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Structural Properties of (l,r)- and (r,l)-Derivations in IUP-Algebras

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Abstract. The notion of derivations in BCI-algebras was initially introduced by Jun and Xin in 2004 [18], providing a foundation for structural analysis in non-classical logics. In this paper, we extend the study of derivations to the framework of IUP-algebras $X = (X, \cdot, 0)$ by introducing and examining two new types: (l,r)-derivations and (r,l)-derivations. These operators are defined via the binary operation $\wedge$ given by $x \wedge y = (y \cdot x) \cdot x$ for all $x, y \in X$, which plays a central role in the algebraic structure. We explore fundamental properties of these derivations, analyze their interaction with IUP-substructures, and establish several characterizations. Additionally, we define two special subsets—$\mathrm{Ker}_{\mathfrak{d}}(X)$ (the kernel) and $\mathrm{Fix}_{\mathfrak{d}}(X)$ (the fixed-point set)—associated with a derivation $\mathfrak{d}$, and investigate conditions under which they exhibit algebraic regularity. Our results enrich the theory of derivations in IUP-algebras and open new directions for the study of morphism-based operations in non-associative systems.

1. Introduction

The notion of a derivation is a vital tool in the study of algebraic structures, providing a mechanism for investigating the internal symmetries and dynamic transformations of various algebraic systems. Initially developed in the context of BCI-algebras, derivations have undergone significant generalizations. Early works such as [1,2,25–27] explored classical and left derivations, while subsequent studies introduced functional variations including f-derivations [17,22,29,39] and (α,β)-derivations [4,25]. Other generalized types include (σ,τ)-derivations [24], (θ,ϕ)-derivations [28], (f,g)-derivations [3,5], and (g,h)-derivations [19]. Derivations have also been applied to various non-classical algebras such as BCC-algebras [31], BCH-algebras [3], BF-algebras [9], MV-algebras [11,38,40], and PU-algebras [20]. More advanced generalizations involve studies in Hilbert algebras [13] and TM-algebras [10], which highlight the widespread relevance of derivations in modern algebraic logic. A prominent trend in recent studies is the use of endomorphisms

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and bi-endomorphisms to induce derivations. In the context of UP-algebras, Sawika et al. [30] laid the foundational theory, while Iampan [12] introduced a new class of derivations via UP-endomorphisms. This morphism-based approach has since been extended to B-algebras [6], d-algebras [23], and Hilbert algebras through structural derivations [15]. These cumulative efforts suggest a unified framework where morphisms are central to generalizing the concept of derivation across algebraic systems.

The class of IUP-algebras was formally introduced by Iampan et al. in 2022 [14], who established its axiomatic foundations and identified fundamental subclasses such as IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals. These structures initiated a research trajectory exploring the algebra's internal organization and subset hierarchies. Subsequent studies have significantly expanded the theory. Chanmanee et al. [8] examined the behavior of (possibly infinite) direct products and weak direct products, while analyzing structural transformations under (anti-)IUP-homomorphisms. A related investigation by Chanmanee et al. [7] focused on the structural properties of external direct products within dual IUP-algebras, thereby enhancing the algebraic foundation for modular construction. In 2024, Kuntama et al. [21] extended the framework by incorporating fuzzy set theory, defining fuzzy variants of subalgebras, ideals, and filters. Parallel work by Suayngam et al. applied intuitionistic fuzzy sets [37], Fermatean fuzzy sets [35], and later generalized to neutrosophic sets [34], Pythagorean fuzzy sets [36], intuitionistic neutrosophic sets [32], and Pythagorean neutrosophic sets [33], producing a rich taxonomy of multi-valued generalizations. Most recently, Inthachot et al. [16] proposed the concept of bipolar fuzzy IUP-substructures, further enriching the interpretive scope. Together, these developments chart a systematic evolution from crisp IUP-theory toward a unified family of uncertainty-aware extensions, wherein homomorphic images, product constructions, and generalized subsets remain central analytic tools.

Motivated by the growing interest in derivation theory for non-associative structures, this paper introduces two novel operators—(l,r)- and (r,l)-derivations—defined directly on IUP-algebras using their intrinsic binary operation $\wedge$. Unlike derivations induced by endomorphic composition, which often rely on associative contexts, the approach here circumvents such dependencies, making it well-suited to the non-associative and non-commutative nature of IUP-algebras. This design enables a more native formulation of internal transformations, highlighting the interplay between the algebra's structure and its generalized derivations. We develop formal properties of these operators, examine their kernels and fixed-point sets, and characterize regularity conditions, thereby extending the scope of derivation theory within the IUP-algebras.

2. Preliminaries

We begin by recalling the basic definition of IUP-algebras—a recently introduced non-classical algebraic structure—along with illustrative examples and essential related notions. These foundational elements not only provide the formal setting for our results but also highlight the intrinsic features of IUP-algebras that motivate the study of generalized derivations such as (l, r)- and (r, l)-types developed in this paper.

Definition 2.1. *[14] An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called an IUP-algebra, where X is a non-empty set, $\cdot$ is a binary operation on X, and 0 is the constant of X if it satisfies the following axioms:*

$$(\forall x \in X)(0 \cdot x = x) \tag{IUP-1}$$

$$(\forall x \in X)(x \cdot x = 0) \tag{IUP-2}$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot (x \cdot z) = y \cdot z) \tag{IUP-3}$$

For clarity and brevity, we shall refer to an IUP-algebra by its underlying structure $X = (X, \cdot, 0)$, unless stated otherwise.

Example 2.1. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with the Cayley table as follows:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	4	0	3	1	5	2
2	2	5	0	4	3	1
3	5	4	1	0	2	3
4	1	3	5	2	0	4
5	3	2	4	5	1	0

Then $X = (X, \cdot, 0)$ is an IUP-algebra.

