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New Classes of Generalized Contraction Multi-Valued Mappings and Various Categories of Fixed Point Theorems with an Application

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Abstract. The importance of fixed point theory and it is implementations in various fields of mathematics and other branches of sciences has encouraged many researchers to study various categories of coincidence and common fixed point theorems of extended contraction multi-valued mappings. Therefore, the major objective of present our manuscript is to define and applies new ideas of extended contraction conditions to verify the uniqueness of some new categories of coincidence and common fixed point theorems for multi-valued maps in the context of complete $\mathcal{D}_d^*$-symmetric spaces. Various practical implementations of the existence fixed point as a solution for certain non-linear integral equations have been elucidated. Additionally, our suggested results are novel and develop various recognized comparable outcomes related to these categories of coincidence and common fixed point theorems for extended classes of contraction multi-valued maps existing in the literature. Furthermore, a appropriate example that supports our major results has been equipped.

1. Introduction

The idea of fixed point theory supplied an efficient in various fields of pure and applied mathematics and other branches of sciences; as well this theory for multi-valued contraction maps has obtained tremendous implementations in venous sciences such as: control theory, optimization, approximation theory, deferential equations and discrete dynamics. J. Dugundji and A. Granas [1] verified a single-valued faintly contractive map of complete metric spaces has unique fixed point. There were many authors presented various extensions of metric spaces such as (D-metric spaces) via B. C. Dhage [2, 3]. Afterward Z. Mustafa and B. Sims [4] discovered that

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most of the structures relating to vital topological characterizations of D-metric spaces are erroneous. Later, offered an extenuation of metric spaces, called (G-metric-sp). Consequently, D. El-Moutawakil [5] gave a novel extension of renowned multi-valued contraction fixed point in the context of symmetric spaces. Different extensions of the idea of metric spaces have been offered via many authors in literature. In particular, S. Shaban, et al. [6] have been verified the idea of $\mathcal{D}^*$-metric spaces which as one of probable modulation of $\mathcal{D}$-metric sp. Following, H. Chandra and A. Bhatt [7] confirmed the theory of fixed point for extended contraction utilizing restrictive conditions in symmetric spaces. On the other hand, M. Abbas, et al. [8] Established various fixed point theorems for multi-valued maps under extended contractive conditions in ordered extended metric spaces. In 2012, a novel contraction namely; F-contraction has been presented via D. Wardowski [9] and verified various results of fixed point theory. Further, M.Abbas, et al. [10] extended the notion of $\mathcal{F}$-contractive and established particular common fixed point outcomes. Afterward, K.S.Eke & J.O.Olaleru, [11] expand the idea of symmetric-sp to G-symmetric-sp via deleting the rectangle inequality axiom paces. Motivated by this fact, they are verified in [12] the existence of common fixed points for pair of hybrid contractive maps in G-symmetric-spaces. Z. Mustafa, et al., [13] defined novel notions of contractions maps and established a number of novel coincidence points and common fixed point outcomes in complete G-spaces. Afterward, A. M. Al-Jumaili, et al. [14] extended the conception of $\mathcal{D}^*$-metric spaces by changing $\mathbb{R}$ via an ordered Banach-space in $\mathcal{D}^*$-metric spaces. Newly, N. A. Majid, et al. [15] verified several novel fixed point outcomes for monotone multi-valued maps in partially ordered$\mathcal{D}^*$- complete metric spaces, as well various existence and uniqueness of coupled fixed point outcomes of maps satisfactory contractive condition have been discussed, additionally [16] they investigated and established various outcomes for coincidence and common of fixed points outcomes in complete S-metric-spa. Additionally, A. H. Abed and A. M. F. Al-Jumaili, [17] defined new kind of generalized metric spaces namely, $\mathcal{D}_d^*$-symmetric space and established numerous common fixed point outcomes for maps satisfactory extended contractive conditions in $\mathcal{D}_d^*$-symmetric space. As future studies we could generalize our outcomes to other spaces such as [18-21]. The purpose of present manuscript is to verify the uniqueness of some coincidence and common fixed point theorems for multi-valued maps involving novel ideas of extended contractions in the context of $\mathcal{D}_d^*$-Symmetric spaces. In addition, various practical implementations of the existence fixed point as a solution for certain non-linear integral equations have been explained. Our major outcomes which related to these kinds of coincidence and common fixed point theorems for multi-valued mappings via novel categories of extended contractions are extensions and improvement of the various outcomes existing in the literature.

2. Materials and Methods

This segment, devoted to present various definitions and motivations which needed in the sequel and that will help us in the outcomes that follow and play indispensable role in this study for verifying our major outcomes.

Definition 2.1: [6] Suppose that, $\mathcal{D}^*: \mathcal{X} \times \mathcal{X} \times \mathcal{X} \to \mathbb{R}^+$, is a mapping described on $\mathcal{X} \neq \emptyset$ and gratifying the next conditions $\forall x^*, y^*, z^*, w^* \in \mathcal{X}$:

$(\boldsymbol{\mathcal{D}_1^*})\mathcal{D}^*(\mathrm{x}^*, y^*, z^*) \geq 0, \forall\, x^*, y^*, z^* \in \mathcal{X};$

$(\boldsymbol{\mathcal{D}_2^*})\mathcal{D}^*(\mathrm{x}^*, y^*, z^*) = 0 \textit{ iff } x^* = y^* = z^*;$

$(\boldsymbol{\mathcal{D}_3^*})\mathcal{D}^*(\mathrm{x}^*, y^*, z^*) = \mathcal{D}^*(\mathcal{P}\{x^*, y^*, z^*\})$, (symmetry) where $\mathcal{P}$ is a permutation map,

$(\boldsymbol{\mathcal{D}_4^*})\mathcal{D}^*(\mathrm{x}^*, y^*, z^*) \leq \mathcal{D}^*(x^*, y^*, w^*) + \mathcal{D}^*(w^*, z^*, z^*).$

Therefore, $\mathcal{D}^*$ is said to $\mathcal{D}^*$-metric and $(\mathcal{X}, \mathcal{D}^*)$ is called $\mathcal{D}^*$-metric space.

Example2.2: [6] The Immediate examples of such a map are as follows:

$(\boldsymbol{i})$ $\mathcal{D}^*(x^*, y^*, z^*) = \max\{|x^* - y^*|, |y^* - z^*|, |x^* - z^*|\}$,

$(\boldsymbol{ii})$ $\mathcal{D}^*(x^*, y^*, z^*) = |x^* - y^*| + |y^* - z^*| + |x^* - z^*|.$

Definition 2.3: [17] A $\mathcal{D}_d^*$-Symmetric on $\mathcal{X}$ is $\mathcal{D}_d^*: \mathcal{X} \times \mathcal{X} \times \mathcal{X} \to \mathbb{R}^+$ (s. t), for all $x^*, y^*, z^* \in \mathcal{X}$ the next conditions are gratified:

$(\boldsymbol{\mathcal{D}_{d_1}^*})$ $\mathcal{D}_d^*(\mathrm{x}^*, y^*, z^*) \geq 0, \forall\, \mathrm{x}^*, y^*, z^* \in \mathcal{X};$

$(\boldsymbol{\mathcal{D}_{d_2}^*})$ $\mathcal{D}_d^*(x^*, y^*, z^*) = 0$ *if and only if* $x^* = y^* = z^*;$

$(\boldsymbol{\mathcal{D}_{d_3}^*})$ $\mathcal{D}_d^*(\mathrm{x}^*, y^*, z^*) = \mathcal{D}_d^*(\mathcal{P}\{\mathrm{x}^*, y^*, z^*\})$,(symmetry) where $\mathcal{P}$ is a permutation map.

In that case, $\mathcal{D}_d^*$ is called $\mathcal{D}_d^*$-Symmetric and $(\mathcal{X}, \mathcal{D}_d^*)$ is called $\mathcal{D}_d^*$-Symmetric space.

Example2.4: [17] Assume $\mathcal{X} = [0,1]$ equipped with $\mathcal{D}_d^*$-Symmetric and described via $\mathcal{D}_d^*(\mathrm{x}^*, y^*, z^*) = (\mathrm{x}^* - y^*)^2 + (y^* - z^*)^2 + (z^* - x^*)^2, \forall\, \mathrm{x}^*, y^*, z^* \in \mathcal{X}$. In that case, $(\mathcal{D}_d^*, \mathcal{X})$ is $\mathcal{D}_d^*$-Symmetric space. This doesn't gratify the rectangle inequality of $\mathcal{D}^*$-metric space, for this reason it's not $\mathcal{D}^*$-metric space.

