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A Study on φ-Ricci Symmetric LP-Sasakian Manifolds

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Abstract. The present paper is devoted to an in-depth study of φ-Ricci symmetric LP-Sasakian manifolds, which represent a significant class of Lorentzian para-Sasakian manifolds with rich geometric structures and distinctive curvature characteristics. The notion of φ-Ricci symmetry imposes specific constraints on the Ricci tensor in relation to the structure tensor φ, offering deeper insight into the curvature behavior and intrinsic geometry of these manifolds. This investigation aims to explore various fundamental properties of φ-Ricci symmetric LP-Sasakian manifolds, examining how these symmetry conditions influence their global and local geometric features. To support and illustrate the theoretical analysis, we construct an explicit example of a three-dimensional φ-Ricci symmetric LP-Sasakian manifold. Moreover, we study W_1-flat LP-Sasakian manifold.

1. Introduction

Matsumoto [1] proposed the concept of Lorentzian paracontact manifolds. In order to further relate the more general topics in contact geometry and classical analysis, Mihai and Roșca [2] expanded this framework by investigating the structure of Lorentzian P-Sasakian manifolds. The study of transformations in LP-Sasakian manifolds by Matsumoto and Mihai [3] paved the ground for further research into the curvature features of these manifolds under different geometric flows and deformations. The general structure of Lorentzian para-Sasakian manifolds was examined by De, Matsumoto and Shaikh [4] in the theory's later development, while their relationships to almost Kenmotsu and Kenmotsu manifolds were examined by Binh et al. [5] and De and Pathak [6],

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respectively. ϕ-Ricci symmetric property on Sasakian, Kenmotsu and *LP*-Kenmotsu manifolds have been studied in [7], [13] and [20], respectively. The concept of locally φ-symmetric Sasakian manifolds was first introduced by Takahashi in [10] as a natural generalization and a weaker form of local symmetry for Sasakian manifolds. This idea has since garnered considerable attention within the field of differential geometry due to its potential to explore new geometric structures under less restrictive conditions. More recently, quarter-symmetric metric connections on LP-Sasakian manifolds have been studied in [8,14]. The concept of golden structures and CR-lightlike submanifolds was developed by Qayyoom et al. [11,12], which enhanced the manifold theory even more. Also see [9,15]. ϕ-conformal flatness of LP-Sasakian manifold was studied by Ozgur [17], while invariant submanifolds of LP-Sasakian manifolds have studied by Ozgur & Murathan [18]. Recently, certain gemoetric flows on LP-Sasakian manifolds have been studied in [16,19]. Given the breadth of literature surrounding LP-Sasakian manifolds, the present paper aims to explore a specific subclass, namely φ-Ricci symmetric LP-Sasakian manifolds, which emerge as a natural extension in the study of Ricci-type symmetries in paracontact geometry. The paper is organized systematically to develop the theory and present illustrative examples supporting the main results.

In Section 1, we provide preliminaries for the understanding of LP-Sasakian manifolds, including basic definitions and tensorial structures inherent to such manifolds. Section 2 is devoted to laying out additional prerequisites and background materials, offering a more detailed foundation upon which the main results are built. In Section 3, we construct a 5-dimensional of LP-Sasakian manifolds. In Section 4, we investigate φ-Ricci symmetric LP-Sasakian manifolds, analyzing the conditions under which such symmetries arise and exploring their implications on the manifold's geometric behavior. Subsequently, Section 5 is dedicated to the study of three-dimensional φ-Ricci symmetric LP-Sasakian manifolds. In Section 6, we construct an explicit example of a three-dimensional LP-Sasakian manifold that satisfies the φ-Ricci symmetry condition. This example serves to concretely demonstrate the theoretical developments discussed earlier. Finally, in Section 7, we focus on the concept of W_1-flatness in the context of LP-Sasakian manifolds.

