Int. J. Anal. Appl. (2025), 23:317

International Journal of Analysis and Applications

Fuzzy Magnified Translations of Fuzzy *Bd*-Ideals in *Bd*-Algebras

Warud Nakkhasen[1,*], Teerapan Jodnok[2], Aiyared Iampan[3]

[1]*Department of Mathematics, Faculty of Science, Mahasarakham University, Maha Sarakham 44150, Thailand*

[2]*Department of Mathematics, Faculty of Science and Technology, Surindra Rajabhat University, Surin 32000, Thailand*

[3]*Department of Mathematics, Faculty of Science, University of Phayao, Phayao 56000, Thailand*

[*]*Corresponding author:* warud.n@msu.ac.th

Abstract. The concept of fuzzy *Bd*-ideals in *Bd*-algebras was introduced by Nakkhasen et al. in 2024. In this study, we consider the notions of fuzzy translations, fuzzy multiplications, and fuzzy magnified translations to investigate fuzzy *Bd*-ideals in *Bd*-algebras. Subsequently, we examine various features and characterizations of fuzzy *Bd*-ideals in *Bd*-algebras through the applications of fuzzy translations, fuzzy multiplications, and fuzzy magnified translations. Finally, we demonstrate how fuzzy translations, fuzzy multiplications, and fuzzy magnified translations of fuzzy *Bd*-ideals in *Bd*-algebras are interconnected through a homomorphism function.

1. Introduction

The fuzzy set idea in a nonempty set X is a function that assigns values from the closed interval $[0,1]$ in the real numbers to elements of X. This notion was established by Zadeh [18] and continues to be an essential tool in several areas, including artificial intelligence, control systems, and decision-making processes. Algebra is another field of study that uses the fuzzy sets to determine their properties. Muhiuddin and Aldhafeeri [8] introduced the notions of uni-hesitant fuzzy algebras and uni-hesitant fuzzy (closed) ideals in *BCK*-algebras and *BCI*-algebras, examining various associated features. Then Ratchakhwan et al. [14] introduced the concept of $(\inf, \sup)$-hesitant fuzzy ideals, which generalizes the concept of interval-valued fuzzy ideals in *BCK*/*BCI*-algebras. They also investigated the related properties of this concept. Meanwhile, Jiang [6] explored the homomorphic and inverse images of hesitant fuzzy dot subalgebras, along

Received: Oct. 17, 2025.

2020 *Mathematics Subject Classification.* 08A72, 46H10.

Key words and phrases. *Bd*-algebra; fuzzy *Bd*-ideal; fuzzy translation; fuzzy multiplication; fuzzy magnified translation.

https://doi.org/10.28924/2291-8639-23-2025-317

ISSN: 2291-8639

with the concept of hesitant fuzzy dot ideals, in the framework of B-algebras. Subsequently, Oli and Tefera [13] introduced the concepts of left and right fuzzy derivations of d-ideals within d-algebras. They also characterized the right and left fuzzy derivations of ideals in d-algebras.

In 2022, Bantaojai et al. [2] integrated some needs from B-algebras and d-algebras to create a novel algebra called Bd-algebras, which explains certain features of Bd-ideals and Bd-subalgebras in Bd-algebras. Following this, Nakkhasen et al. examined the concept of fuzzy sets within the structure of Bd-algebras in 2024, introducing fuzzy Bd-subalgebras and fuzzy Bd-ideals, and investigating their characteristics as detailed in [12] and [11], respectively. In the same year, Iampan and Nakkhasen [5] also researched the characterizations of prime fuzzy Bd-ideals in Bd-algebras. Moreover, in 2025, Nakkhasen et al. [9] presented the general idea of fuzzy Bd-subalgebras in Bd-algebras as the fuzzy dot Bd-subalgebras and studied the relationship of this idea in terms of a homomorphism. Recently, Yilmaz [17] applied the concept of hybrid structures, which are a generalization of fuzzy sets, to the Bd-algebras. He presented various properties of hybrid subalgebras within these algebras.

The concepts of fuzzy translations [16], fuzzy multiplications [16], and fuzzy magnified translations [15] represent another generalization of fuzzy sets. In 2009, Lee et al. [7] investigated fuzzy translations and fuzzy multiplications in BCK/BCI-algebras. Thereafter, in 2013, Chandramouleeswaran et al. [3] discussed the concepts of fuzzy translations and fuzzy multiplications in BF/BG-algebras. In addition, Guntasow et al. [4] explored fuzzy translations of fuzzy sets within UP-algebras. They introduced the notions of fuzzy α-translations and fuzzy β-translations for fuzzy sets, examining their fundamental properties and providing illustrative examples. Subsequently, Alshehri [1] also used the fuzzy translations and the fuzzy multiplication concepts to describe BRK-algebras. In 2024, Nakkhasen et al. [12] contributed to the investigation of fuzzy translations, fuzzy multiplications, and fuzzy magnified translations by classifying the properties of fuzzy Bd-subalgebras in Bd-algebras. This research is focused on using the concepts of fuzzy translations, fuzzy multiplications, and fuzzy magnified translations to examine fuzzy Bd-ideals in Bd-algebras. We also consider different properties and description of fuzzy Bd-ideals in Bd-algebras by using fuzzy translations, fuzzy multiplications, and fuzzy magnified translations. In the end, we show the connections of fuzzy translations, fuzzy multiplications, and fuzzy magnified translations of fuzzy Bd-ideals in Bd-algebras by a homomorphism function.