The following properties hold in any IUP-algebra X (see [14]).

$$(\forall x, y \in X)((x \cdot 0) \cdot (x \cdot y) = y) \tag{2.1}$$

$$(\forall x \in X)((x \cdot 0) \cdot (x \cdot 0) = 0) \tag{2.2}$$

$$(\forall x, y \in X)((x \cdot y) \cdot 0 = y \cdot x) \tag{2.3}$$

$$(\forall x \in X)((x \cdot 0) \cdot 0 = x) \tag{2.4}$$

$$(\forall x, y \in X)(x \cdot ((x \cdot 0) \cdot y) = y) \tag{2.5}$$

$$(\forall x, y \in X)(((x \cdot 0) \cdot y) \cdot x = y \cdot 0) \tag{2.6}$$

$$(\forall x, y, z \in X)(x \cdot y = x \cdot z \Leftrightarrow y = z) \tag{2.7}$$

$$(\forall x, y \in X)(x \cdot y = 0 \Leftrightarrow x = y) \tag{2.8}$$

$$(\forall x \in X)(x \cdot 0 = 0 \Leftrightarrow x = 0) \tag{2.9}$$

$$(\forall x, y, z \in X)(y \cdot x = z \cdot x \Leftrightarrow y = z) \tag{2.10}$$

$$(\forall x, y \in X)(x \cdot y = y \Rightarrow x = 0) \tag{2.11}$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot 0 = (z \cdot y) \cdot (z \cdot x)) \tag{2.12}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (z \cdot x) \cdot (z \cdot y) = 0) \tag{2.13}$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (x \cdot z) \cdot (y \cdot z) = 0) \tag{2.14}$$

the right and the left cancellation laws hold (2.15)

In the study of IUP-algebras, four distinguished types of subsets play a central role in the development of the theory. These are IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals, defined as follows.

Definition 2.2. *[14] A non-empty subset S of X is called*

(i) *an IUP-subalgebra of X if it satisfies the following condition:*

$$(\forall x, y \in S)(x \cdot y \in S) \tag{2.16}$$

(ii) *an IUP-filter of X if it satisfies the following conditions:*

the constant 0 *of X is in S* (2.17)

$$(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S) \tag{2.18}$$

(iii) *an IUP-ideal of X if it satisfies the condition* (2.17) *and the following condition:*

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S) \tag{2.19}$$

(iv) *a strong IUP-ideal of X if it satisfies the following condition:*

$$(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S) \tag{2.20}$$

According to [14], the notion of an IUP-filter generalizes both IUP-ideals and IUP-subalgebras, while IUP-ideals and IUP-subalgebras themselves generalize strong IUP-ideals. Furthermore, in the specific case of X, the only strong IUP-ideal is the whole algebra X itself.

3. Main Results

In this section, we introduce and develop the core notions of (l, r)- and (r, l)-derivations within the framework of IUP-algebras. These operators represent two distinct generalizations of classical derivations, adapted to the non-associative and non-commutative nature of IUP-algebras. Our approach relies on the internal binary operation $\wedge$, and does not presuppose any endomorphic structure. We examine their algebraic behavior, define associated subsets such as $\mathrm{Ker}_{\mathsf{d}}(X)$ and $\mathrm{Fix}_{\mathsf{d}}(X)$, and establish conditions under which such derivations preserve or reflect structural properties of the algebra. The results presented here not only extend existing derivation theory but also shed light on the deeper internal symmetries of IUP-algebras.

Definition 3.1. *A self-map $\mathsf{d} : X \to X$ is called an (l, r)-derivation of X if it satisfies the identity $\mathsf{d}(x \cdot y) = (\mathsf{d}(x) \cdot y) \wedge (x \cdot \mathsf{d}(y))$ for all $x, y \in X$. Similarly, a self-map $\mathsf{d} : X \to X$ is called an (r, l)-derivation of X if it satisfies the identity $\mathsf{d}(x \cdot y) = (x \cdot \mathsf{d}(y)) \wedge (\mathsf{d}(x) \cdot y)$ for all $x, y \in X$. Moreover, if d is both an (l, r)-derivation and an (r, l)-derivation of X, it is called a derivation of X.*

Example 3.1. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be an IUP-algebra in which the binary operation $\cdot$ is defined as follows:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	2	0	1	4	5	3
2	1	2	0	5	3	4
3	3	5	4	0	2	1
4	5	4	3	1	0	2
5	4	3	5	2	1	0

Define a self-map $\boldsymbol{d} : X \to X$ by

$$\boldsymbol{d} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 0 & 2 & 1 \end{pmatrix}$$

Then $\boldsymbol{d}$ is both an (l, r)-derivation and an (r, l)-derivation of X. In particular, $\boldsymbol{d}$ is an (r, l)-derivation and $\boldsymbol{d}(0) = 3 \neq 0$.

Define a self-maps $1_X : X \to X$ by $1_X(x) = x$ for all $x \in X$. Then, for any $x, y \in X$,

$$\begin{aligned} 1_X(x \cdot y) &= x \cdot y \\ &= (x \cdot y) \wedge (x \cdot y), && \text{(by Proposition 3.1 (3))} \end{aligned}$$

so $1_X(x \cdot y) = (1_X(x) \cdot y) \wedge (x \cdot 1_X(y)) = (x \cdot 1_X(y)) \wedge (1_X(x) \cdot y)$. Hence, 1_X is an (l, r)-derivation and an (r, l)-derivation of X.

To illustrate the abstract notions introduced above, we now present several examples of (l, r)- and (r, l)-derivations on concrete IUP-algebras. Examples 3.2, 3.3, 3.4, and 3.5 demonstrate the behavior of these operators under different structural settings. These examples are not only useful for verifying the definitions in explicit terms, but they also reveal subtle distinctions between (l, r)- and (r, l)-derivations, provide insight into their regularity, and help identify conditions under which kernels and fixed-point sets exhibit algebraic structure. As such, these examples serve as an essential bridge from definition to theorem, motivating the results established in the subsequent sections.