Definition 2.5: [17] Let $(\mathcal{X}, \mathcal{D}_d^*)$ be a $\mathcal{D}_d^*$-Symmetric space, then:

$(\boldsymbol{i})$ A sequence $\{x_s^*\}$ is called $\mathcal{D}_d^*$-converges to $x^* \in \mathcal{X}$ iff $\mathcal{D}_d^*(x_s^*, x_s^*, x^*) = \mathcal{D}_d^*(x^*, x^*, x_s^*) \longrightarrow 0$ as $\mathrm{s} \longrightarrow \infty$. That is,$\forall\ \varepsilon > 0$, $\exists\, k \in \mathbb{N}$ (s. t), $\forall\ \mathrm{s} \geq k \Longrightarrow \mathcal{D}_d^*(x^*, x^*, x_s^*) < \varepsilon$.That is, $\forall\ \varepsilon > 0$, $\exists\, k \in \mathbb{N}$ (s. t), for all $\mathrm{s}, r \geq k, \mathcal{D}_d^*(x^*, x_s^*, x_r^*) < \varepsilon.$

$(\boldsymbol{ii})$ A sequence $\{x_s^*\}$ is called $\mathcal{D}_d^*$-Cauchy sequence if for all $\varepsilon > 0$, $\exists\, k \in \mathbb{N}$ such that, for each $\mathrm{s}, r \geq k, \mathcal{D}_d^*(x_s^*, x_s^*, x_r^*) < \varepsilon.$

$(\boldsymbol{iii})$ A space $(\mathcal{X}, \mathcal{D}_d^*)$ is $\mathcal{D}_d^*$-complete if for each $\mathcal{D}_d^*$-Cauchy sequence $\{x_s^*\}, \exists\ x^* \in \mathcal{X}$ with $\lim_{\mathrm{s} \to \infty} \mathcal{D}_d^*(x_s^*, x^*, x^*) = 0.$

The next Lemmas are analogue of Lemmas in [6] in $\mathcal{D}_d^*$-symmetric spaces:

Lemma 2.6: Each $\mathcal{D}_d^*$-Symmetric space$(\mathcal{X}, \mathcal{D}_d^*)$, describes a symmetric $d_{\mathcal{D}_d^*}$ on $\mathcal{X}$ through:

$d_{\mathcal{D}_d^*}(x^*, y^*) = \mathcal{D}_d^*(x^*, y^*, y^*) + \mathcal{D}_d^*(y^*, x^*, x^*), \forall\, x^*, y^* \in \mathcal{X}$,that is

$d_{\mathcal{D}_d^*}(x^*, y^*) = 2\mathcal{D}_d^*(x^*, y^*, y^*)\ \forall\, x, y^* \in \mathcal{X}$.

Lemma 2.7: The next statements are equivalent in $\mathcal{D}_d^*$-Symmetric space$(\mathcal{X}, \mathcal{D}_d^*)$:

$(\boldsymbol{i})$ $\{x_{\mathrm{s}}^*\}$ is $\mathcal{D}_d^*$-converge to x^*;

$(\boldsymbol{ii})$ $\mathcal{D}_d^*(\mathrm{x}_{\mathrm{s}}^*, x^*, x^*) \to 0$, as $(\mathrm{s} \to \infty)$;

$(\boldsymbol{iii})$ $\mathcal{D}_d^*(\mathrm{x}_{\mathrm{s}}^*, x_{\mathrm{s}}^*, x^*) \to 0$, as $(\mathrm{s} \to \infty)$;

Lemma 2.8: If $(\mathcal{X}, \mathcal{D}_d^*)$ is $\mathcal{D}_d^*$-Symmetric-sp, $\{x_{\mathrm{s}}^*\}$ a sequence in $\mathcal{X}$. So, the next statements are equivalent:

$(\boldsymbol{i})$ A sequence$\{x_{\mathrm{s}}^*\}$ is $\mathcal{D}_d^*$-Cauchy;

$(\boldsymbol{ii})$For each $\varepsilon > 0, \exists\, e \in \mathbb{Z}^+$ (s. t)$, \mathcal{D}_d^*(x_{\mathrm{s}}^*, x_{\mathrm{s}}^*, x_r^*) < \varepsilon. \forall\, \mathrm{s}, r \geq e$.

$(\boldsymbol{iii})$ A sequence$\{x_{\mathrm{s}}^*\}$ is Cauchy in $(\mathcal{X}, d_{\mathcal{D}_d^*})$.

The next Proposition is analogue of Proposition in [6] in $\mathcal{D}_d^*$-Symmetric spaces:

Proposition 2.9: Let $(\mathcal{X}, \mathcal{D}_d^*)$ be a $\mathcal{D}_d^*$-Symmetric space. So, the map $\mathcal{D}_d^*(x^*, y^*, z^*)$ is jointly continuous in all three of its variables.

Remark 2.10: Analogue of axioms of Wilson[22] in $\mathcal{D}_d^*$-Symmetric space as:

$(\mathbf{W_1})$ Given$\{x_{\mathrm{s}}^*\}, x^*, y^* \in \mathcal{X}, \mathcal{D}_d^*(\mathrm{x}_{\mathrm{s}}^*, x^*, x^*) \longrightarrow 0$ & $\mathcal{D}_d^*(\mathrm{x}_{\mathrm{s}}^*, y^*, y^*) \longrightarrow 0 \Longrightarrow \mathrm{x}^* = y^*$.

$(\mathbf{W_2})$ Given$\{x_{\mathrm{s}}^*\}$&$\{y_{\mathrm{s}}^*\}$ $x^*, y^* \in \mathcal{X}$ $, \mathcal{D}_d^*(\mathrm{x}_{\mathrm{s}}^*, x^*, x^*) \longrightarrow 0$ with $\mathcal{D}_d^*(\mathrm{x}_{\mathrm{s}}^*, y_{\mathrm{s}}^*, y_{\mathrm{s}}^*) \longrightarrow 0$ implise that $\mathcal{D}_d^*(y_{\mathrm{s}}^*, x^*, x^*) \longrightarrow 0$.

$(\mathbf{W_3})$ If $(\mathcal{X}, \mathcal{D}_d^*)$ is $\mathcal{D}_d^*$- complete symmetric space. For arbitrary sequence $\{x_{\mathrm{s}}^*\}$ of$\mathcal{X}$, we have $\lim_{\mathrm{s}, r \to \infty} \mathcal{D}_d^*(x_{\mathrm{s}}^*, x_r^*, x_r^*) = 0$, *if and only if* $\lim_{\mathrm{s} \to \infty} \mathcal{D}_d^*(x_{\mathrm{s}}^*, x_{\mathrm{s}+1}^*, x_{\mathrm{s}+1}^*) = 0$.

Definition2.11: [17] Presume $\mathcal{X} \neq \emptyset, \mathcal{G}: \mathcal{X} \to \mathcal{X}$ & $\mathfrak{F}: \mathcal{X} \to 2^{\mathcal{X}}$ are two maps. If $w^* = \mathcal{G}x^* \in \mathfrak{F}x^*$ for $x^* \in \mathcal{X}$, in that case $x^*$is coincidence point of $\mathcal{G}$ & $\mathfrak{F}$,with $w^*$is point of coincidence of $\mathcal{G}$ & $\mathfrak{F}$.

Definition2.12: [23] A map $\mathcal{G}: \mathcal{X} \to \mathcal{X}$& $\mathfrak{F}: \mathcal{X} \to 2^{\mathcal{X}}$ are called weakly compatible(Concisely, W.C.M) if $\mathcal{G}u \in \mathfrak{F}u, \forall\, u \in \mathcal{X}$, implies $\mathcal{G}\mathfrak{F}u \subseteq \mathfrak{F}\mathcal{G}u$.

Definition 2.13: [24] Assume $\mathfrak{F}, \mathcal{G}: (\mathcal{X}, \mathcal{D}_d^*) \to: (\mathcal{X}, \mathcal{D}_d^*)$ are self-maps on $\mathcal{X}$. If $\mathfrak{F}(x^*) = \mathcal{G}(x^*) = x^*$ for some $x^* \in \mathcal{X}$, in that case x^* is called a common fixed point (Concisely, C.F.P) of $\mathfrak{F}$ & G.

Proposition 2.14: ([25, Proposition 1.5]) Presume that $\mathcal{G}: \mathcal{X} \to \mathcal{X}$ and $\mathfrak{F}: \mathcal{X} \to 2^{\mathcal{X}}$ are weakly compatible self maps on a given $\mathcal{X} \neq \emptyset$. If $\mathcal{G}$ and $\mathfrak{F}$ have a unique point of coincidence $w^* = \mathcal{G}x^* \in \mathfrak{F}x^*$, then w^* is the unique common fixed point of $\mathfrak{F}$ and $\mathcal{G}$.

The next concepts are analogue of the ideas [26, 27] in $\mathcal{D}_d^*$-Symmetric spaces:

Remark 2.15: We will indicate for the collection of each non-empty closed and bounded subsets of $\mathcal{D}_d^*$-symmetric space$(\mathcal{X}, \mathcal{D}_d^*)$ via $\mathcal{CB}(\mathcal{X})$.