2. Preliminaries

This section reviews some fundamental definitions used throughout this article. A mapping $\vartheta : X \to [0,1]$ from a nonempty set X to a unit interval is referred to as a *fuzzy set* [18]. Let ϑ and δ be any two arbitrary fuzzy sets of a nonempty set X. We further denote:

(i) $(\vartheta \cap \delta)(x) = \min\{\vartheta(x), \delta(x)\}$ for all $x \in X$;
(ii) $(\vartheta \cup \delta)(x) = \max\{\vartheta(x), \delta(x)\}$ for all $x \in X$.

Let ζ be a fuzzy set of a nonempty set X and $T_\zeta = 1 - \sup\{\zeta(x) \mid x \in X\}$. Suppose that $r \in [0, T_\zeta]$ and $m \in [0, 1]$. Then:

(i) a fuzzy set ζ_r^T of X is called a *fuzzy translation* [16] of ζ if $\zeta_r^T(x) = \zeta(x) + r$ for all $x \in X$;
(ii) a fuzzy set ζ_m^M of X is called a *fuzzy multiplication* [16] of ζ if $\zeta_m^M(x) = m\zeta(x)$ for all $x \in X$;
(iii) a fuzzy set ζ_{mr}^{MT} of X is called a *fuzzy magnified translation* [15] of ζ if $\zeta_{mr}^{MT}(x) = m\zeta(x) + r$ for all $x \in X$.

Definition 2.1. *[2] An algebra $(X, *, 0)$ is called a Bd-algebra if it satisfies the following axioms: for every $x, y \in X$,*

(i) *$x * 0 = x$;*
(ii) *if $x * y = 0$ and $y * x = 0$, then $x = y$.*

In this article, we denote a *Bd*-algebra $(X, *, 0)$ simply as X, unless stated differently.

Example 2.1. *[10] Let $\mathbb{Z}$ be the set of integers. It is clear that $(\mathbb{Z}, -, 0)$ constitutes a Bd-algebra under the usual subtraction operation on integers.*

Example 2.2. *Let $X = \{0, a, b, c\}$. Define a binary operation $*$ on the set X assuming the following table:*

$*$	0	a	b	c
0	0	0	c	c
a	a	0	b	c
b	b	b	b	c
c	c	b	b	c

Through straightforward computations, we have $(X, *, 0)$ is a Bd-algebra.

Definition 2.2. *[2] A nonempty subset A of a Bd-algebra X is considered a Bd-ideal of X if it satisfies the following requirement:*

(i) *$0 \in A$;*
(ii) *if for any $x, y \in X$, $x * y \in A$ and $y \in A$, then $x \in A$;*
(iii) *$x * y \in A$, for all $x \in A$ and $y \in X$.*

Example 2.3. *In Example 2.2, we have $A = \{0, a\}$ is a Bd-ideal of X.*

Definition 2.3. *[11] Let X be a Bd-algebra. A fuzzy set ϑ of X is known as a fuzzy Bd-ideal of X if it satisfies the following inequality for every $x, y \in X$:*

(i) *$\vartheta(0) \geq \vartheta(x)$;*
(ii) *$\vartheta(x) \geq \min\{\vartheta(x * y), \vartheta(y)\}$;*
(iii) *$\vartheta(x * y) \geq \vartheta(x)$.*

Example 2.4. *Let $X = \{0, a, b, c\}$ be a Bd-algebra as defined in Example 2.2. Now, we define a fuzzy set ϑ of X by:*

X	0	a	b	c
ϑ	0.9	0.5	0.1	0.1

By means of simple computations, we have ϑ is a fuzzy Bd-ideal of X.

3. Fuzzy magnified translations of fuzzy Bd-ideals

In this section, we examine and characterize fuzzy Bd-ideals of Bd-algebras related to fuzzy translations, fuzzy multiplications, and fuzzy magnified translations.

Proposition 3.1. *Let X be a Bd-algebra, and let ζ and ξ be fuzzy sets of X. Suppose that $\zeta^T_{r_1}$ and $\xi^T_{r_2}$ are fuzzy translations of ζ and ξ, respectively, such that $r_1 \in [0, T_\zeta]$, and $r_2 \in [0, T_\xi]$. If $\zeta^T_{r_1}$ and $\xi^T_{r_2}$ are fuzzy Bd-ideals of X, then $\zeta^T_{r_1} \cap \xi^T_{r_2}$ is a fuzzy Bd-ideal of X.*

Proof. Assume that $\zeta^T_{r_1}$ and $\xi^T_{r_2}$ are fuzzy Bd-ideals of X. Let $x, y \in X$. Then we have

$$\begin{aligned}\left(\zeta^T_{r_1} \cap \xi^T_{r_2}\right)(0) &= \min\{\zeta^T_{r_1}(0), \xi^T_{r_2}(0)\}\\ &\geq \min\{\zeta^T_{r_1}(x), \xi^T_{r_2}(x)\}\\ &= \left(\zeta^T_{r_1} \cap \xi^T_{r_2}\right)(x),\end{aligned}$$

$$\begin{aligned}\left(\zeta^T_{r_1} \cap \xi^T_{r_2}\right)(x) &= \min\{\zeta^T_{r_1}(x), \xi^T_{r_2}(x)\}\\ &\geq \min\{\min\{\zeta^T_{r_1}(x*y), \zeta^T_{r_1}(y)\}, \min\{\xi^T_{r_2}(x*y), \xi^T_{r_2}(y)\}\}\\ &= \min\{\min\{\zeta^T_{r_1}(x*y), \xi^T_{r_2}(x*y)\}, \min\{\zeta^T_{r_1}(y), \xi^T_{r_2}(y)\}\}\\ &= \min\{\left(\zeta^T_{r_1} \cap \xi^T_{r_2}\right)(x*y), \left(\zeta^T_{r_1} \cap \xi^T_{r_2}\right)(y)\},\end{aligned}$$

$$\begin{aligned}\left(\zeta^T_{r_1} \cap \xi^T_{r_2}\right)(x*y) &= \min\{\zeta^T_{r_1}(x*y), \xi^T_{r_2}(x*y)\}\\ &\geq \min\{\zeta^T_{r_1}(x), \xi^T_{r_2}(x)\}\\ &= \left(\zeta^T_{r_1} \cap \xi^T_{r_2}\right)(x).\end{aligned}$$

