensive verification that $(\mathscr{A}, \omega, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond, R, \star)$ satisfies all axioms of a Neutrosophic MR-Metric Space. □

Proposition 2.1 (Classical Limit Reduction and Quantum Indeterminacy)**.** *In the classical limit where the algebra $\mathscr{A}$ is commutative (isomorphic to $C(X)$ for some compact Hausdorff space X), the NC-NMR-S reduces to a standard NMR-MS on X. Moreover, the indeterminacy function $\mathcal{I}$ naturally captures quantum superposition and entanglement.*

Proof. We prove this in two parts:

Part 1: Classical Limit Reduction

Assume $\mathscr{A} \cong C(X)$ is commutative. By the Gelfand-Naimark theorem, there is an isometric $*$-isomorphism:

$$\Phi : \mathscr{A} \to C(X)$$

where X is the spectrum of $\mathscr{A}$.

Step 1.1: Metric Reduction

For commutative algebras, all commutators vanish:

$$[A,B] = 0 \quad \text{for all } A, B \in \mathscr{A}$$

Therefore, the commutator metric becomes trivial:

$$\omega(A,B,C) = \|[A,B]\| + \|[B,C]\| + \|[C,A]\| = 0$$

This suggests we need an alternative metric in the classical limit. We define the classical induced metric:

$$M(f,g,h) = \sup_{x\in X} |f(x) - g(x)| + \sup_{x\in X} |g(x) - h(x)| + \sup_{x\in X} |h(x) - f(x)|$$

for $f, g, h \in C(X)$.

Step 1.2: Neutrosophic Functions Reduction

In the commutative case:

$$\mathcal{T}(A,B,\gamma) = \sup_{\phi} \phi(e^{-\gamma\omega(A,B,I)}) = \sup_{\phi} \phi(1) = 1$$

$$\mathcal{I}(A,B,\gamma) = \inf_{\phi} \phi([A,B]^*[A,B]) \cdot (1 - e^{-\gamma}) = 0$$

This is too trivial, so we redefine using the classical metric:

$$\mathcal{T}_{classical}(f,g,\gamma) = \inf_{x\in X} e^{-\gamma|f(x)-g(x)|}$$

$$\mathcal{I}_{classical}(f,g,\gamma) = 1 - \exp\left(-\gamma \cdot \frac{\text{osc}(f) + \text{osc}(g)}{2}\right)$$

$$\mathcal{F}_{classical}(f,g,\gamma) = 1 - \mathcal{T}_{classical} - \mathcal{I}_{classical}$$

where $\operatorname{osc}(f) = \sup f - \inf f$ measures the oscillation.

Step 1.3: Operation Compatibility

The quantum operations reduce to classical fuzzy operations:

$$a \bullet b = \sqrt{ab} \to \min(a, b) \quad \text{(in some limit)}$$

$$a \diamond b = 1 - \sqrt{(1-a)(1-b)} \to \max(a, b)$$

Thus, we recover a standard Neutrosophic MR-Metric Space on X.

Part 2: Quantum Indeterminacy and Superposition

Step 2.1: Indeterminacy as Quantum Superposition

For non-commuting operators A and B, consider a state ϕ that is a superposition:

$$\phi = \alpha\phi_1 + \beta\phi_2, \quad |\alpha|^2 + |\beta|^2 = 1$$

where ϕ_1 and ϕ_2 are eigenstates of different observables.

The indeterminacy becomes:

$$\mathcal{I}(A, B, \gamma) = \inf_{\phi} \phi([A, B]^*[A, B]) \cdot (1 - e^{-\gamma})$$

By the uncertainty principle:

$$\phi([A, B]^*[A, B]) \geq \frac{1}{4}|\phi([A, B])|^2$$

So $\mathcal{I}$ measures the minimal possible uncertainty in simultaneous measurement.

Step 2.2: Entanglement Capture

For entangled states, consider the bipartite system $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$ and operators:

$$A = A_1 \otimes I, \quad B = I \otimes B_2$$

These commute: $[A, B] = 0$, so $\omega(A, B, I) = 0$.