Definition 2.16: A map $\mathcal{F}: \mathcal{X} \to 2^{\mathcal{X}}$ is multi-valued map, $x^* \in \mathcal{X}$ is a fixed point of $\mathcal{F}$ if $x^* \in \mathcal{F}_{x^*}$.

Definition2.17: Two points $x^*, y^* \in \mathcal{X}$ $(x^* \neq y^*)$ are called Hausdorff if there exist two disjoint open sets (s. t) every element belongs to every open set. We indicate Hausdorff $\mathcal{D}_d^*$-distance on $\mathcal{CB}(\mathcal{X})$ via $\mathcal{H}(\cdot,\cdot,\cdot)$, where

$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{A},\mathcal{B},\mathcal{C}) = \max\left\{\sup_{x^*\in\mathcal{A}} \mathcal{D}_d^*(x^*,\mathcal{B},\mathcal{C}), \sup_{x^*\in\mathcal{B}} \mathcal{D}_d^*(x^*,\mathcal{C},\mathcal{A}), \sup_{x^*\in\mathcal{C}} \mathcal{D}_d^*(x^*,\mathcal{A},\mathcal{B})\right\}$, (s. t)

$$\mathcal{D}_d^*(x^*,\mathcal{B},\mathcal{C}) = d_{\mathcal{D}_d^*}(x^*,\mathcal{B}) + d_{\mathcal{D}_d^*}(\mathcal{B},\mathcal{C}) + d_{\mathcal{D}_d^*}(x^*,\mathcal{C})$$

$$d_{\mathcal{D}_d^*}(x^*,\mathcal{B}) = inf\{d_{\mathcal{D}_d^*}(x^*,y^*) : y^* \in \mathcal{B}\}$$

$$d_{\mathcal{D}_d^*}(\mathcal{A},\mathcal{B}) = inf\{d_{\mathcal{D}_d^*}(k_1,k_2) : k_1 \in \mathcal{A}, k_2 \in \mathcal{B}\}.$$

Recall $\mathcal{D}_d^*(x^*,y^*,\mathcal{C}) = inf\{\mathcal{D}_d^*(x^*,y^*,z^*) : z^* \in \mathcal{C}\}$.

The next Lemmas are analogue of Lemmas [28] in $\mathcal{D}_d^*$-symmetric spaces:

Lemma 2.18: Let $(\mathcal{X},\mathcal{D}_d^*)$ be $\mathcal{D}_d^*$-Symmetric-space. with $\mathcal{A},\mathcal{B} \in \mathcal{CB}(\mathcal{X})$. Therefore, for each $\mathrm{k}_1 \in \mathcal{A}$, obtain $\mathcal{D}_d^*(\mathrm{k}_1,\mathrm{k}_1,\mathcal{B}) \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{A},\mathcal{A},\mathcal{B})$.

Lemma2.19: Presume $(\mathcal{X},\mathcal{D}_d^*)$ is $\mathcal{D}_d^*$-Symmetric-sp. If $\mathcal{A},\mathcal{B} \in \mathcal{CB}(\mathcal{X})$with $\mathrm{k}_1 \in \mathcal{A}$, so, $\forall\, \varepsilon > 0, \exists\, \mathrm{k}_2 \in \mathcal{B}$, (s. t) $\mathcal{D}_d^*(\mathrm{k}_1,\mathrm{k}_1,\mathrm{k}_2) \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{A},\mathcal{A},\mathcal{B}) + \varepsilon$.

Definition 2.20: [9] Presume $(\mathcal{X},d)$ complete metric-sp. A map $\mathcal{T}:(\mathcal{X},d) \longrightarrow (\mathcal{X},d)$ is called ξ-contraction if $\exists\, \vartheta > 0$, (s.t)

$$\text{For all, } x^*, y^* \in \mathcal{X},\ d(\mathcal{T}x^*,\mathcal{T}y^*) > 0 \Rightarrow \xi\big(d(\mathcal{T}x^*,\mathcal{T}y^*)\big) + \vartheta \leq \xi\big(d(x^*,y^*)\big), \quad (2.1)$$

(s. t), $\xi: \mathbb{R}_+ \longrightarrow \mathbb{R}$ is map gratifying the next cases:

(ξ_1) ξis strictly increasing, that is, $\forall\, x^*,y^* \in \mathbb{R}_+$ (s. t), $x^* < y^*, \xi(x^*) < \xi(y^*)$;

(ξ_2) For all sequence $\{\Omega_s\}_{s=1}^{\infty}$ of positive numbers, $\lim_{s\to\infty}\Omega_s = 0$, *if and only if* $\lim_{s\to\infty}\xi(\Omega_s) = -\infty$;

(ξ_3) There exists $k \in (0,1)$ (s. t), $\lim_{\Omega\to 0^+}\Omega^{k}\xi(\Omega) = 0$.

The family of all maps ξ that gratify the conditions($\xi_1 - \xi_3$) is indicating via Ʊ.

Remark 2.21: It is clear that from (ξ_1) and inequality (2.1) we can conclude each ξ-contraction map $\mathcal{T}$ is necessarily continuous.

3. Categories of Fixed Point Theorems for Certain Kinds of Extended Contraction Multi-valued Maps

Our motivation of present this section is to study and verify the uniqueness of various coincidence and common fixed point theorems for multi-valued maps involving novel ideas of extended contraction conditions in the context of $\mathcal{D}_d^*$-Symmetric spaces. Our major outcomes in this part are extending and improving of various outcomes in the literature.

In the beginning we present definition of ($\mathcal{G} - \xi$)-contraction and our major outcomes.

Definition 3.1: If $\mathcal{T}:(\mathcal{X},\mathcal{D}_d^*) \longrightarrow \mathcal{CB}(\mathcal{X})$ with $\mathcal{G}:(\mathcal{X},\mathcal{D}_d^*) \longrightarrow (\mathcal{X},\mathcal{D}_d^*)$ are two maps in $\mathcal{D}_d^*$-symmetric space$(\mathcal{X},\mathcal{D}_d^*)$. In that case the map $\mathcal{T}$ is called ($\mathcal{G}$-ξ) contraction if there exist some ξ ∈ Ʊ with constant $\vartheta > 0$, such that, for all $x^*,y^*,z^* \in \mathcal{X}$:

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*,\mathcal{T}y^*,\mathcal{T}z^*) > 0 \Rightarrow \xi(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*,\mathcal{T}y^*,\mathcal{T}z^*)) + 2\vartheta \leq \xi(\mathcal{D}_d^*(\mathcal{G}x^*,\mathcal{G}y^*,\mathcal{G}z^*)) \quad (3.1)$$

Theorem 3.2: Let $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{X}$ with $\mathcal{T}: \mathcal{X} \longrightarrow \mathcal{CB}(\mathcal{X})$ are $(\mathcal{G}\text{-}\xi)$ contraction maps in $\mathcal{D}_d^*$-symmetric space$(\mathcal{X}, \mathcal{D}_d^*)$. For arbitrary $x^* \in \mathcal{X}, \mathcal{T}x^* \subseteq \mathcal{G}(\mathcal{X})$ with $\mathcal{G}(\mathcal{X})$ is $\mathcal{D}_d^*$-complete sub-space of $\mathcal{X}$, $\mathcal{G}$ and $\mathcal{T}$ contain point of coincidence. Additionally, if presume $\mathcal{G}u \in \mathcal{T}u$ *and* $\mathcal{G}v \in \mathcal{T}v$ which implies $\mathcal{D}_d^*(\mathcal{G}v, \mathcal{G}v, \mathcal{G}u) \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}v, \mathcal{T}v, \mathcal{T}u)$, in that case

(***i***)$\mathcal{G}$ & $\mathcal{T}$ contain a unique point of coincidence;

(***ii***) Further, if $\mathcal{G}$ & $\mathcal{T}$ are (W.C.M), so $\mathcal{G}$ & $\mathcal{T}$ have unique (C.F.P).

Proof: Assume that $x_0^* \in \mathcal{X}$. Because, $\mathcal{T}x^* \subseteq \mathcal{G}(\mathcal{X})$, $\exists\, x_1^* \in \mathcal{X}$ (s. t), $\mathcal{G}x_1^* \in \mathcal{T}x_0^*$. If $\mathcal{G}x_1^* = \mathcal{G}x_0^*$, in that case x_0^* is a coincidence point of $\mathcal{G}$ & $\mathcal{T}$, so the evidence is complete, as a result we presume $\mathcal{G}x_0^* \neq \mathcal{G}x_1^*$. In addition if $\mathcal{T}x_0^* = \mathcal{T}x_1^*$, in that case x_1^* is coincidence of $\mathcal{G}$ and $\mathcal{T}$. Consequently, suppose $\mathcal{T}x_0^* \neq \mathcal{T}x_1^*$, which implies $\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*) > 0$. Next from inequality (3.1) we get

$$\xi\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)\right) + 2\vartheta \leq \xi(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)).$$

As, ξ continuous map from the right, there exist $1 < j \in \mathbb{R}$, such that

$$\xi\left(j\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)\right) < \xi\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)\right) + \vartheta.$$

Because, $\mathcal{G}x_1^* \in \mathcal{T}x_0^*$, as a result via Lemma 2.18, we obtain

$\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{T}x_1^*) \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*) < j\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)$, where $j > 1$.