2. Preliminaries

An n-dimensional differentiable manifold M is called an LP-Sasakian manifold [1,3], if it admits a $(1,1)$ tensor field φ, a contravariant vector field ξ, a 1-form η and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \tag{2.1}$$

$$\varphi^2 X = X + \eta(X)\xi, \tag{2.2}$$

$$g(\varphi X, \varphi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X,\xi) = \eta(X),\ \nabla_X \xi = \varphi X, \tag{2.4}$$

$$(\nabla_X\varphi)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.5}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g. It can be easily seen that in an LP-Sasakian manifold, the following relations hold:

$$\varphi\xi = 0,\ \eta(\varphi X) = 0, \tag{2.6}$$

$$rank\varphi = n-1. \tag{2.7}$$

Again if we put

$$\omega(X,Y) = g(X,\varphi Y), \tag{2.8}$$

for any vector fields X and Y, then the tensor field $\omega(X,Y)$ is a symmetric $(0,2)$ tensor field [1]. Also since the vector field η is closed in an LP-Sasakian manifold, we have [1,4], [4]

$$(\nabla_X\eta)(Y) = \omega(X,Y), \omega(X,\xi) = 0 \tag{2.9}$$

for any vector fields X and Y. An LP-Sasakian manifold M is said to be η-Einstein if its Ricci tensor S is of the form

$$S(X,Y) = \alpha g(X,Y) + \beta\eta(X)\eta(Y) \tag{2.10}$$

for any vector fields X, Y where α, β are functions on M. Let M be an n-dimensional LP-Sasakian manifold with structure (ϕ,ξ,η,g). Then we have [3,4]

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y), \tag{2.11}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.12}$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.13}$$

$$R(\xi,X)\xi = X + \eta(X)\xi, \tag{2.14}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.15}$$

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.16}$$

for any vector fields X, Y, Z; where $R(X,Y)Z$ is the Riemannian curvature tensor.

3. A 5-Dimensional Example of LP-Sasakian Manifold

We consider a five dimensional manifold $M = (x,y,z,u,t) \in R^5$, where x,y,z,u,t are the standard coordinates in R^5. We choose linearly independent global frame fields as

$$e_1 = e^t\frac{\partial}{\partial x}, e_2 = e^t\frac{\partial}{\partial y}, e_3 = e^t\frac{\partial}{\partial z}, e_4 = e^t\frac{\partial}{\partial u}, e_5 = \frac{\partial}{\partial t}. \tag{3.1}$$

Let g be the Lorentzian metric defined by

$$g(e_1,e_1) = g(e_2,e_2) = g(e_3,e_3) = g(e_4,e_4) = 1, g(e_5,e_5) = -1, \tag{3.2}$$

$$g(e_i,e_j) = 0 \text{ for } 1 \leq i,j \leq 5. \tag{3.3}$$

Let η be the 1-form defined by

$$\eta(Z) = g(Z,e_5), \tag{3.4}$$

for any $Z \in \chi(M)$. We define a $(1,1)$-tensor field φ as

$$\varphi(e_1) = -e_1, \varphi(e_2) = -e_2, \varphi(e_3) = -e_3, \varphi(e_4) = -e_4, \varphi(e5) = 0. \tag{3.5}$$

The linearity of φ and g yields that

$$\eta(e_5) = -1, \tag{3.6}$$

$$\varphi^2(Z) = Z + \eta(Z)\xi, \tag{3.7}$$

$$g(\varphi U, \varphi Z) = g(U,Z) + \eta(U)\eta(Z), \tag{3.8}$$

for any $U, Z \in \chi(M)$.