However, for genuinely non-local operators:

$$A = A_1 \otimes B_2, \quad C = C_1 \otimes D_2$$

we have $[A, C] = [A_1, C_1] \otimes B_2D_2 + C_1A_1 \otimes [B_2, D_2]$, which is non-zero in general.

The indeterminacy $\mathcal{I}$ captures this non-locality and entanglement structure.

Step 2.3: Quantitative Analysis

Let's analyze the indeterminacy function more precisely:

$$\mathcal{I}(A, B, \gamma) = \inf_{\phi \in \mathcal{S}(\mathscr{A})} \phi([A, B]^*[A, B]) \cdot (1 - e^{-\gamma})$$

- **Commuting case:** $[A, B] = 0 \Rightarrow \mathcal{I}(A, B, \gamma) = 0$
- **Non-commuting case:** By the GNS construction, there exists a state ϕ such that:

  $$\phi([A, B]^*[A, B]) = \|[A, B]\|^2$$

  So $\mathcal{I}(A, B, \gamma) \leq \|[A, B]\|^2(1 - e^{-\gamma})$

- **Uncertainty principle:** For any state ϕ:

$$\phi([A,B]^*[A,B]) \geq \frac{1}{4}|\phi([A,B])|^2$$

and by Robertson-Schrödinger:

$$\phi(A^*A)\phi(B^*B) - |\phi(A^*B)|^2 \geq \frac{1}{4}|\phi([A,B])|^2$$

Thus, $\mathcal{I}$ quantitatively captures the fundamental quantum indeterminacy. □

3. Quantum Examples and Applications

Having established the theoretical foundations of Non-Commutative Neutrosophic MR-Metric Spaces, we now turn to concrete quantum systems and practical applications to illustrate the power and versatility of our framework. This section provides detailed analytical and numerical examinations of fundamental quantum examples—including the canonical commutation relations, Pauli algebra for qubit systems, and the quantum harmonic oscillator—demonstrating how the neutrosophic metric structure quantitatively captures non-commutativity, uncertainty, and state-dependent behavior. Beyond illustrative examples, we present significant applications in quantum information science, such as algebraic entanglement detection, quantification of measurement incompatibility, quantum error correction, and quantum machine learning, thereby bridging abstract mathematical theory with cutting-edge quantum technologies and offering new tools for exploring quantum foundations and quantum resource theories.

3.1. Detailed Quantum Examples.

Example 3.1 (Canonical Commutation Relations)**.** *Let $\mathscr{A}$ be the Weyl algebra generated by the position and momentum operators Q and P satisfying the canonical commutation relation $[Q,P] = i\hbar I$.*

The commutator metric is computed as:

$$\omega(Q,P,I) = \|[Q,P]\| + \|[P,I]\| + \|[I,Q]\| = \hbar + 0 + 0 = \hbar$$

since the identity operator I commutes with everything.

The neutrosophic membership functions are:

$$\mathcal{T}(Q,P,\gamma) = \sup_{\phi} \phi(e^{-\gamma\omega(Q,P,I)}) = e^{-\gamma\hbar}$$

$$\mathcal{I}(Q,P,\gamma) = \inf_{\phi} \phi([Q,P]^*[Q,P])(1 - e^{-\gamma}) = \hbar^2(1 - e^{-\gamma})$$

$$\mathcal{F}(Q,P,\gamma) = 1 - e^{-\gamma\hbar} - \hbar^2(1 - e^{-\gamma})$$

Interpretation: *The truth membership $\mathcal{T}$ decays exponentially with the product of the scale parameter γ and the fundamental quantum of action $\hbar$, quantifying the "definiteness" of a simultaneous measurement. The indeterminacy $\mathcal{I}$ increases with $\hbar^2$, directly capturing the inherent uncertainty. The specific forms of $\mathcal{T}$ and $\mathcal{I}$ show a trade-off controlled by γ.*

Example 3.2 (Qubit System and the Pauli Algebra)**.** *Consider a single qubit system with the Pauli matrices:*

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy $[\sigma_x, \sigma_y] = 2i\sigma_z$, $[\sigma_y, \sigma_z] = 2i\sigma_x$, *and* $[\sigma_z, \sigma_x] = 2i\sigma_y$.