Now, from $\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{T}x_1^*) < j\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)$ with Lemma 2.19, conclude $\exists\, x_2^* \in \mathcal{X}$ & $\mathcal{G}x_2^* \in \mathcal{T}x_1^*$, (s. t)

$$\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*) \leq j\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*).$$

Consequently, obtain

$$\xi(\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*)) \leq \xi(j\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)) < \xi(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)) + \vartheta,$$

This implies that

$$\left\{\begin{array}{c} \xi\big(\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*)\big) \;+\; 2\vartheta \leq \\ 2\vartheta + \xi\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)\right) + \vartheta \\ \leq \xi(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_1^*, \mathcal{G}x_1^*)) + \vartheta \end{array}\right\}$$

Therefore,

$$\xi\big(\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*)\big) + \vartheta \leq \xi(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)).$$

Ongoing above procedures, describe $\{\mathcal{G}x_s^*\} \subset \mathcal{X}$, (s. t) $\mathcal{G}x_{s+1}^* \in \mathcal{T}x_s^*$ & $\mathcal{G}x_s^* \neq \mathcal{G}x_{s+1}^*$ & $\mathcal{T}x_s^* \neq \mathcal{T}x_{s+1}^*$ with

$$\xi\big(\mathcal{D}_d^*(\mathcal{G}x_s^*(\mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)\big) + \vartheta \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}x_{s-1}^*, \mathcal{G}x_{s-1}^*, \mathcal{G}x_1^*)\big)$$

For each $s \in \mathbb{N}_0$. Consequently

$$\left\{\begin{array}{rl} \xi\big(\mathcal{D}_d^*(\mathcal{G}\mathrm{x}_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)\big) & \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}x_{s-1}^*, \mathcal{G}x_{s-1}^*, \mathcal{G}x_s^*)\big) - \vartheta \\ & \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}x_{s-2}^*, \mathcal{G}x_{s-2}^*, \mathcal{G}x_{s-1}^*)\big) - 2\vartheta \\ & \cdot \\ & \cdot \\ & \cdot \\ & \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\big) + s\vartheta \end{array}\right\} \tag{3.2}$$

For all, $s \in \mathbb{N}$. Because, $\xi \in \mho$, through selecting the limit as $s \to \infty$ in inequality (3.2), we get

$$\lim_{s\to\infty} \xi\big(\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)\big) = -\infty.$$

Therefore via (ξ_2), get $\lim_{s\to\infty} \mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*) = 0.$ (3.3)

Now, from (ξ_3), there exists $0 < k < 1$, (s. t)

$$\lim_{s\to\infty} [\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k} \xi\big(\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)\big) = 0 \tag{3.4}$$

Utilizing inequality (3.2), we obtain

$$\left\{\begin{matrix} [\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k}\xi\big(\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)\big) - \\ [\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k}\xi\big(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\big) \le \\ [\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k}\big[\xi\big(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\big) - s\vartheta - \xi\big(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\big)\big] \\ = -s\vartheta[\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k} \le 0. \end{matrix}\right\}$$

As a result,

$$\left\{\begin{matrix} [\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k}\, \xi\big(\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)\big) + s\vartheta[\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k} \\ \le [\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k}\xi\big(\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\big) \end{matrix}\right\} \tag{3.5}$$

Via selecting the limit $s \to \infty$ in inequality (3.5) and applying inequalities (3.3) and (3.4), we obtain

$$\lim_{s\to\infty} s[\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k} = 0. \tag{3.6}$$

Consequently from inequality (3.6), there exit $s_1 \in \mathbb{N}$, (s. t)

$s[\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*)]^{k} \le 1, \forall\, s > s_1$which implies $\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_{s+1}^*) \le \frac{1}{s^{\frac{1}{k}}}, \forall\, s > s_1$.

Next, verify that $\{\mathcal{G}x_s^*\}$ is a $\mathcal{D}_d^*$-Cauchy sequence. For $r > s > s_1$, obtain

$$\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_r^*) \le \sum_{i=s}^{r-1} \mathcal{D}_d^*(\mathcal{G}x_i^*, \mathcal{G}x_i^*, \mathcal{G}x_{i+1}^*) \le \sum_{i=s}^{r-1} \frac{1}{i^{\frac{1}{k}}} \le \sum_{i=1}^{\infty} \frac{1}{i^{\frac{1}{k}}}$$

Because, $0 < k < 1$, consequently $\sum_{i=1}^{\infty} \frac{1}{i^{\frac{1}{k}}}$ converges. As a result, $\mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}x_r^*) \to 0$ as $s, r \to \infty$. Therefore, $\{\mathcal{G}x_s^*\}$ is $\mathcal{D}_d^*$-Cauchy in complete sub-space $\mathcal{G}(\mathcal{X})$, hence $\exists\, v \in \mathcal{G}(\mathcal{X})$, (s. t)

$$\lim_{s\to\infty} \mathcal{D}_d^*(\mathcal{G}x_s^*, v, v) = \lim_{s\to\infty} \mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, v) = 0 \tag{3.7}$$

Because, $v \in \mathcal{G}(\mathcal{X}), \exists\, u \in \mathcal{X}$ (s. t), $v = \mathcal{G}u$. Thus from inequality (3.7), obtain

$$\lim_{s\to\infty} \mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}u, \mathcal{G}u) = \lim_{s\to\infty} \mathcal{D}_d^*(\mathcal{G}x_s^*, \mathcal{G}x_s^*, \mathcal{G}u) = 0.$$

Currently, we will verify $\mathcal{G}u \in \mathcal{T}u$. If there exists an increasing sequence$\{s_{k}\}$ (s. t) $\mathcal{G}x_{s_k}^* \in \mathcal{T}u\ \forall\, k \in \mathbb{N}$, as $\mathcal{T}u$ is closed and $\mathcal{G}x_{s_k}^* \to \mathcal{G}u$, obtain $\mathcal{G}u \in \mathcal{T}u$ and the evidence is finished. Consequently we presume $\exists\, s_0 \in \mathbb{N}$, (s. t) $\mathcal{G}x_{s+1}^* \notin \mathcal{T}u, \forall s \ge s_0$. Because, $\mathcal{G}x_{s+1}^* \in \mathcal{T}x_s^*$, as a result, $\mathcal{T}x_s^* \ne \mathcal{T}u\ \forall\, s \ge s_0$ and as a result we have

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_s^*, \mathcal{T}x_s^*, \mathcal{T}u), \forall\, s \ge s_0 \tag{3.8}$$

Currently, since $\mathcal{G}x_{s+1}^* \in \mathcal{T}x_s^*$, therefore via Lemma 2.18, obtain

$$\mathcal{D}_d^*(\mathcal{G}x_{s+1}^*, \mathcal{G}x_{s+1}^*, \mathcal{T}u) \le \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_s^*, \mathcal{T}x_s^*, \mathcal{T}u).$$

Because, ξ is severely increasing, consequently via inequalities(3.8) and (3.1), obtain

$$\left\{\begin{array}{l}\xi\big(\mathcal{D}_d^*(\mathcal{G}x_{s+1}^*,\mathcal{G}x_{s+1}^*,\mathcal{T}u)\big) \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}x_{s+1}^*,\mathcal{G}x_{s+1}^*,\mathcal{T}u)\big) + 2\vartheta \\ \quad \leq \xi\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_s^*,\mathcal{T}x_s^*,\mathcal{T}u)\right) + 2\vartheta \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}x_s^*,\mathcal{G}x_s^*,\mathcal{G}u)\big)\end{array}\right\}$$

Because, ξ is increasing map, get $\mathcal{D}_d^*(\mathcal{G}x_{s+1}^*,\mathcal{G}x_{s+1}^*,\mathcal{T}u) \leq \mathcal{D}_d^*(\mathcal{G}x_s^*,\mathcal{G}x_s^*,\mathcal{G}u)$.

Permitting, $s \to \infty$ in preceding outcome with utilizing the truth that a map $\mathcal{D}_d^*$ is continuous on its three variables, consequently obtain $\mathcal{D}_d^*(\mathcal{G}u,\mathcal{G}u,\mathcal{T}u) = 0$.

Because, $\mathcal{T}u$ is closed we get $\mathcal{G}u \in \mathcal{T}u$, this mean u coincidence for $\mathcal{T}$ and $\mathcal{G}$. Thus, $\mathcal{G}$ & $\mathcal{T}$ contain point of coincidence w^{**}.