This shows that $\zeta^T_{r_1} \cap \xi^T_{r_2}$ is a fuzzy Bd-ideal of X. □

The following propositions can be proved to be similar to Proposition 3.1.

Proposition 3.2. *Let X be a Bd-algebra, and let ζ and ξ be fuzzy sets of X. Suppose that $\zeta^M_{m_1}$ and $\xi^M_{m_2}$ are fuzzy multiplications of ζ and ξ, respectively, such that $m_1, m_2 \in [0, 1]$. If $\zeta^M_{m_1}$ and $\xi^M_{m_2}$ are fuzzy Bd-ideals of X, then $\zeta^M_{m_1} \cap \xi^M_{m_2}$ is a fuzzy Bd-ideal of X.*

Proposition 3.3. *Let X be a Bd-algebra, and let ζ and ξ be fuzzy sets of X. Suppose that $\zeta^{MT}_{m_1r_1}$ and $\xi^{MT}_{m_2r_2}$ are fuzzy magnified translations of ζ and ξ, respectively, such that $r_1 \in [0, T_\zeta]$, $r_2 \in [0, T_\xi]$, and $m_1, m_2 \in [0, 1]$. If $\zeta^{MT}_{m_1r_1}$ and $\xi^{MT}_{m_2r_2}$ are fuzzy Bd-ideals of X, then $\zeta^{MT}_{m_1r_1} \cap \xi^{MT}_{m_2r_2}$ is a fuzzy Bd-ideal of X.*

On the other hand, the union of any two fuzzy translations (or multiplications, or magnified translations) that are fuzzy Bd-ideals of a Bd-algebra X may not constitute a fuzzy Bd-ideal of X, as seen in the following example.

Example 3.1. *Let $X = \{0, a, b, c\}$ be a Bd-algebra with the binary operation $*$ on X, see [11], by the following table:*

$*$	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	b	b	b	b
c	c	c	a	a

Define the fuzzy sets ζ and ξ of X by

X	0	a	b	c
ζ	0.4	0.2	0	0
ξ	0.5	0.1	0.3	0.1

Then $T_\zeta = 1 - 0.4 = 0.6$ and $T_\xi = 1 - 0.5 = 0.5$. Now, choose $r_1 = 0.3 \in [0, 0.6]$, $r_2 = 0.2 \in [0, 0.5]$, $m_1 = 0.1 \in [0, 1]$, and $m_2 = 0.4 \in [0, 1]$. So, the mapping $\zeta^{MT}_{(0.1)(0.3)} : X \to [0, 1]$ and $\xi^{MT}_{(0.4)(0.2)} : X \to [0, 1]$ are defined by

X	0	a	b	c
$\zeta^{MT}_{(0.1)(0.3)}$	0.34	0.32	0.30	0.30

X	0	a	b	c
$\xi^{MT}_{(0.4)(0.2)}$	0.40	0.24	0.32	0.24

Through standard computations, we have $\zeta^{MT}_{(0.1)(0.3)}$ and $\xi^{MT}_{(0.4)(0.2)}$ are fuzzy Bd-ideals of X. But $\zeta^{MT}_{(0.1)(0.3)} \cup \xi^{MT}_{(0.4)(0.2)}$ is not a fuzzy Bd-ideal of X, since

$$\left(\zeta^{MT}_{(0.1)(0.3)} \cup \xi^{MT}_{(0.4)(0.2)}\right)(c) = 0.30 \not\geq 0.32 = \min\left\{\left(\zeta^{MT}_{(0.1)(0.3)} \cup \xi^{MT}_{(0.4)(0.2)}\right)(c * b), \left(\zeta^{MT}_{(0.1)(0.3)} \cup \xi^{MT}_{(0.4)(0.2)}\right)(b)\right\}.$$

Next, we characterize fuzzy *Bd*-ideals of *Bd*-algebras in relation to fuzzy translations, fuzzy multiplications, and fuzzy magnified translations.

Theorem 3.1. *Let X be a Bd-algebra, ζ be a fuzzy set of X, $r \in [0, T_\zeta]$, and ζ^T_r be the fuzzy translation of ζ with respect to r. Then ζ is a fuzzy Bd-ideal of X if and only if ζ^T_r is a fuzzy Bd-ideal of X.*