Example 3.2. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be an IUP-algebra in which the operation $\cdot$ is defined as follows:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	2	0	1	4	5	3
2	1	2	0	5	3	4
3	3	5	4	0	2	1
4	5	4	3	1	0	2
5	4	3	5	2	1	0

Define a self-map $0_X : X \to X$ *by*

$$0_X = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Then 0_X *is not both an* (l, r)*-derivation and an* (r, l)*-derivation of* X*. Indeed,*

$$0_X(2 \cdot 5) = 0_X(4) = 0 \neq 1 = 3 \cdot 5 = (1 \cdot 5) \cdot 5 = 5 \wedge 1 = 0 \cdot 5 \wedge 2 \cdot 0 = 0_X(2) \cdot 5 \wedge 2 \cdot 0_X(5)$$

and

$$0_X(5 \cdot 2) = 0_X(5) = 0 \neq 2 = 3 \cdot 4 = (2 \cdot 4) \cdot 4 = 4 \wedge 2 = 5 \cdot 0 \wedge 0 \cdot 2 = 5 \cdot 0_X(2) \wedge 0_X(5) \cdot 2.$$

Example 3.3. *[14] Let* $\mathbb{R}$ *be the set of all real numbers. Then* $(\mathbb{R}, \cdot, 0)$ *is an IUP-algebra, where* $\cdot$ *is the binary operation on* $\mathbb{R}$ *defined by* $x \cdot y = y - x$ *for all* $x, y \in \mathbb{R}$*. Define a self-map* $d : \mathbb{R} \to \mathbb{R}$ *by* $d(x) = x + 2$ *for all* $x \in \mathbb{R}$*. Then, for any* $x, y \in \mathbb{R}$*,*

$$d(x \cdot y) = d(y - x) = y - x + 2$$

and

$$\begin{aligned} (d(x) \cdot y) \wedge (x \cdot d(y)) &= ((x + 2) \cdot y) \wedge (x \cdot (y + 2)) \\ &= (y - x - 2) \wedge (y + 2 - x) \\ &= ((y + 2 - x) \cdot (y - x - 2)) \cdot (y - x - 2) \\ &= (y - x - 2 - y - 2 + x) \cdot (y - x - 2) \\ &= y - x - 2 - y + x + 2 + y + 2 - x \\ &= y - x + 2, \end{aligned}$$

so $d(x \cdot y) = d(x) \cdot y \wedge x \cdot d(y)$*. Therefore,* d *is an* (l, r)*-derivation of* $\mathbb{R}$*.*

Assume that d *is an* (r, l)*-derivation of* $\mathbb{R}$*. Then* $d(y - x) = d(x \cdot y) = x \cdot d(y) \wedge d(x) \cdot y = (d(y) - x) \wedge (y - d(x))$ *for all* $x, y \in \mathbb{R}$*. Then, for any* $x \in \mathbb{R}$*,*

$$\begin{aligned} d(x - x) &= (d(x) - x) \wedge (x - d(x)) \\ &= ((x - d(x)) \cdot (d(x) - x)) \cdot (d(x) - x) \\ &= ((d(x) - x) - (x - d(x))) \cdot (d(x) - x) \\ &= (d(x) - x) - ((d(x) - x) - (x - d(x))) \\ &= d(x) - x - d(x) + x + x - d(x) \\ &= x - d(x), \end{aligned}$$

so $d(0) + d(x) = d(x - x) + d(x) = x$*. In particular,* $d(0) + d(0) = 0$ *and so* $d(0) = 0$*. Hence,* $d(x) = x$ *for all* $x \in \mathbb{R}$*. Therefore,* $d = 1_{\mathbb{R}}$*.*

Example 3.4. *[14] Let* $(G, \cdot, e)$ *be a group where each element is self-inverse. Then* $(G, \cdot, e)$ *is an IUP-algebra. Let* $e \neq a \in G$*. Define a self-map* $d_a : G \to G$ *by* $d_a(x) = ax$ *for all* $x \in G$*. Then* $d_a(xy) = axy$ *for all* $x, y \in G$*. Let* $x, y \in G$*. Then*

$$\begin{aligned} \mathbf{d}_a(x)y \wedge x\mathbf{d}_a(y) &= axy \wedge xay \\ &= (xayaxy)axy \\ &= (xay)((axy)(axy)) \\ &= (xay)e \\ &= xay \\ &= axy && (G \text{ is commutative}) \\ &= \mathbf{d}_a(xy). \end{aligned}$$

Hence, $\mathbf{d}_a$ is an (l, r)-derivation of G. Also,

$$\begin{aligned} x\mathbf{d}_a(y) \wedge \mathbf{d}_a(x)y &= xay \wedge axy \\ &= (axyxay)xay \\ &= (axy)((xay)(xay)) \\ &= (axy)e \\ &= axy \\ &= \mathbf{d}_a(xy). \end{aligned}$$

Hence, $\mathbf{d}_a$ is an (r, l)-derivation of G.

Example 3.5. *[14] Let $\mathbb{R}^*$ be the set of all nonzero real numbers. Then $(\mathbb{R}^*, \cdot, 1)$ is an IUP-algebra, where $\cdot$ is the binary operation on $\mathbb{R}^*$ define by $x \cdot y = \frac{y}{x}$ for all $x \in \mathbb{R}^*$. Define a self-map $\mathbf{d} : \mathbb{R}^* \to \mathbb{R}^*$ by $\mathbf{d}(x) = 2x$ for all $x \in \mathbb{R}^*$. Then, for any $x, y \in \mathbb{R}^*$,*

$$\mathbf{d}(x \cdot y) = \mathbf{d}(\frac{y}{x}) = 2(\frac{y}{x})$$

and

$$\begin{aligned} (\mathbf{d}(x) \cdot y) \wedge (x \cdot \mathbf{d}(y)) &= (2x \cdot y) \wedge (x \cdot 2y) \\ &= \frac{y}{2x} \wedge \frac{2y}{x} \\ &= (\frac{2y}{x} \cdot \frac{y}{2x}) \cdot \frac{y}{2x} \\ &= (\frac{y}{2x}\frac{x}{2y}) \cdot \frac{y}{2x} \\ &= \frac{y}{2x}\frac{2x2y}{xy} \\ &= \frac{2y}{x} \\ &= 2(\frac{y}{x}), \end{aligned}$$

so $d(x \cdot y) = (d(x) \cdot y) \wedge (x \cdot d(y))$. *Hence,* d *is an* (l, r)*-derivation of* $\mathbb{R}^*$.