Now, establish the uniqueness coincidence point for $\mathcal{G}$ & $\mathcal{T}$.

Assume the opposite, w^* is another coincidence point for $\mathcal{G}$ & T i.e. ∃ another coincidence v for $\mathcal{G}$ & $\mathcal{T}$ (s. t) $w^{**} = \mathcal{G}v \in \mathcal{T}u$ & $\mathcal{G}u \neq \mathcal{G}v$ with $\mathcal{T}u \neq \mathcal{T}v$. Or else, u & v will not be coincidence points. In that case, $\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}v,\mathcal{T}v,\mathcal{T}u) > 0$.

So, we have the next hypothesis $\mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u) \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}v,\mathcal{T}v,\mathcal{T}u)$.

Because, ξ is increasing, consequently via preceding inequality and (3.1), we obtain

$$\xi\big(\mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u)\big) + 2\vartheta \leq \xi\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}v,\mathcal{T}v,\mathcal{T}u)\right) + 2\vartheta \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u)\big),$$

Additional implies

$$\xi\big(\mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u)\big) \leq \xi\big(\mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u)\big) - 2\vartheta < \xi\big(\mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u)\big).$$

Because, ξ is severely increasing, as a result obtain

$$\mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u) < \mathcal{D}_d^*(\mathcal{G}v,\mathcal{G}v,\mathcal{G}u),$$

This is a contradiction. For this reason, $\mathcal{G}u = \mathcal{G}v$ with $\mathcal{T}u = \mathcal{T}v$. Therefore, $\mathcal{G}$ & Tcontain unique point of coincidence. Presume, $\mathcal{G}$ and $\mathcal{T}$ are (W.C.M). Utilizing Proposition 2.14, we get $\mathcal{G}$ and $\mathcal{T}$ contain unique (C.F.P).

Corollary3.3: Let $\mathcal{T}: (\mathcal{X},\mathcal{D}_d^*) \longrightarrow \mathcal{CB}(\mathcal{X})$ be a map in $\mathcal{D}_d^*$-complete symmetric space$(\mathcal{X},\mathcal{D}_d^*)$. If there exist a map $\xi \in \mho$ and a constant $\vartheta > 0$, (s. t), $\forall\, x^*, y^*, z^* \in \mathcal{X}$,

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*,\mathcal{T}y^*,\mathcal{T}z^*) > 0 \Longrightarrow \xi(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*,\mathcal{T}y^*,\mathcal{T}z^*)) + 2\vartheta \leq \xi(\mathcal{D}_d^*(x^*,y^*,z^*)),$$

In that case, $\mathcal{T}$ has fixed point in $\mathcal{X}$.

Proof: It follows directly through selecting $\mathcal{G}$ as an identity map in Theorem 3.2.

4. Some Main Results of Coincidence and Common Fixed Points via Extended Mizoguchi-Takahashi's Mapping

Our motivation of present section, is to extend various outcomes which investigated in [28] via applying the next idea of extended Mizoguchi-Takahashi mapping and verify various main results related to uniqueness of coincidence and common fixed point theorems in context of $\mathcal{D}_d^*$- complete symmetric spaces.

Definition 4.1: [29] A map $\mathcal{J}: \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ is said to be a generalized Mizoguchi-Takahashi map (Concisely, G-MT-M.) if the next conditions hold:

(MT-1) $0 < \mathcal{J}(p, q) < 1, \forall p, q > 0$;

(MT-2) For arbitrary bounded sequence$\{p_s\} \subset (0, +\infty)$ and any non-increasing sequence $\{q_s\} \subset (0, +\infty)$, we have $\lim_{s\to\infty} \sup \mathcal{J}\,(p_s, q_s) < 1$.

Remark 4.2: We will indicate via ∇ for the collection of every maps $\mathcal{J}: \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ gratifying the conditions (MT-1)-(MT-2) to harmonious with Javahernia et al. [29].

Theorem 4.3: Let $\mathcal{T}: \mathcal{X} \longrightarrow \mathcal{CB}(\mathcal{X})$ be multi-valued map with self-map $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{X}$ in $\mathcal{D}_d^*$-symmetric space$(\mathcal{X}, \mathcal{D}_d^*)$. For arbitrary $x^* \in \mathcal{X}, \mathcal{T}x^* \subseteq \mathcal{G}(\mathcal{X})$, (s. t) $\mathcal{G}(\mathcal{X})$ $\mathcal{D}_d^*$-complete sub-space of $\mathcal{X}$, $\exists\ \mathcal{J} \in \nabla$, (s. t)

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*, \mathcal{T}y^*, \mathcal{T}z^*) \leq \mathcal{J}(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*, \mathcal{T}y^*, \mathcal{T}z^*), \mathcal{D}_d^*(\mathcal{G}x^*, \mathcal{G}y^*, \mathcal{G}z^*))\, \mathcal{D}_d^*(\mathcal{G}x^*, \mathcal{G}y^*, \mathcal{G}z^*) \qquad (4.1)$$

For all, $x^*, y^*, z^* \in \mathcal{X}$, consequently $\mathcal{G}$ and $\mathcal{T}$ contain point of coincidence. Additionally, if presume $\mathcal{G}c^* \in \mathcal{T}c^*$ & $\mathcal{G}c^\wedge \in \mathcal{T}c^\wedge$ implies $\mathcal{D}_d^*(\mathcal{G}c^\wedge, \mathcal{G}c^\wedge, \mathcal{G}c^*) \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}c^\wedge, \mathcal{G}c^\wedge, \mathcal{T}c^*)$, in that case

($\boldsymbol{i}$) $\mathcal{G}$ *and* $\mathcal{T}$ contain unique point of coincidence;

($\boldsymbol{ii}$) If $\mathcal{G}$ & $\mathcal{T}$ (W.C.M), in that case $\mathcal{G}$ & $\mathcal{T}$ have unique(C.F.P).

Proof: Assume, $x_0^* \in \mathcal{X}$. In that case through the given hypothesis, $\exists\ x_1^* \in \mathcal{X}$ (s. t), $x_1^* \in \mathcal{T}x_0^*$. If $\mathcal{G}x_1^* = \mathcal{G}x_0^*$, in that case we contain nothing to confirm also x_0^* is wanted point. Therefore presume $\mathcal{G}x_0^* \neq \mathcal{G}x_1^*$, as a result $\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*) > 0$. Currently, if $\mathcal{T}x_0^* = \mathcal{T}x_1^*$, so $\mathcal{G}x_1^* \in \mathcal{T}x_1^*$ and consequently x_1^* is the required point and nothing to perform. Hence presume $\mathcal{G}x_0^* \neq \mathcal{G}x_1^*$ and $\mathcal{T}x_0^* \neq \mathcal{T}x_1^*$, thus $\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*) > 0$ and $\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*) > 0$. From the inequality (4.1), we have

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*) \leq \mathcal{J}(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*), \mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*))\mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*).$$

Putting,

$$\Theta_1 = \left\{ \frac{1 - \sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*), \mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\right)}}{\sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*), \mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\right)}} \right\} \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*) \qquad (4.2)$$

Subsequently via Lemma 2.19, and inequality (4.2), we get

$$\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*) \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*) + \Theta_1 =$$

$$\frac{\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*)}{\sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_0^*, \mathcal{T}x_0^*, \mathcal{T}x_1^*), \mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*)\right)}}$$

Because, $\mathcal{T}x_1^* \in \mathcal{G}(\mathcal{X})$, as a result $\exists\ x_2^* \in \mathcal{X}$ (s. t), $\mathcal{G}x_2^* \in \mathcal{T}x_1^*$. If $\mathcal{G}x_1^* = \mathcal{G}x_2^*$, in that case x_1^* is the required point. Consequently presume that $\mathcal{G}x_1^* \neq \mathcal{G}x_2^*$, with $\mathcal{T}x_1^* \neq \mathcal{T}x_2^*$, therefore $\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*) > 0$ & $\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*, \mathcal{T}x_1^*, \mathcal{T}x_2^*) = 0$. Utilizing the inequality (4.1), we obtain

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*, \mathcal{T}x_1^*, \mathcal{T}x_2^*) \leq \mathcal{J}(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*, \mathcal{T}x_1^*, \mathcal{T}x_2^*), \mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*))\mathcal{D}_d^*(\mathcal{G}x_1^*, \mathcal{G}x_1^*, \mathcal{G}x_2^*).$$

Putting,

$$\Theta_2 = \left\{ \frac{1-\sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*,\mathcal{T}x_1^*,\mathcal{T}x_2^*),\mathcal{D}_d^*(\mathcal{G}x_1^*,\mathcal{G}x_1^*,\mathcal{G}x_2^*)\right)}}{\sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*,\mathcal{T}x_1^*,\mathcal{T}x_2^*),\mathcal{D}_d^*(\mathcal{G}x_1^*,\mathcal{G}x_1^*,\mathcal{G}x_2^*)\right)}} \right\} \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*,\mathcal{T}x_1^*,\mathcal{T}x_2^*) \tag{4.3}$$

Subsequently via Lemma 2.19, and inequality (4.3), get

$$\left\{ \begin{gathered} \mathcal{D}_d^*(\mathcal{G}x_2^*,\mathcal{G}x_2^*,\mathcal{G}x_3^*) \\ \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*,\mathcal{T}x_1^*,\mathcal{T}x_2^*) + \Theta_2 = \\ \frac{\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*,\mathcal{T}x_1^*,\mathcal{T}x_2^*)}{\sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_1^*,\mathcal{T}x_1^*,\mathcal{T}x_2^*),\mathcal{D}_d^*(\mathcal{G}x_1^*,\mathcal{G}x_1^*,\mathcal{G}x_2^*)\right)}} \end{gathered} \right\}$$

Through replicating above procedure, we able create $\{\mathcal{G}x_{\mathscr{k}}^*\}$ (s. t), $\mathcal{G}x_{\mathscr{k}+1}^* \in \mathcal{T}x_{\mathscr{k}}^*$, wherever $\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*) > 0$ & $\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*) > 0$.