Proof. Assume that ζ is a fuzzy *Bd*-ideal of X. Let $x, y \in X$. Then we have

$$\zeta^T_r(0) = \zeta(0) + r \geq \zeta(x) + r = \zeta^T_r(x),$$

$$\begin{aligned}\zeta^T_r(x) &= \zeta(x) + r \\ &\geq \min\{\zeta(x * y), \zeta(y)\} + r \\ &= \min\{\zeta(x * y) + r, \zeta(y) + r\} \\ &= \min\{\zeta^T_r(x * y), \zeta^T_r(y)\},\end{aligned}$$

$$\zeta^T_r(x * y) = \zeta(x * y) + r \geq \zeta(x) + r = \zeta^T_r(x).$$

Hence, ζ_r^T is a fuzzy Bd-ideal of X. Conversely, assume that ζ_r^T is a fuzzy Bd-ideal of X. Let $x, y \in X$. Then we consider

$$\zeta(0) + r = \zeta_r^T(0) \geq \zeta_r^T(x) = \zeta(x) + r,$$

$$\begin{aligned}\zeta(x) + r &= \zeta_r^T(x) \\ &\geq \min\{\zeta_r^T(x * y), \zeta_r^T(y)\} \\ &= \min\{\zeta(x * y) + r, \zeta(y) + r\} \\ &= \min\{\zeta(x * y), \zeta(y)\} + r,\end{aligned}$$

$$\zeta(x * y) + r = \zeta_r^T(x * y) \geq \zeta_r^T(x) = \zeta(x) + r.$$

Since $r \geq 0$, we have $\zeta(0) \geq \zeta(x)$, $\zeta(x) \geq \min\{\zeta(x * y), \zeta(y)\}$, and $\zeta(x * y) \geq \zeta(x)$. This shows that ζ is a fuzzy Bd-ideal of X. □

Theorem 3.2. *Let X be a Bd-algebra, ζ be a fuzzy set of X, $m \in (0, 1]$, and ζ_m^M be the fuzzy multiplication of ζ with respect to m. Then ζ is a fuzzy Bd-ideal of X if and only if ζ_m^M is a fuzzy Bd-ideal of X.*

Proof. Assume that ζ is a fuzzy Bd-ideal of X. For any $x, y \in X$, we have

$$\zeta_m^M(0) = m\zeta(0) \geq m\zeta(x) = \zeta_m^M(x),$$

$$\begin{aligned}\zeta_m^M(x) &= m\zeta(x) \\ &\geq m \min\{\zeta(x * y), \zeta(y)\} \\ &= \min\{m\zeta(x * y), m\zeta(y)\} \\ &= \min\{\zeta_m^M(x * y), \zeta_m^M(y)\},\end{aligned}$$

$$\zeta_m^M(x * y) = m\zeta(x * y) \geq m\zeta(x) = \zeta_m^M(x).$$

This implies that ζ_m^M is a fuzzy Bd-ideal of X. Conversely, assume that ζ_m^M is a fuzzy Bd-ideal of X. Let $x, y \in X$. Then we get

$$m\zeta(0) = \zeta_m^M(0) \geq \zeta_m^M(x) = m\zeta(x),$$

$$\begin{aligned}m\zeta(x) &= \zeta_m^M(x) \\ &\geq \min\{\zeta_m^M(x * y), \zeta_m^M(y)\} \\ &= \min\{m\zeta(x * y), m\zeta(y)\} \\ &= m \min\{\zeta(x * y), \zeta(y)\},\end{aligned}$$

$$m\zeta(x * y) = \zeta_m^M(x * y) \geq \zeta_m^M(x) = m\zeta(x).$$

Since $m > 0$, we have $\zeta(0) \geq \zeta(x)$, $\zeta(x) \geq \min\{\zeta(x * y), \zeta(y)\}$, and $\zeta(x * y) \geq \zeta(x)$. Therefore, ζ is a fuzzy Bd-ideal of X. □

Theorem 3.3. *Let X be a Bd-algebra, ζ be a fuzzy set of X, $r \in [0, T_\zeta]$, $m \in (0,1]$, and ζ_{mr}^{MT} be the fuzzy magnified translation of ζ with respect to m and r. Then ζ is a fuzzy Bd-ideal of X if and only if ζ_{mr}^{MT} is a fuzzy Bd-ideal of X.*

Proof. Assume that ζ is a fuzzy Bd-ideal of X. For every $x, y \in X$, we have

$$\zeta_{mr}^{MT}(0) = m\zeta(0) + r \geq m\zeta(x) + r = \zeta_{mr}^{MT}(x),$$

$$\begin{aligned}\zeta_{mr}^{MT}(x) &= m\zeta(x) + r \\ &\geq m \min\{\zeta(x * y), \zeta(y)\} + r \\ &= \min\{m\zeta(x * y) + r, m\zeta(y) + r\} \\ &= \min\{\zeta_{mr}^{MT}(x * y), \zeta_{mr}^{MT} y)\},\end{aligned}$$

$$\zeta_{mr}^{MT}(x * y) = m\zeta(x * y) + r \geq m\zeta(x) + r = \zeta_{mr}^{MT}(x).$$

Thus, ζ_{mr}^{MT} is a fuzzy Bd-ideal of X. Conversely, assume that ζ_{mr}^{MT} is a fuzzy Bd-ideal of X. Let $x, y \in X$. Then we have

$$m\zeta(0) + r = \zeta_{mr}^{MT}(0) \geq \zeta_{mr}^{MT}(x) = m\zeta(x) + r,$$

$$\begin{aligned}m\zeta(x) + r &= \zeta_{mr}^{MT}(x) \\ &\geq \min\{\zeta_{mr}^{MT}(x * y), \zeta_{mr}^{MT}(y)\} \\ &= \min\{m\zeta(x * y) + r, m\zeta(y) + r\} \\ &= m \min\{\zeta(x * y), \zeta(y)\} + r,\end{aligned}$$

$$m\zeta(x * y) + r = \zeta_{mr}^{MT}(x * y) \geq \zeta_{mr}^{MT}(x) = m\zeta(x) + r.$$

Since $m > 0$ and $r \geq 0$, we have $\zeta(0) \geq \zeta(x)$, $\zeta(x) \geq \min\{\zeta(x * y), \zeta(y)\}$, and $\zeta(x * y) \geq \zeta(x)$. It follows that ζ is a fuzzy Bd-ideal of X. □

4. Homomorphism on fuzzy magnified translations of fuzzy Bd-ideals

This section studies the relationships on fuzzy translations, fuzzy multiplications, and fuzzy magnified translations of fuzzy Bd-ideals in Bd-algebras under a homomorphism function.