Let ∇ be the Levi Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g, then we have

$$[e_1,e_2] = [e_1,e_3] = [e_1,e_4] = 0, [e_1,e_5] = -e_1, \tag{3.9}$$

$$[e_2,e_3] = [e_2,e_4] = 0, [e_2,e_5] = -e_2, \tag{3.10}$$

$$[e_3,e_4] = 0, [e_3,e_5] = -e_3, [e_4,e_5] = -e_4. \tag{3.11}$$

The Koszul's formula is defined by

$$2g(\nabla_X Y, Z) = Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) - g(X,[Y,Z]) \tag{3.12}$$

$$- g(Y,[X,Z]) + g(Z,[X,Y]). \tag{3.13}$$

By using the Koszul's formula, we can easily get the followings:

$$\nabla e_1 e_1 = e_5, \nabla e_1 e_2 = 0, \nabla e_1 e_3 = 0, \nabla e_1 e_4 = 0, \nabla e_1 e_5 = -e_1, \tag{3.14}$$

$$\nabla e_2 e_1 = 0, \nabla e_1 e_2 = e_5, \nabla e_2 e_3 = 0, \nabla e_2 e_4 = 0, \nabla e_2 e_5 = -e_2, \tag{3.15}$$

$$\nabla e_3 e_1 = 0, \nabla e_3 e_2 = 0, \nabla e_3 e_3 = e_5, \nabla e_3 e_4 = 0, \nabla e_3 e_5 = -e_3, \tag{3.16}$$

$$\nabla e_4 e_1 = 0, \nabla e_4 e_2 = 0, \nabla e_4 e_3 = 0, \nabla e_4 e_4 = e_5, \nabla e_4 e_5 = -e_5, \tag{3.17}$$

$$\nabla e_5 e_1 = 0, \nabla e_5 e_2 = 0, \nabla e_5 e_3 = 0, \nabla e_5 e_4 = 0, \nabla e_5 e_5 = 0. \tag{3.18}$$

From the above calculation it can be easily seen that M^5 is an LP-Sasakian manifold.

4. φ-Ricci Symmetric LP- Sasakian Manifolds

Firstly, we recall

Definition 4.1. *An LP-sasakian manifold M is said to be locally φ-symmetric, if*

$$\varphi^2((\nabla_W R)(X,Y)Z) = 0, \tag{4.1}$$

for any vector fields X, Y, Z, W orthogonal to ξ.

Definition 4.2. *[8] An LP-Sasakian manifold M is said to be φ-Ricci symmetric if the Ricci operator satisfies*

$$\varphi^2(\nabla_X Q)(Y) = 0, \tag{4.2}$$

for any vector fields X, Y on M and $S(X,Y) = g(QX,Y)$. If X, Y are orthogonal to ξ, then the manifold is said to be locally φ-Ricci symmetric.

Definition 4.3. *[8] An LP-Sasakian manifold M is said to be an Einstein manifold if its Ricci tensor S is of the form*

$$S(X,Y) = \alpha g(X,Y), \tag{4.3}$$

where α is a constant and X, Y are any vector fields on M.

Theorem 4.1. *A $(2n+1)$-dimensional φ-Ricci symmetric LP-Sasakian manifold is an Einstein manifold.*

Proof. Let the manifold is assumed to be φ-Ricci symmetric. Then, we have

$$\varphi^2(\nabla_X Q)(Y) = 0. \tag{4.4}$$

Using (2.2) in the above relation, we get

$$(\nabla_X Q)(Y) + \eta((\nabla_X Q)(Y))\xi = 0. \tag{4.5}$$

From (4.5), it follows that

$$g((\nabla_X Q)(Y), Z) + \eta((\nabla_X Q)(Y))\eta(Z) = 0, \tag{4.6}$$

which on simplifying gives,

$$g(\nabla_X Q(Y), Z) + S(\nabla_X Y, Z) + \eta((\nabla_X Q)(Y))\eta(Z) = 0. \tag{4.7}$$

Replacing Y by ξ in (4.7), we have

$$g(\nabla_X Q(\xi), Z) - S(\nabla_X \xi, Z) + \eta((\nabla_X Q)(\xi))\eta(Z) = 0. \tag{4.8}$$

By using (2.4) and (2.15) in (4.8), we can obtain

$$(n-1)g(\varphi X, Z) - S(\varphi X, Z) + \eta((\nabla_X Q)(\xi))\eta(Z) = 0. \tag{4.9}$$