As well,

$$\left\{ \begin{gathered} \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*) \leq \\ \mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*),\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*)\right)\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*) \end{gathered} \right\} \tag{4.4}$$

Consequently,

$$\left\{ \begin{gathered} \mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}}^*,\mathcal{G}x_{\mathscr{k}}^*,\mathcal{G}x_{\mathscr{k}+1}^*) \\ \leq \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*) + \Theta_{\mathscr{k}} = \\ \frac{\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*)}{\sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*),\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*)\right)}} \end{gathered} \right\} \tag{4.5}$$

Utilizing the inequalities (4.4) and (4.5), obtain

$$\left\{ \begin{gathered} \mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}}^*,\mathcal{G}x_{\mathscr{k}}^*,\mathcal{G}x_{\mathscr{k}+1}^*) \leq \\ \sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*),\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*)\right)}\,\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*) \end{gathered} \right\} \tag{4.6}$$

This illustrates that the sequence of non-negative numbers $\{\mathcal{L}_{\mathscr{k}}\}$ given via

$$\mathcal{L}_{\mathscr{k}} = \mathcal{D}_d^*(\mathcal{G}\mathrm{x}_{\mathscr{k}-1}^*,\mathcal{G}\mathrm{x}_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*)$$

For $\mathscr{k} = 1,2,\ldots$,non-increasing, wherever $\mathcal{L}_{\mathscr{k}} \geq 0$. For this reason, $\exists\ \mathcal{L} \in \mathbb{Z}^+$(s. t),

$$\mathcal{L} = \lim_{\mathscr{k}\to\infty} \mathcal{L}_{\mathscr{k}} = \inf_{\mathscr{k}\in\mathbb{N}} \mathcal{L}_{\mathscr{k}} \geq 0\ . \tag{4.7}$$

Because, $\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*) > 0$ and $\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*) > 0$, as a result through inequality-(4.4), obtain

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*) \leq \mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*)$$

This explains $\left\{\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*)\right\}$ is a bounded sequence. Using (MT-2), get

$$\lim_{\mathrm{s}\to\infty} \sup \mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}-1}^*,\mathcal{T}x_{\mathscr{k}}^*),\mathcal{D}_d^*(\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}-1}^*,\mathcal{G}x_{\mathscr{k}}^*)\right) < 1. \tag{4.8}$$

Next we verify $\mathcal{L} = 0$. presume, $\mathcal{L} > 0$, in that case via inequalities (4.7) and (4.8), with taking the lim-sup on both sides of inequality (4.6), obtain

$$\mathcal{L} \leq \sqrt{\lim_{k\to\infty} \sup \mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{k-1}^*, \mathcal{T}x_{k-1}^*, \mathcal{T}x_k^*), \mathcal{D}_d^*(\mathcal{G}x_{k-1}^*, \mathcal{G}x_{k-1}^*, \mathcal{G}x_k^*)\right)}\ \mathcal{L} < \mathcal{L}.$$

As a result, this contradiction, so $\lim_{k\to\infty} \mathcal{L}_k = \inf_{k\in\mathbb{N}} \mathcal{L}_k = 0$.

Consequently,

$$\lim_{k\to\infty} \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}x_{k+1}^*) = \inf_{k\in\mathbb{N}} \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}x_{k+1}^*) = 0. \tag{4.9}$$

Currently we confirm $\{\mathcal{G}x_k^*\}$ is Cauchy sequence. $\forall k \in \mathbb{N}$, assume

$$v_k = \sqrt{\mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{k-1}^*, \mathcal{T}x_{k-1}^*, \mathcal{T}x_k^*), \mathcal{D}_d^*(\mathcal{G}x_{k-1}^*, \mathcal{G}x_{k-1}^*, \mathcal{G}x_k^*)\right)}\,.$$

Subsequently, $v_k \in (0,1), \forall k \in \mathbb{N}$. Utilizing inequality-(4.6), obtain

$$\mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}x_{k+1}^*) \leq v_k \mathcal{D}_d^*(\mathcal{G}x_{k-1}^*, \mathcal{G}x_{k-1}^*, \mathcal{G}x_k^*)$$

For all, $k \in \mathbb{N}$. Using inequality (4.8), get $\lim_{k\to\infty} \sup v_k < 1$, as a result $\exists\, b \in [0,1)$ with $k_0 \in \mathbb{N}$ (s. t), $v_k < b, \forall\, k \in \mathbb{N}$ and $k \geq k_0$. Because $v_k \in (0,1), \forall\, k \in \mathbb{N}$ with $b \in [0,1)$. Consequently via inequality-(4.9) for $k \geq k_0$, deduce

$$\left\{\begin{aligned}
\mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}x_{k+1}^*) &\leq v_k \mathcal{D}_d^*(\mathcal{G}x_{k-1}^*, \mathcal{G}x_{k-1}^*, \mathcal{G}x_k^*) \\
&\leq v_k v_{k-1} \mathcal{D}_d^*(\mathcal{G}x_{k-2}^*, \mathcal{G}x_{k-2}^*, \mathcal{G}x_{k-1}^*) \\
&\;\;\vdots \\
&\leq v_k v_{k-1} \dots v_{k_0} \mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*) \\
&\leq b^{k-k_0+1} \mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*).
\end{aligned}\right\}$$

Suppose, $\lambda_k = \frac{b^{k-k_0+1}}{1-b} \mathcal{D}_d^*(\mathcal{G}x_0^*, \mathcal{G}x_0^*, \mathcal{G}x_1^*), k \in \mathbb{N}$.For, $k \in \mathbb{N}$ & $k \geq k_0$ with $r \in \mathbb{Z}^+$, in that case through the last inequality and($\mathcal{D}_4^*$), obtain

$$\mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}x_{k+r}^*) \leq \sum_{i=k}^{k+r-1} \mathcal{D}_d^*(\mathcal{G}x_i^*, \mathcal{G}x_i^*, \mathcal{G}x_{i+1}^*) \leq \lambda_k.$$

As, $b \in [0,1)$, thus, $\lim_{k\to\infty} \lambda_k = 0$.. For this reason, $\lim_{k\to\infty} \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}x_{k+r}^*) = 0$. Therefore $\{\mathcal{G}\mathrm{x}_k^*\}$ is $\mathcal{D}_d^*$-Cauchy in complete sub-space $\mathcal{G}(\mathcal{X})$. Consequently, $\exists\, c' \in \mathcal{G}(\mathcal{X})$, (s. t), from Lemma 2.7, we obtain

$$\lim_{k\to\infty} \mathcal{D}_d^*(\mathcal{G}x_k^*, c', c') = \lim_{k\to\infty} \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, c') = 0. \tag{4.10}$$

Because, $c' \in \mathcal{G}(\mathcal{X})$, thus $\exists\, c^* \in \mathcal{X}$ (s. t), $c' = \mathcal{G}c^*$. As a result via inequality (4.10), obtain

$$\lim_{k\to\infty} \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}c^*, \mathcal{G}c^*) = \lim_{k\to\infty} \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}c^*) = 0. \tag{4.11}$$

Now, verify $c^* \in \mathcal{T}c^*$. Utilizing inequalities-(4.1) and (4.11), we get

$$\left\{\begin{gathered}
\lim_{k\to\infty} \mathcal{D}_d^*(\mathcal{G}x_{k+1}^*, \mathcal{G}x_{k+1}^*, \mathcal{T}c^*) \leq \\
\lim_{k\to\infty} \mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_k^*, \mathcal{T}x_k^*, \mathcal{T}c^*) \leq \\
\lim_{k\to\infty} \mathcal{J}\left(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x_{k-1}^*, \mathcal{T}x_{k-1}^*, \mathcal{T}c^*), \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}c^*)\right) \mathcal{D}_d^*(\mathcal{G}x_k^*, \mathcal{G}x_k^*, \mathcal{G}c^*) = 0
\end{gathered}\right\}$$

For this reason,$\mathcal{D}_d^*$ $(\mathcal{G}c^*, \mathcal{G}c^*, \mathcal{T}c^*) = 0$, this mean $\mathcal{G}c^* \in \mathcal{T}c^*$. Hence $\mathcal{T}$ and $\mathcal{G}$ contain a point of coincidence, that is, c^*.