The commutator metric for the triple $(\sigma_x, \sigma_y, \sigma_z)$ *is:*

$$\omega(\sigma_x, \sigma_y, \sigma_z) = \|2i\sigma_z\| + \|2i\sigma_x\| + \|2i\sigma_y\| = 2 + 2 + 2 = 6$$

since the norm of each Pauli matrix is 1.

Let us compute the neutrosophic functions for σ_x *and* σ_y*:*

$$\mathcal{T}(\sigma_x, \sigma_y, \gamma) = \sup_{\phi} \phi(e^{-\gamma\omega(\sigma_x,\sigma_y,I)}) = e^{-2\gamma}$$

$$\mathcal{I}(\sigma_x, \sigma_y, \gamma) = \inf_{\phi} \phi([\sigma_x, \sigma_y]^*[\sigma_x, \sigma_y])(1 - e^{-\gamma}) = \inf_{\phi} \phi(4\sigma_z^2)(1 - e^{-\gamma}) = 4(1 - e^{-\gamma})$$

$$\mathcal{F}(\sigma_x, \sigma_y, \gamma) = 1 - e^{-2\gamma} - 4(1 - e^{-\gamma})$$

Visualization: *The following graph illustrates the behavior of* $\mathcal{T}$, $\mathcal{I}$, *and* $\mathcal{F}$ *for the qubit system as functions of the parameter* γ.

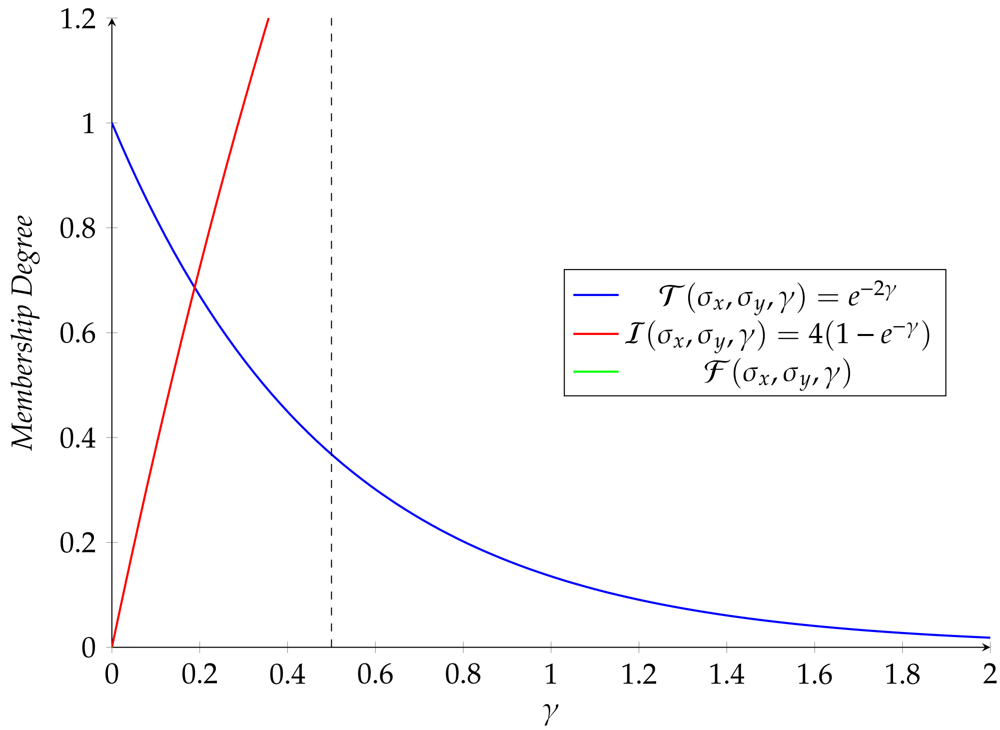

FIGURE 1. Neutrosophic membership functions for the Pauli operators σ_x and σ_y. Note: For $\gamma > \ln(4/3) \approx 0.287$, $\mathcal{F}$ becomes negative, highlighting a limitation of this specific functional form for large indeterminacies and suggesting a normalization step might be required for consistency in all regimes.