Let $\Psi : X \to Y$ be a mapping of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ϑ be a fuzzy set of Y. The fuzzy set ϑ^Ψ of X is defined by $\vartheta^\Psi(x) = \vartheta(\Psi(x))$ for all $x \in X$. The mapping Ψ is called a *homomorphism* (see, [9]) of Bd-algebras X and Y if $\Psi(0) = 0'$ and $\Psi(x * y) = \Psi(x) \circ \Psi(y)$ for all $x, y \in X$. In addition, the homomorphism Ψ is said to be an *epimorphism* if Ψ is onto.

Theorem 4.1. *Let $\Psi : X \to Y$ be a homomorphism of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_r^T is a fuzzy translation of ζ with respect to $r \in [0, T_\zeta]$. If ζ_r^T is a fuzzy Bd-ideal of Y, then $\left(\zeta_r^T\right)^\Psi$ is a fuzzy Bd-ideal of X.*

Proof. Assume that ζ_r^T is a fuzzy Bd-ideal of Y. For any $x, y \in X$, we have

$$\left(\zeta_r^T\right)^\Psi(0) = \zeta_r^T(\Psi(0)) = \zeta_r^T(0') \geq \zeta_r^T(\Psi(x)) = \left(\zeta_r^T\right)^\Psi(x),$$

$$\begin{aligned}\left(\zeta_r^T\right)^\Psi(x) &= \zeta_r^T(\Psi(x)) \\ &\geq \min\{\zeta_r^T(\Psi(x) \circ \Psi(y)), \zeta_r^T(\Psi(y))\} \\ &= \min\{\left(\zeta_r^T\right)^\Psi(x * y), \left(\zeta_r^T\right)^\Psi(y)\},\end{aligned}$$

$$\left(\zeta_r^T\right)^\Psi(x * y) = \zeta_r^T(\Psi(x * y)) = \zeta_r^T(\Psi(x) \circ \Psi(y)) \geq \zeta_r^T(\Psi(x)) = \left(\zeta_r^T\right)^\Psi(x).$$

Hence, $\left(\zeta_r^T\right)^\Psi$ is a fuzzy Bd-ideal of X. □

Theorem 4.2. *Let $\Psi : X \to Y$ be a homomorphism of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_m^M is a fuzzy multiplication of ζ with respect to $m \in [0, 1]$. If ζ_m^M is a fuzzy Bd-ideal of Y, then $\left(\zeta_m^M\right)^\Psi$ is a fuzzy Bd-ideal of X.*

Proof. Assume that ζ_m^M is a fuzzy Bd-ideal of Y. Let $x, y \in X$. Then we have

$$\left(\zeta_m^M\right)^\Psi(0) = \zeta_m^M(\Psi(0)) = \zeta_m^M(0') \geq \zeta_m^M(\Psi(x)) = \left(\zeta_m^M\right)^\Psi(x),$$

$$\begin{aligned}\left(\zeta_m^M\right)^\Psi(x) &= \zeta_m^M(\Psi(x)) \\ &\geq \min\{\zeta_m^M(\Psi(x) \circ \Psi(y)), \zeta_m^M(\Psi(y))\} \\ &= \min\{\left(\zeta_m^M\right)^\Psi(x * y), \left(\zeta_m^M\right)^\Psi(y)\},\end{aligned}$$

$$\left(\zeta_m^M\right)^\Psi(x * y) = \zeta_m^M(\Psi(x * y)) = \zeta_m^M(\Psi(x) \circ \Psi(y)) \geq \zeta_m^M(\Psi(x)) = \left(\zeta_m^M\right)^\Psi(x).$$

Thus, $\left(\zeta_m^M\right)^\Psi$ is a fuzzy Bd-ideal of X. □

Theorem 4.3. *Let $\Psi : X \to Y$ be a homomorphism of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_{mr}^{MT} is a fuzzy magnified translation of ζ with respect to $m \in [0, 1]$ and $r \in [0, T_\zeta]$. If ζ_{mr}^{MT} is a fuzzy Bd-ideal of Y, then $\left(\zeta_{mr}^{MT}\right)^\Psi$ is a fuzzy Bd-ideal of X.*

Proof. Assume that ζ_{mr}^{MT} is a fuzzy Bd-ideal of Y. For every $x, y \in X$, we have

$$\left(\zeta_{mr}^{MT}\right)^\Psi(0) = \zeta_{mr}^{MT}(\Psi(0)) = \zeta_{mr}^{MT}(0') \geq \zeta_{mr}^{MT}(\Psi(x)) = \left(\zeta_{mr}^{MT}\right)^\Psi(x),$$

$$\begin{aligned}\left(\zeta_{mr}^{MT}\right)^\Psi(x) &= \zeta_{mr}^{MT}(\Psi(x)) \\ &\geq \min\{\zeta_{mr}^{MT}(\Psi(x) \circ \Psi(y)), \zeta_{mr}^{MT}(\Psi(y))\} \\ &= \min\{\left(\zeta_{mr}^{MT}\right)^\Psi(x * y), \left(\zeta_{mr}^{MT}\right)^\Psi(y)\},\end{aligned}$$

$$\left(\zeta_{mr}^{MT}\right)^\Psi(x * y) = \zeta_{mr}^{MT}(\Psi(x * y)) = \zeta_{mr}^{MT}(\Psi(x) \circ \Psi(y)) \geq \zeta_{mr}^{MT}(\Psi(x)) = \left(\zeta_{mr}^{MT}\right)^\Psi(x).$$

Therefore, $\left(\zeta_{mr}^{MT}\right)^{\Psi}$ is a fuzzy Bd-ideal of X. $\square$

The following example shows that the converses of Theorems 4.1, 4.2, and 4.3 are not true.