Assume that d *is an* (r, l)*-derivation of* $\mathbb{R}^*$. *Then* $d(\frac{y}{x}) = d(x \cdot y) = y \cdot d(x) \wedge d(x) \cdot y = \frac{d(x)}{y} \wedge \frac{y}{d(x)}$ *for all* $x, y \in \mathbb{R}^*$. *Then, for any* $x \in \mathbb{R}^*$,

$$
\begin{aligned}
d(\frac{x}{x}) &= \frac{d(x)}{x} \wedge \frac{x}{d(x)} \\
&= (\frac{x}{d(x)} \cdot \frac{d(x)}{x}) \cdot \frac{d(x)}{x} \\
&= (\frac{d(x)}{x}\frac{d(x)}{x}) \cdot \frac{d(x)}{x} \\
&= \frac{d(x)}{x}\frac{xx}{d(x)d(x)} \\
&= \frac{x}{d(x)},
\end{aligned}
$$

so $d(1)d(x) = d(\frac{x}{x})d(x) = x$. *In particular,* $d(1)d(1) = 1$ *and so* $d(1) = \pm 1$.

If $d(1) = 1$, *then* $d(x) = x$ *for all* $x \in \mathbb{R}^*$. *Hence,* $d = 1_{\mathbb{R}^*}$.

If $d(1) = -1$, *then* $d(x) = -x$ *for all* $x \in \mathbb{R}^*$. *Hence,* $d = -1_{\mathbb{R}^*}$.

Building on the preceding examples, we now turn to a more systematic investigation of algebraic properties associated with (l, r)- and (r, l)-derivations. In particular, we explore conditions under which these derivations exhibit regularity, and how such regularity constrains or characterizes the structure of the IUP-algebra. The following propositions lay the groundwork for understanding the interplay between derivations and the distinguished subsets of X, such as kernels and fixed-point sets.

Proposition 3.1. *In an IUP-algebra* X*, the following properties hold: for any* $x, y, z \in X$,

(1) $0 \wedge x = x$
(2) $x \wedge 0 = 0$
(3) $x \wedge x = x$
(4) $x \wedge y = x \wedge z \Rightarrow y = z$.

Proof. (1) Let $x \in X$. Then

$$
\begin{aligned}
0 \wedge x &= (x \cdot 0) \cdot 0 \\
&= x. && \text{(by (2.4))}
\end{aligned}
$$

(2) Let $x \in X$. Then

$$
\begin{aligned}
x \wedge 0 &= (0 \cdot x) \cdot x \\
&= x \cdot x && \text{(by (IUP-1))} \\
&= 0. && \text{(by (IUP-2))}
\end{aligned}
$$

(3) Let $x \in X$. Then

$$
\begin{aligned}
x \wedge x &= (x \cdot x) \cdot x \\
&= 0 \cdot x && \text{(by (IUP-2))} \\
&= x. && \text{(by (IUP-1))}
\end{aligned}
$$

(4) Let $x, y, z \in X$ be such that $x \wedge y = x \wedge z$. Then $(y \cdot x) \cdot x = (z \cdot x) \cdot x$. By (2.10), we have $y \cdot x = z \cdot x$. By (2.10) again, we have $y = z$. □

As we continue our investigation of (l, r)- and (r, l)-derivations, a particularly important subclass arises: those derivations that act trivially on at least one element of the IUP-algebra. This motivates the notion of regular derivations, which serve as a focal point in several of the results to follow. We now give the formal definition.

Definition 3.2. *An (l, r)-derivation (resp., (r, l)-derivation, derivation) $\mathtt{d}$ of X is called regular if $\mathtt{d}(0) = 0$.*

Theorem 3.1. *In an IUP-algebra X, the following statements hold:*

(1) *if $\mathtt{d}$ is an (l, r)-derivation of X such that $\mathtt{d}(x) = x$ for some $x \in X$, then $\mathtt{d}$ is regular*
(2) *if $\mathtt{d}$ is an (r, l)-derivation of X such that $\mathtt{d}(x) = x$ for some $x \in X$, then $\mathtt{d}$ is regular.*

Proof. (1) Assume that $\mathtt{d}$ is an (l, r)-derivation of X such that $\mathtt{d}(x) = x$ for some $x \in X$. Then

$$
\begin{aligned}
\mathtt{d}(0) &= \mathtt{d}(x \cdot x) && \text{(by (IUP-2))} \\
&= (\mathtt{d}(x) \cdot x) \wedge (x \cdot \mathtt{d}(x)) \\
&= (x \cdot x) \wedge (x \cdot x) && \text{(by the assumption)} \\
&= 0 \wedge 0 && \text{(by (IUP-2))} \\
&= 0. && \text{(by Proposition 3.1 (3))}
\end{aligned}
$$

Hence, $\mathtt{d}$ is regular.

(2) Assume that $\mathtt{d}$ is an (r, l)-derivation of X such that $\mathtt{d}(x) = x$ for some $x \in X$. Then

$$
\begin{aligned}
\mathtt{d}(0) &= \mathtt{d}(x \cdot x) && \text{(by (IUP-2))} \\
&= (x \cdot \mathtt{d}(x)) \wedge (\mathtt{d}(x) \cdot x) \\
&= (x \cdot x) \wedge (x \cdot x) && \text{(by the assumption)} \\
&= 0 \wedge 0 && \text{(by (IUP-2))} \\
&= 0. && \text{(by Proposition 3.1 (3))}
\end{aligned}
$$

Hence, $\mathtt{d}$ is regular.

□

Corollary 3.1. *If $\mathtt{d}$ is a derivation of X such that $\mathtt{d}(x) = x$ for some $x \in X$, then $\mathtt{d}$ is regular.*

Theorem 3.2. *In an IUP-algebra X, the following statements hold:*

(1) *if* $\mathtt{d}$ *is a regular* (l,r)*-derivation of* X*, then* $\mathtt{d}(x) = x \wedge \mathtt{d}(x)$ *for all* $x \in X$
(2) *if* $\mathtt{d}$ *is a regular* (r,l)*-derivation of* X*, then* $\mathtt{d}(x) = \mathtt{d}(x) \wedge x$ *for all* $x \in X$*.*

Proof. (1) Assume that $\mathtt{d}$ is a regular (l,r)-derivation of X. For all $x \in X$,

$$\begin{aligned} \mathtt{d}(x) &= \mathtt{d}(0 \cdot x) && \text{(by (IUP-1))} \\ &= (\mathtt{d}(0) \cdot x) \wedge (0 \cdot \mathtt{d}(x)) \\ &= (0 \cdot x) \wedge \mathtt{d}(x) && \text{(by (IUP-1))} \\ &= x \wedge \mathtt{d}(x). && \text{(by (IUP-1))} \end{aligned}$$