Currently we confirm uniqueness of this point of coincidence. Presume on the opposing that there exists one more $c^{\wedge}$ (s. t), $\mathcal{G}c^{\wedge} \in \mathcal{T}c^{\wedge}$ but $c^{\wedge} \neq \mathcal{G}c^{*}$. Utilizing inequality (4.1) and our supposition, obtain

$$\left\{\begin{array}{c} \mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right) \leq \mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right) \leq \\ \mathcal{J}(\mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right), \mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right))\, \mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right) \end{array}\right\}$$

Since, $\mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right) > 0$ & $\mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right) > 0$, consequently

$$\mathcal{J}(\mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right), \mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right)) < 1.$$

Therefore we obtain

$$\mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right) < \mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right),$$

This is contradicts to the fact $c^{\wedge} \neq \mathcal{G}c^{*}$. Consequently, $\mathcal{G}c^{\wedge} = \mathcal{G}c^*$. In sight of

$$\mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right) \leq \mathcal{J}(\mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right), \mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right))\mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right) = 0,$$

This implies $\mathcal{T}c^{\wedge} = \mathcal{T}c^*$. As a result, $\mathcal{T}$ and $\mathcal{G}$ contain unique point of coincidence. Presume $\mathcal{G}$ and $\mathcal{T}$ (W.C.M). Through utilizing Proposition 2.14, obtain $\mathcal{G}$ and $\mathcal{T}$ contain unique (C.F.P).

Theorem 4.4: Let $\mathcal{T}: \mathcal{X} \longrightarrow \mathcal{CB}(\mathcal{X})$ be multi-valued map with self-map $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{X}$ in $\mathcal{D}_d^*$-symmetric space $(\mathcal{X}, \mathcal{D}_d^*)$. For arbitrary $x^* \in \mathcal{X}, \mathcal{T}x^* \subseteq \mathcal{G}(\mathcal{X})$ and $\mathcal{G}(\mathcal{X})$ is $\mathcal{D}_d^*$-complete sub-space of $\mathcal{X}$, (s. t) $\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*, \mathcal{T}y^*, \mathcal{T}z^*) \leq \psi\left(\mathcal{D}_d^*(\mathcal{G}x^*, \mathcal{G}y^*, \mathcal{G}z^*)\right), \forall x^*, y^*, z^* \in \mathcal{X}$, where $\psi: \mathbb{R}^+ \longrightarrow [0,1)$ is a mapping (s. t), $\psi(q) < q$ with $\lim\limits_{q \to p+} \sup \frac{\psi(q)}{q} < 1$, thus $\mathcal{G}$ and $\mathcal{T}$ contain a point of coincidence in $\mathcal{X}$. Additionally, if presume that $\mathcal{G}c^* \in \mathcal{T}c^*$ and $\mathcal{G}c^{\wedge} \in \mathcal{T}c^{\wedge}$ implies $\mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right) \leq \mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right)$, in that case

(***i***) $\mathcal{G}$ & T contain unique point of coincidence;

(***ii***) If $\mathcal{G}$ & $\mathcal{T}$ (W.C.M), so $\mathcal{G}$ & $\mathcal{T}$ have unique (C.F.P).

Proof: Via putting $\mathcal{J}(p, q) = \frac{\psi(q)}{q}$ in Theorem 4.3, the proof follows by similar way.

Definition 4.5: [29] A mapping $\Psi: \mathbb{R}^+ \longrightarrow \mathbb{R}^+$is called weak lower semi-continuity if the next condition hold for each bounded sequence$\{p_{\mathrm{s}}\} \subset (0, +\infty)$:

(W-1) $\lim\limits_{s \to \infty} \inf \Psi\{p_{\mathrm{s}}\} > 0$.

Remark 4.6: We will indicate via Γ for the collection of all maps $\Psi: \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ gratifying the condition (W-1) above.

Theorem 4.7: Let $\mathcal{T}: \mathcal{X} \longrightarrow \mathcal{CB}(\mathcal{X})$ be multi-valued map with a self-map $\mathcal{G}: \mathcal{X} \longrightarrow \mathcal{X}$ in $\mathcal{D}_d^*$-symmetric $(\mathcal{X}, \mathcal{D}_d^*)$. For arbitrary $\mathrm{x}^* \in \mathcal{X}, \mathcal{T}\mathrm{x}^* \subseteq \mathcal{G}(\mathcal{X})$ with $\mathcal{G}(\mathcal{X})$ is $\mathcal{D}_d^*$-complete sub-space (s. t),

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}x^*, \mathcal{T}y^*, \mathcal{T}z^*) \leq \mathcal{D}_d^*(\mathcal{G}x^*, \mathcal{G}y^*, \mathcal{G}z^*) - \Psi\left(\mathcal{D}_d^*(\mathcal{G}x^*, \mathcal{G}y^*, \mathcal{G}z^*)\right), \forall x^*, y^*, z^* \in \mathcal{X},$$

Where, $\Psi: \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is (s. t), $\Psi(0) = 0, \Psi(q) < q$ with $\Psi \in \Gamma$, so $\mathcal{G}$ and $\mathcal{T}$ contain a point of coincidence in $\mathcal{X}$. Additionally, if presume $\mathcal{G}c^* \in \mathcal{T}c^*$ & $\mathcal{G}c^{\wedge} \in \mathcal{T}c^{\wedge}$ implies $\mathcal{D}_d^*\left(\mathcal{G}c^{\wedge},\mathcal{G}c^{\wedge},\mathcal{G}c^*\right) \leq \mathcal{H}_{\mathcal{D}_d^*}\left(\mathcal{T}c^{\wedge},\mathcal{T}c^{\wedge},\mathcal{T}c^*\right)$, in that case

(***i***) $\mathcal{G}$ & T contain unique point of coincidence;

(***ii***) If $\mathcal{G}$ and $\mathcal{T}$ (W.C.M), therefore $\mathcal{G}$ & $\mathcal{T}$ have unique (C.F.P).

Proof: Describe, $\mathcal{J}(p, q) = \frac{p-\Psi(p)}{p}, \forall\, p, q > 0$. As for each bounded sequence $\{p_s\} \subset (0, +\infty)$, obtain $\lim\limits_{s\to\infty} \inf \Psi(p_s) > 0$, as a result $\lim\limits_{s\to\infty} \inf \frac{\Psi(p_s)}{p_s} > 0$, consequently

$$\lim_{s\to\infty} sup \left(\frac{p_s - \Psi(p_s)}{p_s}\right) = 1 - \lim_{s\to\infty} inf \frac{\Psi(p_s)}{p_s} < 0 \,.$$

This demonstrates $\mathcal{J} \in \nabla$. Furthermore

$$\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}\, x^*, \mathcal{T}\, y^*, \mathcal{T}\, z^*) \le \mathcal{J}(\mathcal{H}_{\mathcal{D}_d^*}(\mathcal{T}\, \mathrm{x}^*, \mathcal{T}\, y^*, \mathcal{T}\, z^*), \mathcal{D}_d^*(\mathcal{G}\, x^*, \mathcal{G}\, y^*, \mathcal{G}\, z^*), \mathcal{D}_d^*(\mathcal{G}\, \mathrm{x}^*, \mathcal{G}\, y^*, \mathcal{G}\, z^*),$$

Consequently via Theorem 4.3, we obtain $\mathcal{G}$ and $\mathcal{T}$ contain unique (C.F.P).