Example 4.1. *Let $X = \{0, u, v\}$ and $Y = \{0', a, b, c\}$. Consider the Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, which are defined in the following tables:*

$*$	0	u	v
0	0	u	v
u	u	0	0
v	v	v	v

$\circ$	$0'$	a	b	c
$0'$	$0'$	$0'$	$0'$	$0'$
a	a	a	a	a
b	b	b	b	b
c	c	c	a	a

Define the mapping $\Psi : X \to Y$ by

$$\Psi(x) = \begin{cases} 0' & \text{if } x \in \{0, u\}, \\ b & \text{otherwise} \end{cases} \text{ for all } x \in X.$$

It's straightforward to check that Ψ is a homomorphism. Now, the fuzzy set ζ of Y is defined by:

Y	$0'$	a	b	c
ζ	0.8	0.2	0.2	0.4

Choose, $r = 0.1 \in [0, T_\zeta]$ and $m = 0.5 \in [0, 1]$. Then the fuzzy magnified translation $\zeta_{(0.5)(0.1)}^{MT}$ of ζ is defined as follows:

Y	$0'$	a	b	c
$\zeta_{(0.5)(0.1)}^{MT}$	0.5	0.2	0.2	0.3

We obtain that the fuzzy set $\left(\zeta_{(0.5)(0.1)}^{MT}\right)^{\Psi}$ of X can be described as follows:

X	0	u	v
$\left(\zeta_{(0.5)(0.1)}^{MT}\right)^{\Psi}$	0.5	0.5	0.2

Moreover, $\left(\zeta_{(0.5)(0.1)}^{MT}\right)^{\Psi}$ is a fuzzy Bd-ideal of X, while $\zeta_{(0.5)(0.1)}^{MT}$ is not a fuzzy Bd-ideal of Y, since $\zeta_{(0.5)(0.1)}^{MT}(c \circ c) = 0.2 \ngeq 0.3 = \zeta_{(0.5)(0.1)}^{MT}(c)$.

We now need to add the following condition for the converses of Theorems 4.1, 4.2, and 4.3 to be valid.

Theorem 4.4. *Let $\Psi : X \to Y$ be an epimorphism of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_r^T is a fuzzy translation of ζ with respect to $r \in [0, T_\zeta]$. If $\left(\zeta_r^T\right)^{\Psi}$ is a fuzzy Bd-ideal of X, then ζ_r^T is a fuzzy Bd-ideal of Y.*

Proof. Assume that $\left(\zeta_r^T\right)^{\Psi}$ is a fuzzy Bd-ideal of X. Let $a, b \in Y$. Since Ψ is onto, we have there exist $x, y \in X$ such that $\Psi(x) = a$ and $\Psi(y) = b$. Then we obtain

$$\zeta_r^T(0') = \zeta_r^T(\Psi(0)) = \left(\zeta_r^T\right)^{\Psi}(0) \geq \left(\zeta_r^T\right)^{\Psi}(x) = \zeta_r^T(\Psi(x)) = \zeta_r^T(a),$$

$$\begin{aligned}\zeta_r^T(a) &= \zeta_r^T(\Psi(x)) = \left(\zeta_r^T\right)^\Psi(x) \\ &\geq \min\{\left(\zeta_r^T\right)^\Psi(x*y), \left(\zeta_r^T\right)^\Psi(y)\} \\ &= \min\{\zeta_r^T(\Psi(x*y)), \zeta_r^T(\Psi(y))\} \\ &= \min\{\zeta_r^T(\Psi(x)\circ\Psi(y)), \zeta_r^T(\Psi(y))\} \\ &= \min\{\zeta_r^T(a\circ b), \zeta_r^T(b))\},\end{aligned}$$

$$\zeta_r^T(a\circ b) = \zeta_r^T(\Psi(x)\circ\Psi(y)) = \zeta_r^T(\Psi(x*y)) = \left(\zeta_r^T\right)^\Psi(x*y) \geq \left(\zeta_r^T\right)^\Psi(x) = \zeta_r^T(\Psi(x)) = \zeta_r^T(a).$$

Therefore, ζ_r^T is a fuzzy Bd-ideal of Y. □

The following theorems can be similarly confirmed.