(2) Assume that $\mathtt{d}$ is a regular (r,l)-derivation of X. For all $x \in X$,

$$\begin{aligned} \mathtt{d}(x) &= \mathtt{d}(0 \cdot x) && \text{(by (IUP-1))} \\ &= (0 \cdot \mathtt{d}(x)) \wedge (\mathtt{d}(0) \cdot x) \\ &= \mathtt{d}(x) \wedge (0 \cdot x) && \text{(by (IUP-1))} \\ &= \mathtt{d}(x) \wedge x. && \text{(by (IUP-1))} \end{aligned}$$

□

Corollary 3.2. *If* $\mathtt{d}$ *is a regular derivation of* X*, then* $\mathtt{d}(x) \wedge x = \mathtt{d}(x) = x \wedge \mathtt{d}(x)$ *for all* $x \in X$*.*

To capture the elements sent to the zero element under a derivation, we define the notion of the kernel. This subset reflects important structural properties of IUP-algebras under derivational mappings.

Definition 3.3. *Let* $\mathtt{d}$ *be an* (l,r)*-derivation (resp.,* (r,l)*-derivation, derivation) of* X*. We define a subset* $\mathrm{Ker}_{\mathtt{d}}(X)$ *of* X *by*

$$\mathrm{Ker}_{\mathtt{d}}(X) = \{x \in X \mid \mathtt{d}(x) = 0\}.$$

Example 3.6. *From Example 3.4, let* $x \in \mathrm{Ker}_{\mathtt{d}_a}(G)$*. Then* $ax = \mathtt{d}_a(x) = e$*, so* $x = a^{-1}e = a^{-1} = a$*. Hence,* $\mathrm{Ker}_{\mathtt{d}_a}(G) = \{a\}$*.*

Proposition 3.2. *Let* $\mathtt{d}$ *be an* (l,r)*-derivation of* X*. Then the following properties hold: for any* $x, y \in X$*,*

(1) $\mathtt{d}(x \cdot \mathtt{d}(x)) = x \cdot \mathtt{d}(\mathtt{d}(x))$
(2) $\mathtt{d}(\mathtt{d}(x) \cdot x) = 0$
(3) $\mathtt{d}(x \cdot y) = 0 \Rightarrow \mathtt{d}(y) = x$*.*

Proof. (1) Let $x \in X$. Then

$$\begin{aligned} \mathtt{d}(x \cdot \mathtt{d}(x)) &= (\mathtt{d}(x) \cdot \mathtt{d}(x)) \wedge (x \cdot \mathtt{d}(\mathtt{d}(x))) \\ &= 0 \wedge (x \cdot \mathtt{d}(\mathtt{d}(x))) && \text{(by (IUP-2))} \\ &= x \cdot \mathtt{d}(\mathtt{d}(x)). && \text{(by Proposition 3.1 (1))} \end{aligned}$$

(2) Let $x \in X$. Then

$$\begin{aligned} \mathtt{d}(\mathtt{d}(x)\cdot x) &= (\mathtt{d}(\mathtt{d}(x))\cdot x)\wedge(\mathtt{d}(x)\cdot \mathtt{d}(x)) \\ &= (\mathtt{d}(\mathtt{d}(x))\cdot x)\wedge 0 && \text{(by (IUP-2))} \\ &= 0. && \text{(by Proposition 3.1 (2))} \end{aligned}$$

(3) Let $x, y \in X$ be such that $\mathtt{d}(x\cdot y) = 0$. Since $\mathtt{d}$ is an (l,r)-derivation of X, we have

$$\begin{aligned} (\mathtt{d}(x)\cdot y)\wedge(x\cdot \mathtt{d}(y)) &= \mathtt{d}(x\cdot y) = 0, \\ ((x\cdot \mathtt{d}(y))\cdot(\mathtt{d}(x)\cdot y))\cdot(\mathtt{d}(x)\cdot y) &= 0 \\ &= (\mathtt{d}(x)\cdot y)\cdot(\mathtt{d}(x)\cdot y), && \text{(by (IUP-2))} \\ (x\cdot \mathtt{d}(y))\cdot(\mathtt{d}(x)\cdot y) &= \mathtt{d}(x)\cdot y && \text{(by (2.15))} \\ &= 0\cdot(\mathtt{d}(x)\cdot y), && \text{(by (IUP-1))} \\ x\cdot \mathtt{d}(y) &= 0, && \text{(by (2.15))} \\ \mathtt{d}(y) &= x. && \text{(by (2.8))} \end{aligned}$$

□

The statement $\mathtt{d}(x)\cdot y = \mathtt{d}(x\cdot y)$ is not hold in general for an (l,r)-derivation. By Example 3.3, let $x = 1$ and $y = 3$. Then $\mathtt{d}(1\cdot 3) = \mathtt{d}(3-1) = \mathtt{d}(2) = 4$ and $\mathtt{d}(1)\cdot 3 = (1+2)\cdot 3 = 3\cdot 3 = 0$. Thus, $\mathtt{d}(1\cdot 3) \neq \mathtt{d}(1)\cdot 3$.

Theorem 3.3. *If $\mathtt{d}$ is a regular (l,r)-derivation of X, then it is the identity function.*

Proof. Assume that $\mathtt{d}$ is a regular (l,r)-derivation of X. Then $\mathtt{d}(0) = 0$. Let $x \in X$. Thus,

$$\begin{aligned} \mathtt{d}(x\cdot x) &= \mathtt{d}(0) && \text{(by (IUP-2))} \\ &= 0, \\ \mathtt{d}(x) &= x. && \text{(by Proposition 3.2 (3))} \end{aligned}$$

Hence, $\mathtt{d}$ is the identity function on X. Moreover, $\mathrm{Ker}_{\mathtt{d}}(X) = \{0\}$ and $\mathrm{Fix}_{\mathtt{d}}(X) = X$. □

Proposition 3.3. *Let d be an (r,l)-derivation of X. Then the following properties hold: for any $x, y \in X$*

(1) $d(x\cdot d(x)) = 0$
(2) $d(0) = d(0)\cdot 0$
(3) $d(x\cdot y) = 0 \Rightarrow d(x) = y$.