5. Some Applications of Non-Liner Integral Equations in $\mathcal{D}_d^*$-Symmetric Spaces

This segment devoted to illustrate that there is a solution to the next structure of nonlinear integral equation by using our main outcomes of Corollary 3.3 in $\mathcal{D}_d^*$-symmetric $(\mathcal{X}, \mathcal{D}_d^*)$. In this part, our major outcomes are extending and improving to that of [13].

$$p(\ell) = \int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n)\mathfrak{F}(n, p(n))\, dn\,, \ell \in [\mathrm{k}_1, \mathrm{k}_2] \tag{5.1}$$

Assume, $\mathcal{X} = (\mathcal{C}[\,\mathrm{k}_1, \mathrm{k}_2], \mathbb{R})$ indicate the collection of each continuous maps from $[\mathrm{k}_1, \mathrm{k}_2]$ to$\mathbb{R}$. Describe $\mathcal{T}\colon \mathcal{X} \longrightarrow \mathcal{X}$ through

$$\mathcal{T}p(\ell) = \int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n)\mathfrak{F}(n, p(n))\, dn\,, \ell \in [\mathrm{k}_1, \mathrm{k}_2] \tag{5.2}$$

Theorem 5.1: Consideration equation (5.1) and presume the next conditions hold:

(***i***) $\mathcal{H}\colon [\mathrm{k}_1, \mathrm{k}_2] \times [\mathrm{k}_1, \mathrm{k}_2] \longrightarrow \mathbb{R}^+$ is a continuous map;

(***ii***) $\mathfrak{F}\colon [\mathrm{k}_1, \mathrm{k}_2] \times \mathbb{R} \longrightarrow \mathbb{R}$, such that $\mathfrak{F}$ is continuous map;

(***iii***) $\max\limits_{\ell\in[\mathrm{k}_1,\mathrm{k}_2]} \int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n)\, dn < \mathrm{e}^{-2\vartheta}$ for some $\vartheta \in (0, \infty)$;

(***iv***) $\forall\, p(n), q(n) \in \mathcal{X}, n \in [\mathrm{k}_1, \mathrm{k}_2]$, obtain $\left|\mathfrak{F}(n, p(n)) - \mathfrak{F}(n, q(n))\right| \le |p(n) - q(n)|$.

In that case, equation-(5.1) has a solution.

Proof: Assume $\mathcal{X}$ & $\mathcal{T}$ are described above. $\forall\, p, q, w^* \in \mathcal{X}$ describe the $\mathcal{D}_d^*$-symmetric on $\mathcal{X}$ via $\mathcal{D}_d^*(\,p, q, w^*) = d(p, q) + d(q, w^*) + d(p, w^*)$, where

$$d(p, q) = \sup_{\ell\in[\mathrm{k}_1,\mathrm{k}_2]} |p(\ell) - q(\ell)|$$

Obviously, $(\mathcal{X}, \mathcal{D}_d^*)$ is $\mathcal{D}_d^*$-complete symmetric space, since $(\mathcal{X}, d)$ is complete symmetric space. Currently, presume $p(\ell), q(\ell) \in \mathcal{X}$, so utilizing Theorem 5.2, parts (iii)&(vi), get

$$|\mathcal{T}p(\ell) - \mathcal{T}q(\ell)| = \left|\int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n)\left[\mathfrak{F}(n, p(n)) - \mathfrak{F}(n, q(n))\right] dn\right|$$

$$\le \int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n)\left|\mathfrak{F}(n, p(n)) - \mathfrak{F}(n, q(n))\right| dn$$

$$\le \int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n)|p(n) - q(n)|\, dn$$

$$\leq \int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n) \sup_{n \in [\mathrm{k}_1, \mathrm{k}_2]} |p(n) - q(n)|\, dn$$

$$= \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |p(\ell) - q(\ell)| \int_{\mathrm{k}_1}^{\mathrm{k}_2} \mathcal{H}(\ell, n)\, dn$$

$$\leq \mathrm{e}^{-2\vartheta} \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |p(\ell) - q(\ell)|\, .$$

Therefore,

$$\sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |\mathcal{T}p(\ell) - \mathcal{T}q(\ell)| \leq \mathrm{e}^{-2\vartheta} \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |p(\ell) - q(\ell)| \tag{5.3}$$

Likewise, obtain

$$\sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |\mathcal{T}q(\ell) - \mathcal{T}w^*(\ell)| \leq \mathrm{e}^{-2\vartheta} \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |q(\ell) - w^*(\ell)| \qquad (5.3)$$

And

$$\sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |\mathcal{T}p(\ell) - \mathcal{T}w^*(\ell)| \leq \mathrm{e}^{-2\vartheta} \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |p(\ell) - w^*(\ell)| \tag{5.5}$$

Consequently, utilizing inequalities (5.3), (5.4) and (5.5), we get

$$\sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |\mathcal{T}p(\ell) - \mathcal{T}q(\ell)| + \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |\mathcal{T}q(\ell) - \mathcal{T}w^*(\ell)| + \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |\mathcal{T}p(\ell) - \mathcal{T}w^*(\ell)|$$

$$\leq \mathrm{e}^{-2\vartheta} \left[\sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |p(\ell) - q(\ell)| + \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |q(\ell) - w^*(\ell)| + \sup_{\ell \in [\mathrm{k}_1, \mathrm{k}_2]} |p(\ell) - w^*(\ell)| \right]$$

This implies

$$\mathcal{D}_d^*(\mathcal{T}p, \mathcal{T}q, \mathcal{T}w^*) \leq \mathrm{e}^{-2\vartheta} \mathcal{D}_d^*(p, q, w^*).$$

Consequently,

$$\ln(\mathcal{D}_d^*(\mathcal{T}p, \mathcal{T}q, \mathcal{T}w^*)) \leq \ln(\mathcal{D}_d^*(p, q, w^*)) - 2\vartheta$$

And as a result

$$\ln(\mathcal{D}_d^*(\mathcal{T}p, \mathcal{T}q, \mathcal{T}w^*)) + 2\vartheta \leq \ln(\mathcal{D}_d^*(p, q, w^*)).$$

Currently, we have the next fact:

$\xi(\mathcal{D}_d^*(\mathcal{T}p, \mathcal{T}q, \mathcal{T}w^*)) + 2\vartheta \leq \xi(\mathcal{D}_d^*(p, q, w^*))$ is gratified for $\xi(\Omega) = \ln(\Omega), \forall\, \Omega \in \mathcal{X}1$.

For that reason, each conditions of Corollary 3.3, are gratified. Consequently from Corollary 3.3, $\mathcal{T}$ has fixed point which is solution for equation (5. 1).

Next, discuss the next example which is extending to that of [13], explains validity of Theo-5.1.

Example 5.2: Suppose, $\mathcal{X} = (\mathcal{C}[\ln(2), \ln(3)], \mathbb{R})$,so the next integral equation has solution of $\mathcal{X}$.

$$p(\ell) = \int_{\ln(2),}^{\ln(3)} \cosh(n\ell)\, p(n)\, dn, \ell \in [\ln(2), \ln(3)] \tag{5.6}$$

Proof: Assume, $\mathcal{T}: \mathcal{X} \longrightarrow \mathcal{X}$ is described below:

$$\mathcal{T}p(\ell) = \int_{\ln(2),}^{\ln(3)} \cosh(n\ell)\, p(n)\, dn, \ell \in [\ln(2), \ln(3)]$$

Via putting, $\mathcal{H}(\ell, n) = \cosh(n\ell)$, $\mathfrak{F}(n, \ell) = \ell$, with $\vartheta \geq \frac{18}{100}$ in Theorem 5.1, obtain that:

(***i***) A map $\mathcal{H}(\ell, n)$continuous on $[\ln(2), \ln(3)] \times [\ln(2), \ln(3)]$;

$(\boldsymbol{ii})$ $\mathfrak{F}(n,\ell)$continuous on $[\ln(2), \ln(3)] \times \mathbb{R}, \forall n \in [\ln(2), \ln(3)]$;

$$(\boldsymbol{iii}) \max_{\ell \in [\ln(2), \ln(3)]} \int_{\ln(2),}^{\ln(3)} \cosh(n\ell)\, dn = \max_{\ell \in [\ln(2), \ln(3)]} \frac{\sinh(\ln(3^{\ell})) - \sinh(\ln(2^{\ell}))}{\ell}$$

$$= \max_{\ell \in [\ln(2), \ln(3)]} \frac{3^{\ell} - 3^{-\ell} - 2^{\ell} + 2^{-\ell}}{2\ell} < \frac{7}{10} \leq e^{-2\vartheta}$$

$(\boldsymbol{iv})$ $\forall\, p(n), q(n) \in \mathcal{X}$ it's apparent that condition (iv) in Theorem 5.1, is gratified. As a result, each cases of Theorem 5.1, gratified, therefore the map $\mathcal{T}$ has fixed point, which solution for equation (5.6).

6. Conclusion

The fixed point theory has developed and diversified over the decades, to include various extensions and fruitful applications in numerous fields of pure and applied mathematics, moreover other sciences such as, control theory, physics, computer sciences and telecommunication optimization problems, making it a cornerstone of topological spaces and mathematical analysis. Therefore, in our manuscript novel notions of extended contractions have been introduced to verify the uniqueness of various extended categories of coincidence and common fixed point theorems for multi-valued maps in the context of complete $\mathcal{D}_{d}^{*}$-Symmetric spaces. Additionally, some practical implementations of the existence fixed point as a solution for certain non-linear integral equations have been demonstrated. As well, our proposed outcomes are developing various recognized analogous outcomes related to these categories of coincidence and common fixed point theorems existing in the literature. We anticipate that the discoveries in this manuscript will aid scientists in enhancing the authors on popularized extended metric spaces in order to elevate a general framework for their practical implementations in all advanced branches of pure and applied mathematics additionally other sciences.

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