Theorem 4.5. *Let $\Psi : X \to Y$ be an epimorphism of Bd-algebras $(X,*,0)$ and $(Y,\circ,0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_m^M is a fuzzy multiplication of ζ with respect to $m \in [0,1]$. If $\left(\zeta_m^M\right)^\Psi$ is a fuzzy Bd-ideal of X, then ζ_m^M is a fuzzy Bd-ideal of Y.*

Proof. Assume that $\left(\zeta_m^M\right)^\Psi$ is a fuzzy Bd-ideal of X. Let $a,b \in Y$. Since Ψ is onto, we have there exist $x,y \in X$ such that $\Psi(x) = a$ and $\Psi(y) = b$. Then we have

$$\zeta_m^M(0') = \zeta_m^M(\Psi(0)) = \left(\zeta_m^M\right)^\Psi(0) \geq \left(\zeta_m^M\right)^\Psi(x) = \zeta_m^M(\Psi(x)) = \zeta_m^M(a),$$

$$\begin{aligned}\zeta_m^M(a) &= \zeta_m^M(\Psi(x)) = \left(\zeta_m^M\right)^\Psi(x) \\ &\geq \min\{\left(\zeta_m^M\right)^\Psi(x*y), \left(\zeta_m^M\right)^\Psi(y)\} \\ &= \min\{\zeta_m^M(\Psi(x*y)), \zeta_m^M(\Psi(y))\} \\ &= \min\{\zeta_m^M(\Psi(x)\circ\Psi(y)), \zeta_m^M(\Psi(y))\} \\ &= \min\{\zeta_m^M(a\circ b), \zeta_m^M(b))\},\end{aligned}$$

$$\zeta_m^M(a\circ b) = \zeta_m^M(\Psi(x)\circ\Psi(y)) = \zeta_m^M(\Psi(x*y)) = \left(\zeta_m^M\right)^\Psi(x*y) \geq \left(\zeta_m^M\right)^\Psi(x) = \zeta_m^M(\Psi(x)) = \zeta_m^M(a).$$

Hence, ζ_m^M is a fuzzy Bd-ideal of Y. □

Theorem 4.6. *Let $\Psi : X \to Y$ be an epimorphism of Bd-algebras $(X,*,0)$ and $(Y,\circ,0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_{mr}^{MT} is a fuzzy magnified translation of ζ with respect to $m \in [0,1]$ and $r \in [0,T_\zeta]$. If $\left(\zeta_{mr}^{MT}\right)^\Psi$ is a fuzzy Bd-ideal of X, then ζ_{mr}^{MT} is a fuzzy Bd-ideal of Y.*

Proof. Assume that $\left(\zeta_{mr}^{MT}\right)^\Psi$ is a fuzzy Bd-ideal of X. Let $a,b \in Y$. Since Ψ is onto, we have there exist $x,y \in X$ such that $\Psi(x) = a$ and $\Psi(y) = b$. Then we get

$$\zeta_{mr}^{MT}(0') = \zeta_{mr}^{MT}(\Psi(0)) = \left(\zeta_{mr}^{MT}\right)^\Psi(0) \geq \left(\zeta_{mr}^{MT}\right)^\Psi(x) = \zeta_{mr}^{MT}(\Psi(x)) = \zeta_{mr}^{MT}(a),$$

$$\begin{aligned}
\zeta_{mr}^{MT}(a) &= \zeta_{mr}^{MT}(\Psi(x)) = \left(\zeta_{mr}^{MT}\right)^{\Psi}(x) \\
&\geq \min\{\left(\zeta_{mr}^{MT}\right)^{\Psi}(x * y), \left(\zeta_{mr}^{MT}\right)^{\Psi}(y)\} \\
&= \min\{\zeta_{mr}^{MT}(\Psi(x * y)), \zeta_{mr}^{MT}(\Psi(y))\} \\
&= \min\{\zeta_{mr}^{MT}(\Psi(x) \circ \Psi(y)), \zeta_{mr}^{MT}(\Psi(y))\} \\
&= \min\{\zeta_{mr}^{MT}(a \circ b), \zeta_{mr}^{MT}(b)\},
\end{aligned}$$

$$\zeta_{mr}^{MT}(a \circ b) = \zeta_{mr}^{MT}(\Psi(x) \circ \Psi(y)) = \zeta_{mr}^{MT}(\Psi(x * y)) = \left(\zeta_{mr}^{MT}\right)^{\Psi}(x * y) \geq \left(\zeta_{mr}^{MT}\right)^{\Psi}(x) = \zeta_{mr}^{MT}(\Psi(x)) = \zeta_{mr}^{MT}(a).$$

Consequently, ζ_{mr}^{MT} is a fuzzy *Bd*-ideal of Y. □

By Theorems 3.1-3.3 in conjunction with Theorems 4.1-4.6, we can deduce the following corollaries.

Corollary 4.1. *Let $\Psi : X \to Y$ be an epimorphism of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_r^T is a fuzzy translation of ζ with respect to $r \in [0, T_\zeta]$. Then $\left(\zeta_r^T\right)^{\Psi}$ is a fuzzy Bd-ideal of X if and only if ζ is a fuzzy Bd-ideal of Y.*

Corollary 4.2. *Let $\Psi : X \to Y$ be an epimorphism of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_m^M is a fuzzy multiplication of ζ with respect to $m \in (0, 1]$. Then $\left(\zeta_m^M\right)^{\Psi}$ is a fuzzy Bd-ideal of X if and only if ζ is a fuzzy Bd-ideal of Y.*

Corollary 4.3. *Let $\Psi : X \to Y$ be an epimorphism of Bd-algebras $(X, *, 0)$ and $(Y, \circ, 0')$, and let ζ be a fuzzy set of Y. Suppose that ζ_{mr}^{MT} is a fuzzy magnified translation of ζ with respect to $m \in (0, 1]$ and $r \in [0, T_\zeta]$. Then $\left(\zeta_{mr}^{MT}\right)^{\Psi}$ is a fuzzy Bd-ideal of X if and only if ζ is a fuzzy Bd-ideal of Y.*