Proof. (1) Let $x \in X$. Then

$$\begin{aligned} \mathtt{d}(x\cdot \mathtt{d}(x)) &= (x\cdot \mathtt{d}(\mathtt{d}(x)))\wedge(\mathtt{d}(x)\cdot \mathtt{d}(x)) \\ &= (x\cdot \mathtt{d}(\mathtt{d}(x)))\wedge 0 && \text{(by (IUP-2))} \\ &= 0. && \text{(by Proposition 3.1 (2))} \end{aligned}$$

(2) Since $\mathtt{d}$ is an (r,l)-derivation of X, we have $\mathtt{d}(0\cdot 0) = (0\cdot \mathtt{d}(0)) \wedge (\mathtt{d}(0)\cdot 0)$. Thus,

$$\begin{aligned}
\mathtt{d}(0) &= (0\cdot \mathtt{d}(0)) \wedge (\mathtt{d}(0)\cdot 0) && \text{(by (IUP-2))}\\
&= \mathtt{d}(0) \wedge (\mathtt{d}(0)\cdot 0) && \text{(by (IUP-1))}\\
&= ((\mathtt{d}(0)\cdot 0)\cdot \mathtt{d}(0))\cdot \mathtt{d}(0), \\
0\cdot \mathtt{d}(0) &= ((\mathtt{d}(0)\cdot 0)\cdot \mathtt{d}(0))\cdot \mathtt{d}(0), && \text{(by (IUP-1))}\\
0 &= (\mathtt{d}(0)\cdot 0)\cdot \mathtt{d}(0), && \text{(by (2.15))}\\
\mathtt{d}(0)\cdot \mathtt{d}(0) &= (\mathtt{d}(0)\cdot 0)\cdot \mathtt{d}(0), && \text{(by (IUP-2))}\\
\mathtt{d}(0) &= \mathtt{d}(0)\cdot 0. && \text{(by (2.15))}
\end{aligned}$$

(3) Let $x, y \in X$ be such that $\mathtt{d}(x\cdot y) = 0$. Since $\mathtt{d}$ is an (r,l)-derivation of X, we have

$$\begin{aligned}
(x\cdot \mathtt{d}(y)) \wedge (\mathtt{d}(x)\cdot y) &= \mathtt{d}(x\cdot y) = 0, \\
((\mathtt{d}(x)\cdot y)\cdot (x\cdot \mathtt{d}(y)))\cdot (x\cdot \mathtt{d}(y)) &= (x\cdot \mathtt{d}(y))\cdot (x\cdot \mathtt{d}(y)), && \text{(by (IUP-2))}\\
(\mathtt{d}(x)\cdot y)\cdot (x\cdot \mathtt{d}(y)) &= x\cdot \mathtt{d}(y) && \text{(by (2.15))}\\
&= 0\cdot (x\cdot \mathtt{d}(y)), && \text{(by (IUP-1))}\\
\mathtt{d}(x)\cdot y &= 0, && \text{(by (2.15))}\\
\mathtt{d}(x) &= y. && \text{(by (2.8))}
\end{aligned}$$

□

Theorem 3.4. *If $\mathtt{d}$ is a regular (r,l)-derivation of X, then it is the identity function.*

Proof. Assume that $\mathtt{d}$ is a regular (r,l)-derivation of X. Then $\mathtt{d}(0) = 0$. Let $x \in X$. Thus,

$$\begin{aligned}
\mathtt{d}(x\cdot x) &= \mathtt{d}(0) && \text{(by (IUP-2))}\\
&= 0, \\
\mathtt{d}(x) &= x. && \text{(by Proposition 3.3 (3))}
\end{aligned}$$

Hence, $\mathtt{d}$ is the identity function on X. Moreover, $\mathrm{Ker}_{\mathtt{d}}(X) = \{0\}$ and $\mathrm{Fix}_{\mathtt{d}}(X) = X$. □

Corollary 3.3. *Let $\mathtt{d}$ be a derivation of X and $x, y \in X$. If $\mathtt{d}(x\cdot y) = 0$, then $\mathtt{d}(\mathtt{d}(y)) = y$ and $\mathtt{d}(\mathtt{d}(x)) = x$.*

Proof. Let $\mathtt{d}(x\cdot y) = 0$. By Proposition 3.2 (3), we have $\mathtt{d}(y) = x$. By Proposition 3.3 (3), we have $\mathtt{d}(x) = y$. Hence, $\mathtt{d}(\mathtt{d}(y)) = y$ and $\mathtt{d}(\mathtt{d}(x)) = x$. □

Corollary 3.4. *If $\mathtt{d}$ is a derivation of X, then $y \wedge x \in \mathrm{Ker}_{\mathtt{d}}(X)$ for all $y \in \mathrm{Ker}_{\mathtt{d}}(X)$ and $x \in X$.*

Theorem 3.5. *In an IUP-algebra X, the following statements hold:*

(1) *if $\mathtt{d}$ is a regular (l,r)-derivation of X, then $\mathrm{Ker}_{\mathtt{d}}(X) = \{0\}$ is an IUP-subalgebra of X*
(2) *if $\mathtt{d}$ is a regular (r,l)-derivation of X, then $\mathrm{Ker}_{\mathtt{d}}(X) = \{0\}$ is an IUP-subalgebra of X.*

Proof. (1) By Theorem 3.3, d is the identity function. Thus $\mathrm{Ker}_{\mathsf{d}}(X) = \{0\}$ is an IUP-subalgebra of X.

(2) By Theorem 3.4, d is the identity function. Thus $\mathrm{Ker}_{\mathsf{d}}(X) = \{0\}$ is an IUP-subalgebra of X. □

By Example 3.1, we have d is not regular and $\mathrm{Ker}_{\mathsf{d}}(X) = \{3\}$. Thus, $\mathrm{Ker}_{\mathsf{d}}(X)$ is not an IUP-subalgebra of X.

Corollary 3.5. *If* d *is a derivation of* X*, then* $\mathrm{Ker}_{d}(X) = \{0\}$ *is IUP-subalgebra of* X*.*

In contrast to the kernel, we now consider the set of elements that are invariant under a derivation. This set, called the fixed-point set, is formally defined as follows.