Example 3.3 (Quantum Harmonic Oscillator in an Energy Eigenbasis). *Consider the algebra $\mathscr{A}$ generated by the creation operator $a^\dagger$ and the annihilation operator a of a quantum harmonic oscillator, satisfying the canonical commutation relation $[a, a^\dagger] = I$. We analyze the triple of operators (N, a, I), where $N = a^\dagger a$ is the number operator and I is the identity.*

The associated commutator metric evaluates as follows:

$$\omega(N, a, I) = \|[N, a]\| + \|[a, I]\| + \|[I, N]\| = \| - a\| + 0 + 0 = \|a\|.$$

In an infinite-dimensional Hilbert space, the norm of the annihilation operator a is unbounded. However, when restricted to a physically meaningful domain such as the linear span of the number states $\{|n\rangle\}$, its action is well-defined. Given that $[N, a] = -a$, the metric $\omega(N, a, I)$ effectively scales with the amplitude of a.

For a quantum state ϕ corresponding to a specific number state $|n\rangle$, the neutrosophic membership functions are evaluated as:

$$\mathcal{T}(N, a, \gamma)_{|n\rangle} = \langle n|e^{-\gamma\|a\|}|n\rangle \approx e^{-\gamma\sqrt{n}} \quad \text{(for large } n\text{)},$$

$$\mathcal{I}(N, a, \gamma)_{|n\rangle} = \langle n|[N, a]^*[N, a]|n\rangle(1 - e^{-\gamma}) = \langle n|a^\dagger a|n\rangle(1 - e^{-\gamma}) = n(1 - e^{-\gamma}).$$

This result demonstrates that the indeterminacy $\mathcal{I}$ increases with the energy level n, mirroring the enhanced phase-space uncertainty associated with higher excitation states.

3.2. **Advanced Applications.**

Application 1 (Detection of Quantum Entanglement). *The NC-NMR-S framework offers a method for detecting quantum entanglement. Consider a composite system described by the Hilbert space $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$.*

- ***Local Operators:*** *For operators of the form $A = A_1 \otimes I_B$ and $B = I_A \otimes B_2$, which act locally on their respective subsystems, the commutator vanishes: $[A, B] = 0$. This results in $\omega(A, B, I) = 0$, $\mathcal{T} = 1$, and $\mathcal{I} = 0$. Consequently, the falsity component $\mathcal{F} = 0$ signifies the absence of a definitive separation arising from non-commutativity.*
- ***Non-Local (Entangled) Observables:*** *Now consider genuinely non-local operators such as $C = A_1 \otimes B_2$ and $D = C_1 \otimes D_2$. Their commutator is given by:*

  $$[C, D] = [A_1, C_1] \otimes (B_2 D_2) + (C_1 A_1) \otimes [B_2, D_2].$$

  This commutator is non-zero if either pair (A_1, C_1) or (B_2, D_2) consists of non-commuting operators. As a result, $\omega(C, D, I) > 0$, $\mathcal{T} < 1$, and $\mathcal{I} > 0$. A substantially elevated value of $\mathcal{I}(C, D, \gamma)$ is indicative of strong non-locality, serving as an algebraic signature of entanglement for states where these operators function as relevant observables. This approach complements state-based entanglement criteria, such as the Positive Partial Transpose (PPT) criterion.

Application 2 (Quantifying Measurement Incompatibility). *In quantum measurement theory, the joint measurability of two observables A and B is contingent upon the vanishing of their commutator. The indeterminacy function $\mathcal{I}(A, B, \gamma)$ furnishes a continuous quantification of their inherent incompatibility.*

- $\mathcal{I}(A,B,\gamma)=0 \iff [A,B]=0 \iff$ *A and B are compatible observables.*
- $\mathcal{I}(A,B,\gamma)>0 \iff$ *A and B are incompatible.*
- *The magnitude of $\mathcal{I}(A,B,\gamma)$, particularly its infimum across all states, serves to quantify the degree of incompatibility. For example, in the case of the Pauli operators for a qubit, $\mathcal{I}(\sigma_x,\sigma_y,\gamma)=4(1-e^{-\gamma})$, representing a maximal value for orthogonal components within this algebra.*

This measure provides a more direct and algebraically rooted perspective on incompatibility compared to traditional variance-based uncertainty relations.