5. Conclusion

In this research, we used the notions of fuzzy translations, fuzzy multiplications, and fuzzy magnified translations to investigate the properties of fuzzy *Bd*-ideals in *Bd*-algebras. We then presented the intersection of fuzzy translations, which includes fuzzy multiplications and fuzzy magnified translations that together constitute fuzzy *Bd*-ideals of *Bd*-algebras. However, the union of these concepts may not hold true, as shown in Example 3.1. Subsequently, we utilized fuzzy translations, fuzzy multiplications, and fuzzy magnified translations to describe the concept of fuzzy *Bd*-ideals in *Bd*-algebras. In the final section, we examined the relationships of fuzzy *Bd*-ideals under a homomorphism of *Bd*-algebras. Our focus was on fuzzy translations, fuzzy multiplications, and fuzzy magnified translations, as discussed in Theorems 4.1-4.6. We also summarized our findings from the previous section as Corollaries 4.1, 4.2, and 4.3. Future research may utilize the concept of intuitionistic fuzzy sets to develop intuitionistic fuzzy *Bd*-subalgebras and intuitionistic fuzzy *Bd*-ideals, thereby generalizing fuzzy *Bd*-subalgebras and fuzzy *Bd*-ideals within the framework of *Bd*-algebras.

Acknowledgements: This research project was financially supported by Mahasarakham University.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] H.A. Alshehri, Fuzzy Translation and Fuzzy Multiplication in BRK-Algebras, Eur. J. Pure Appl. Math. 14 (2021), 737–745. https://doi.org/10.29020/nybg.ejpam.v14i3.3971.

[2] T. Bantaojai, C. Suanoom, J. Phuto, A. Iampan, On *Bd*-Algebras, Int. J. Math. Comput. Sci. 17 (2022), 731–737.

[3] M. Chandramouleeswaran, Fuzzy Translation and Fuzzy Multiplication in BF/BG-Algebras, Indian J. Sci. Technol. 6 (2013), 1–4. https://doi.org/10.17485/ijst/2013/v6i9.2.

[4] T. Guntasow, S. Sajak, A. Jomkham, A. Iampan, Fuzzy Translations of a Fuzzy Set in *UP*-Algebras, J. Indones. Math. Soc. 23 (2017), 1–19. https://doi.org/10.22342/jims.23.2.371.1-19.

[5] A. Iampan, W. Nakkhasen, Prime Fuzzy *Bd*-Ideals of *Bd*-Algebras, Int. J. Math. Comput. Sci. 19 (2024), 941–948.

[6] M. Jiang, Hesitant Fuzzy Dot Subalgebra and Dot Ideals of B-Algebra, J. Intell. Fuzzy Syst. 43 (2022), 6203–6212. https://doi.org/10.3233/jifs-220158.

[7] K. Lee, Y. Jun, M. Doh, Fuzzy Translations and Fuzzy Multiplications of BCK/BCI-Algebras, Commun. Korean Math. Soc. 24 (2009), 353–360. https://doi.org/10.4134/ckms.2009.24.3.353.

[8] G. Muhiuddin, S. Aldhafeeri, Subalgebras and Ideals in BCK/BCI-Algebras Based on Uni-Hesitant Fuzzy Set Theory, Eur. J. Pure Appl. Math. 11 (2018), 417–430. https://doi.org/10.29020/nybg.ejpam.v11i2.3246.

[9] W. Nakkhasen, N. Jaroenwan, P. Huekkhunthod, A. Chada, Applications of Strongest Fuzzy Dot Bd-Subalgebras in Bd-Algebras, Eur. J. Pure Appl. Math. 18 (2025), 6189. https://doi.org/10.29020/nybg.ejpam.v18i3.6189.

[10] W. Nakkhasen, J. Panyachanawong, N. Khambai, T. Jodnok, Derivations of Bd-Algebras Were Constructed into Semigroups, Int. J. Anal. Appl. 23 (2025), 69. https://doi.org/10.28924/2291-8639-23-2025-69.

[11] W. Nakkhasen, S. Phimkota, K. Phoemkhuen, A. Iampan, Characterizations of Fuzzy *Bd*-Ideals in *Bd*-Algebras, Int. J. Math. Comput. Sci. 19 (2024), 757–764.

[12] W. Nakkhasen, P. Srisarakham, W. Sakuntanat, A Study on Fuzzy *Bd*-Subalgebras of *Bd*-Algebras, ICIC Express Lett. 18 (2024), 777–783. https://doi.org/10.24507/icicel.18.08.777.

[13] A. Oli, G. Tefera, Fuzzy Derivations of d-Ideals of d-Algebras and Cartesian Product of Fuzzy Derivation of d-Ideals of d-Algebras, Appl. Artif. Intell. 37 (2023), 2157938. https://doi.org/10.1080/08839514.2022.2157938.

[14] N. Ratchakhwan, P. Julatha, T. Gaketem, P. Khamrot, R. Prasertpong, et al., (Inf, Sup)-Hesitant Fuzzy Ideals of BCK/BCI-Algebras, Int. J. Anal. Appl. 20 (2022), 34. https://doi.org/10.28924/2291-8639-20-2022-34.

[15] S.K. Majumdar, S.K. Sardar, Fuzzy Magnified Translation on Groups, J. Math. North Bengal Univ. 1 (2008), 117–124.

[16] W.B.V. Kandasamy, Smarandache Fuzzy Algebra, American Research Press, Rehoboth, 2003.

[17] D. Yılmaz, On the Structure of Hybrid Subalgebras in Bd-Algebras, J. New Results Sci. 14 (2025), 73–81. https://doi.org/10.54187/jnrs.1611847.

[18] L. Zadeh, Fuzzy Sets, Inf. Control. 8 (1965), 338–353. https://doi.org/10.1016/s0019-9958(65)90241-x.