Definition 3.4. *Let* d *be an* (l, r)*-derivation (resp.,* (r, l)*-derivation, derivation) of* X*. We define a subset* $\mathrm{Fix}_{d}X$ *of* X *by*

$$\mathrm{Fix}_{d}(X) = \{x \in X \mid d(x) = x\}.$$

Example 3.7. *From Example 3.4, let* $x \in \mathrm{Fix}_{d_a}(G)$*. Then* $ax = d_a(x) = x$*, so* $a = e$*, which is impossible. Hence,* $\mathrm{Fix}_{d_a}(G) = \emptyset$*.*

Theorem 3.6. *In an IUP-algebra* X*, the following statements hold:*

(1) *if* d *is a regular* (l, r)*-derivation of* X*, then* $\mathrm{Fix}_{d}(X) = X$ *is an IUP-subalgebra of* X
(2) *if* d *is a regular* (r, l)*-derivation of* X*, then* $\mathrm{Fix}_{d}(X) = X$ *is an IUP-subalgebra of* X*.*

Proof. (1) Assume that d is a regular (l, r)-derivation of X. By Theorem 3.3, d is the identity function. Thus $\mathrm{Ker}_{\mathsf{d}}(X) = X$ is an IUP-subalgebra of X.

(2) Assume that d is a regular (r, l)-derivation of X. By Theorem 3.4, d is the identity function. Thus $\mathrm{Ker}_{\mathsf{d}}(X) = X$ is an IUP-subalgebra of X. □

Corollary 3.6. *If* d *is a regular derivation of* X*, then* $\mathrm{Fix}_{d}(X)$ *is an IUP-subalgebra of* X*.*

4. Conclusion

From our investigation, the following results were obtained:

(1) We introduced three derivation concepts—(l, r)-derivations, (r, l)-derivations, and general derivations—on an IUP-algebra $X = (X, \cdot, 0)$, defined in terms of the binary operation $\wedge$ given by $x \wedge y = (y \cdot x) \cdot x$ for all $x, y \in X$.
(2) We proved that both (l, r)- and (r, l)-derivations become *regular* when the condition $\mathsf{d}(x) = x$ holds for some element $x \in X$ (Theorem 3.1).
(3) We showed that if a derivation is both regular and of type (r, l) or (l, r), then it must coincide with the identity map on X (Theorems 3.3 and 3.4).
(4) Under regularity, the kernel $\mathrm{Ker}_{\mathsf{d}}(X) = \{0\}$ forms an IUP-subalgebra of X (Theorem 3.5).
(5) Similarly, the fixed-point set $\mathrm{Fix}_{\mathsf{d}}(X) = X$ forms an IUP-subalgebra of X when d is regular (Theorem 3.6).

Future research may extend (l, r)- and (r, l)-derivations to generalized IUP-algebras equipped with fuzzy, intuitionistic fuzzy, or neutrosophic structures. Additionally, characterizing these derivations under relaxed regularity conditions or exploring their categorical properties presents valuable directions for further investigation.

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References

[1] H.A.S. Abujabal, N.O. Al-Sehri, Some Results on Derivations of BCI-Algebras, J. Nat. Sci. Math. 46 (2006), 13–19.

[2] H.A.S. Abujabal, N.O. Al-Shehri, On Left Derivations of BCI-Algebras, Soochow J. Math. 33 (2007), 435–444.

[3] S. Afriastuti, S. Gemawati, Syamsudhuha, On (f, g)-Derivation in BCH-Algebra, SINTECHCOM: Sci. Technol. Commun. J. 1 (2021), 89–94.

[4] R.M. Al-Omary, On (α, β)-Derivations in d-Algebras, Boll. Unione Mat. Ital. 12 (2019), 549–556. https://doi.org/10.1007/s40574-018-00190-5.

[5] L.K. Ardakani, B. Davvaz, f-Derivations and (f, g)-Derivations of MV-Algebras, J. Algebr. Syst. 1 (2013), 11–31. https://doi.org/10.22044/jas.2013.167.

[6] T. Bantaojai, C. Suanoom, J. Phuto, A. Iampan, A Bi-Endomorphism Induces a New Type of Derivations on B-Algebras, Ital. J. Pure Appl. Math. 48 (2022), 336–348.

[7] C. Chanmanee, W. Nakkhase, R. Prasertpong, P. Julatha, A. Iampan, Notes on External Direct Products of Dual IUP-Algebras, South East Asian J. Math. Math. Sci. 19 (2023), 13–30. https://doi.org/10.56827/seajmms.2023.1903.2.

[8] C. Chanmanee, R. Prasertpong, P. Julatha, N. Lekkoksung, A. Iampan, On External Direct Products of IUP-Algebras, Int. J. Innov. Comput. Inf. Control 19 (2023), 775–787.

[9] G.T. Dejen, T. Gebrekidan, On Fuzzy Derivation of Ideals of BF-Algebras, J. Math. 2022 (2022), 5470197. https://doi.org/10.1155/2022/5470197.

[10] T. Ganeshkumar, M. Chandramouleeswaran, Generalized Derivation on TM-Algebras, Int. J. Algebr. 7 (2013), 251–258. https://doi.org/10.12988/ija.2013.13025.

[11] S. Ghorbani, L. Torkzadeh, S. Motamed, $(\odot, \oplus)$-Derivations and $(\ominus, \odot)$-Derivations on MV-Algebras, Iran. J. Math. Sci. Inform. 8 (2013), 75–90. http://dx.doi.org/10.7508/ijmsi.2013.01.008.

[12] A. Iampan, Derivations of UP-Algebras by Means of UP-Endomorphisms, Algebr. Struct. Appl. 3 (2016), 1–20.

[13] A. Iampan, R. Alayakkaniamuthu, P. Gomathi Sundari, N. Rajesh, Derivations of Hilbert Algebras, Int. J. Anal. Appl. 21 (2023), 39. https://doi.org/10.28924/2291-8639-21-2023-39.

[14] A. Iampan, P. Julatha, P. Khamrot, D.A. Romano, Independent UP-Algebras, J. Math. Comput. Sci. 27 (2022), 65–76. https://doi.org/10.22436/jmcs.027.01.06.