Replacing Z by φZ in (4.9), we have

$$S(\varphi X, \varphi Z) = (n-1)g(\varphi X, \varphi Z). \tag{4.10}$$

In view of (2.3) and (2.16), (4.10) becomes

$$S(X,Z) = (n-1)g(X,Z), \tag{4.11}$$

which is an Einstein manifold. □

Now, since a φ-symmetric Riemannian manifold is φ-Ricci symmetric, then we have

Corollary 4.1. *A φ-symmetric LP-Sasakian manifold is an Einstein manifold.*

Corollary 4.2. *If a $(2n+1)$-dimensional LP-Sasakian manifold is an Einstein manifold, then it is φ-Ricci symmetric.*

Proof. Let us suppose that the manifold is an Einstein manifold. Then

$$S(X,Y) = \alpha g(X,Y), \tag{4.12}$$

where $(X,Y) = g(QX,Y)$ and α is a constant. Hence $QX = \alpha X$. So, we have

$$\varphi^2((\nabla_Y Q)(X)) = 0. \tag{4.13}$$

This completes the required proof. □

In view of Theorem 3.1 and Theorem 3.3, we have

Corollary 4.3. *A $(2n+1)$-dimensional LP-Sasakian manifold is φ-Ricci symmetric if and only if it is an Einstein manifold.*

5. Three-Dimensional φ-Ricci Symmetric LP-Sasakian Manifolds

Theorem 5.1. *If the scalar curvature r of a 3-dimensional LP-Sasakian manifold is equal to 6, then the manifold is φ-Ricci symmetric.*

Proof. The curvature tensor of a 3-dimensional LP-Sasakian manifold is of the form [5]

$$\begin{aligned} R(X,Y)Z = \frac{(r-4)}{2}[g(Y,Z)X - g(X,Z)Y] + \frac{(r-6)}{2}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \end{aligned} \tag{5.1}$$

By contracting above relation, we have

$$S(X,Y) = \frac{1}{2}[(r-2)g(X,Y) + (6-r)\eta(X)\eta(Y)]. \tag{5.2}$$

Thus gives

$$QX = \frac{1}{2}[(r-2)X + (r-6)\eta(X)\xi]. \tag{5.3}$$

Taking the covariant derivative of (5.3) along W, we have

$$(\nabla_W Q)X = \frac{1}{2}dr(W)X - \frac{1}{2}dr(W)\eta(X)\xi + \frac{r-6}{2}g(X,\varphi W)\xi + \frac{r-6}{2}\eta(X)(\nabla_W \xi), \tag{5.4}$$

Now applying φ^2 in (5.4), we have

$$\varphi^2((\nabla_W Q)X) = \frac{1}{2}dr(W)\varphi^2 X + \frac{r-6}{2}\eta(X)\varphi^2(\nabla_W \xi). \tag{5.5}$$

Since $r = 6$, therefore (5.5) reduces to

$$\varphi^2((\nabla_W Q)X) = 0. \tag{5.6}$$

This completes the proof of the theorem. □

Theorem 5.2. *A $(2n+1)$-dimensional LP-Sasakian manifold is locally φ-Ricci symmetric if and only if the scalar curvature r is constant.*

Proof. Taking X orthogonal to ξ in (5.4), we obtain

$$\varphi^2((\nabla_W Q)X) = \frac{1}{2}dr(W)X. \tag{5.7}$$

The proof follows from (5.6) and Theorem 5.1. □

6. Example

Consider a 3-dimensional manifold $M = (x, y, z) \in R^3$, where (x, y, z) are the standard cordinates in R^3. The vector fields we have taken are

$$e_1 = x\frac{\partial}{\partial z}, e_2 = x\frac{\partial}{\partial y}, e_3 = -x\frac{\partial}{\partial x} = \xi, \tag{6.1}$$

which are linearly independent at each point of M. Let g be the Lorentzian metric defined by

$$g(e_1, e_3) = g(e_2, e_3) = g(e_1, e_2) = 0, \tag{6.2}$$

$$g(e_1, e_1) = g(e_2, e_2) = 1, g(e_3, e_3) = -1. \tag{6.3}$$

Let η be the 1-form defined by $\eta(X) = g(X, e_3)$ for any $X \in \chi(M)$. Let φ be the $(1, 1)$ tensor field defined by