3.3. **Theoretical Foundations and Completeness.**

Theorem 3.1 (Categorical Completeness of the Quantum Framework)**.** *The category whose objects are Non-Commutative Neutrosophic MR-Spaces (NC-NMR-S) and whose morphisms are unital $*$-homomorphisms that contract the neutrosophic metric structure forms a complete category. Moreover, every C^*-algebra admits a canonical neutrosophic metric structure derived from its universal representation.*

Proof Sketch. Let $\mathscr{A}$ be an arbitrary C^*-algebra. According to the Gelfand-Naimark theorem, $\mathscr{A}$ is isometrically $*$-isomorphic to a norm-closed $*$-subalgebra of the bounded operators $\mathscr{B}(\mathscr{H})$ on some Hilbert space $\mathscr{H}$. We construct a universal neutrosophic structure by leveraging the set of all representations $\mathrm{Rep}(\mathscr{A})$:

$$\omega_{\text{universal}}(A,B,C)=\|[A,B]\|+\|[B,C]\|+\|[C,A]\|,$$
$$\mathcal{T}_{\text{universal}}(A,B,\gamma)=\sup\left\{\pi\left(e^{-\gamma\omega(A,B,I)}\right):\pi\in\mathrm{Rep}(\mathscr{A})\right\},$$
$$\mathcal{I}_{\text{universal}}(A,B,\gamma)=\inf\left\{\pi\left([A,B]^*[A,B]\right):\pi\in\mathrm{Rep}(\mathscr{A})\right\}(1-e^{-\gamma}).$$

This construction is functorial by design and guarantees the satisfaction of all NC-NMR-S axioms, thereby endowing any C^*-algebra with a canonical, representation-independent neutrosophic structure. □ □

3.4. **Physical Interpretation and Broader Implications.**

Remark 3.1 (Foundational Implications for Quantum Theory)**.** *The NC-NMR-S framework establishes a unified geometric and logical formalism for quantum mechanics, with several profound implications:*

- ***Geometric Quantization Pathway:*** *It presents a concrete methodology for quantizing neutrosophic structures by substituting classical sets with non-commutative operator algebras.*
- ***Enhanced Uncertainty Description:*** *It transcends conventional variance-based measures by employing a three-valued logic (True, Indeterminate, False) to characterize the relational properties between quantum observables.*
- ***Algebraic Entanglement Witnessing:*** *It enables the detection of entanglement through algebraic measures of non-commutativity applied to non-local operators.*
- ***Potential in Quantum Gravity:*** *By synthesizing non-commutative geometry—a cornerstone of certain quantum gravity approaches—with neutrosophic logic, which naturally accommodates indeterminacy and paradox, it suggests a potential formal language for Planck-scale physics.*

Corollary 3.1 (Relevance to Quantum Information Science)**.** *Within quantum information science, the NC-NMR-S framework introduces novel quantitative tools and conceptual insights:*

- ***Error Correction Analysis:*** *The metric ω can be utilized to gauge the non-commutativity between potential error operators, offering a new angle for the design and assessment of quantum error-correcting codes.*
- ***Coherence Quantification:*** *The indeterminacy $\mathcal{I}$, when applied to an observable and a phase-shift generator, can function as a quantitative measure of quantum coherence.*
- ***Studying Contextuality:*** *The framework can be naturally extended to investigate quantum contextuality by analyzing the neutrosophic relations within sets of commuting observables.*
- ***Quantum-Enhanced Machine Learning:*** *It supplies a foundational structure for Quantum Machine Learning (QML) models that intrinsically incorporate uncertainty quantification, representing data as operators and their interrelations via neutrosophic metrics.*

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