[15] A. Iampan, N. Rajesh, C. Arivazhagi, R. Vennila, Structural Derivations in Hilbert Algebras by Endomorphisms, Int. J. Anal. Appl. 22 (2024), 213. https://doi.org/10.28924/2291-8639-22-2024-213.

[16] C. Inthachot, K. Moonnon, M. Visutho, P. Julatha, A. Iampan, Bipolar Fuzzy Sets in IUP-Algebras: Concepts and Analysis, Trans. Fuzzy Sets Syst. 5 (2026), 39–58. https://doi.org/10.71602/TFSS.2026.1194460.

[17] M.A. Javed, M. Aslam, A Note on f-Derivations of BCI-Algebras, Commun. Korean Math. Soc. 24 (2009), 321–331. https://doi.org/10.4134/ckms.2009.24.3.321.

[18] Y.B. Jun, X.L. Xin, On Derivations of BCI-Algebras, Inf. Sci. 159 (2004), 167–176. https://doi.org/10.1016/j.ins.2003.03.001.

[19] N. Kandaraj, P. Ganesan, On Generalized (g, h)-Derivations of BH-Algebras, J. Math. Comput. Sci. 11 (2021), 8400–8414. https://doi.org/10.28919/jmcs/6732.

[20] M. Khan, D. Khan, K. Mir Aalm, Characterization of Associative PU-Algebras by the Notion of Derivations, Am. J. Math. Comput. Model. 6 (2021), 14–18. https://doi.org/10.11648/j.ajmcm.20210601.13.

[21] K. Kuntama, P. Krongchai, P. Prasertpong, P. Julatha, A. Iampan, Fuzzy Set Theory Applied to IUP-Algebras, J. Math. Comput. Sci. 34 (2024), 128–143. https://doi.org/10.22436/jmcs.034.02.03.

[22] S.M. Lee, K.H. Kim, A Note on f-Derivations of BCC-Algebras, Pure Math. Sci. 1 (2012), 87–93.

[23] P. Muangkarn, C. Suanoom, A. Iampan, New Derivations of d-Algebras Based on Endomorphisms, Int. J. Math. Comput. Sci. 17 (2022), 1025–1032.

[24] G. Muhiuddin, On (σ, τ)-Derivations of BCI-Algebras, Southeast Asian Bull. Math. 43 (2019), 715–723.

[25] G. Muhiuddin, A.M. Al-Roqi, On (α, β)-Derivations in BCI-Algebras, Discret. Dyn. Nat. Soc. 2012 (2012), 403209. https://doi.org/10.1155/2012/403209.

[26] G. Muhiuddin, A.M. Al-roqi, On t-Derivations of BCI-Algebras, Abstr. Appl. Anal. 2012 (2012), 872784. https://doi.org/10.1155/2012/872784.

[27] G. Muhiuddin, A.M. Al-Roqi, On Generalized Left Derivations in BCI-Algebras, Appl. Math. Inf. Sci. 8 (2014), 1153–1158. https://doi.org/10.12785/amis/080326.

[28] G. Muhiuddin, A.M. Al-roqi, On Left (θ, ϕ)-Derivations in BCI-Algebras, J. Egypt. Math. Soc. 22 (2014), 157–161. https://doi.org/10.1016/j.joems.2013.07.013.

[29] F. Nisar, On F-Derivations of BCI-Algebras, J. Prime Res. Math. 5 (2009), 176–191.

[30] K. Sawika, R. Intasan, A. Kaewwasri, A. Iampan, Derivations of UP-Algebras, Korean J. Math. 24 (2016), 345–367. https://doi.org/10.11568/kjm.2016.24.3.345.

[31] K. So, S. Ahn, Complicated BCC-Algebras and Its Derivation, Honam Math. J. 34 (2012), 263–271. https://doi.org/10.5831/hmj.2012.34.2.263.

[32] K. Suayngam, P. Julatha, W. Nakkhasen, A. Iampan, Structural Insights into IUP-Algebras via Intuitionistic Neutrosophic Set Theory, Eur. J. Pure Appl. Math. 18 (2025), 5857. https://doi.org/10.29020/nybg.ejpam.v18i2.5857.

[33] K. Suayngam, P. Julatha, W. Nakkhasen, R. Prasertpong, A. Iampan, Pythagorean Neutrosophic IUP-Algebras: Theoretical Foundations and Extensions, Eur. J. Pure Appl. Math. 18 (2025), 6171. https://doi.org/10.29020/nybg.ejpam.v18i3.6171.

[34] A. Aiyared, P. Julatha, R. Prasertpong, A. Iampan, Neutrosophic Sets in IUP-Algebras: A New Exploration, Int. J. Neutrosophic Sci. 25 (2025), 540–560. https://doi.org/10.54216/ijns.250343.

[35] K. Suayngam, R. Prasertpong, N. Lekkoksung, P. Julatha, A. Iampan, Fermatean Fuzzy Set Theory Applied to IUP-Algebras, Eur. J. Pure Appl. Math. 17 (2024), 3022–3042. https://doi.org/10.2139/ssrn.4898342.

[36] K. Suayngam, R. Prasertpong, W. Nakkhasen, P. Julatha, A. Iampan, Pythagorean Fuzzy Sets: A New Perspective on IUP-Algebras, Int. J. Innov. Comput. Inf. Control 21 (2025), 339–357.

[37] K. Suayngam, T. Suwanklang, P. Julatha, R. Prasertpong, A. Iampan, New Results on Intuitionistic Fuzzy Sets in IUP-Algebras, Int. J. Innov. Comput. Inf. Control 20 (2024), 1125–1141.

[38] J. Wang, P. He, Y. She, Some Results on Derivations of MV-Algebras, Appl. Math. J. Chin. Univ. 38 (2023), 126–143. https://doi.org/10.1007/s11766-023-4054-8.

[39] J. Thomys, f-Derivation of weak BCC-Algebras, Int. J. Algebra 5 (2011), 325–334.

[40] H. Yazarli, A Note on Derivations in MV-Algebras, Miskolc Math. Notes 14 (2013), 345–354. https://doi.org/10.18514/mmn.2013.420.