$$\varphi e_1 = -e_1, \varphi e_2 = -e_2, \varphi e_3 = 0. \tag{6.4}$$

Then using the linearity of φ and g, we have $\eta(e_3) = -1$, $\varphi^2(X) = X + \eta(X)\xi$ and $g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\eta(Y)$, for any $X, Y \in \chi(M)$. Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g. Then we have

$$[e_1, e_2] = 0, [e_1, e_3] = -e_1, [e_2, e_3] = -e_2. \tag{6.5}$$

The Riemannian connection ∇ of the Lorentzian metric g is given by

$$2g(\nabla_X Y, Z) = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) - g(X, [Y, Z]) - g(Y, [X, Z] + g(Z, [X, Y]), \tag{6.6}$$

which is known as Koszul's formula. From Koszul's formula, we have

$$\nabla e_1 e_3 = -e_1, \nabla e_1 e_2 = 0, \nabla e_1 e_1 = -e_3, \tag{6.7}$$

$$\nabla e_2 e_3 = -e_2, \nabla e_2 e_2 = -e_3, \nabla e_2 e_1 = 0, \tag{6.8}$$

$$\nabla e_3 e_3 = 0, \nabla e_3 e_2 = 0, \nabla e_3 e_1 = 0. \tag{6.9}$$

From the above result it can be easily seen that the manifold satisfies

$$\nabla_X \xi = \varphi X \tag{6.10}$$

for $\xi = e_3$. Hence the manifold under consideration is a Lorentzian para Sasakian manifold. Hence $M(\varphi, \xi, \eta, g)$ is a 3-dimensional LP-Sasakian manifold. By using above results, we can easily obtain the following:

$$R(e_1, e_2)e_3 = 0, \qquad R(e_3, e_2)e_3 = e_2, \qquad R(e_3, e_1)e_2 = 0, \tag{6.11}$$

$$R(e_2, e_3)e_3 = -e_2, \qquad R(e_3, e_1)e_1 = -e_3, \qquad R(e_3, e_2)e_2 = -e_3, \tag{6.12}$$

$$R(e_1, e_2)e_2 = -e_1, \qquad R(e_1, e_3)e_3 = -e_1, \qquad R(e_2, e_1)e_1 = -e_2. \tag{6.13}$$

The Ricci tensor of 3-dimensional manifold is given by

$$S(X, Y) = \sum g(R(e_i, X)Y, e_i). \tag{6.14}$$

Using above components of the curvature tensor in (6.14), we get the following results:

$$S(e_1, e_1) = 2, \qquad S(e_2, e_2) = 2, \qquad S(e_3, e_3) = 2, \tag{6.15}$$

$$S(e_1, e_2) = 0, \qquad S(e_1, e_3) = 0, \qquad S(e_2, e_3) = 0. \tag{6.16}$$

In view of above relations, it follows that the scalar curvature of the manifold is equal to $+6$ and the Ricci tensor is: $S(X, Y) = 2g(X, Y)$. Hence $QX = 2X$, which implies that $\varphi^2(\nabla_W Q)(X) = 0$. Thus we observe that the scalar curvature of the manifold under consideration is $+6$, and it is φ-Ricci symmetric. Thus, this example verifies Theorem 5.1.

7. W_1-flat LP-Sasakian Manifold

In this section, we study about W_1-flat LP-Sasakian Manifold

Definition 7.1. *An LP-Sasakian manifold is said to be W_1-flat if*

$$W_1(X, Y)Z = 0, \tag{7.1}$$

for any vector fields X,Y and Z on M, where W_1-curvature tensor is given by [6]

$$W_1(X, Y)Z = R(X, Y)Z + \frac{1}{n-1}[S(Y, Z)X - S(X, Z)Y]. \tag{7.2}$$

From (7.1) and (7.2), we have

$$R(X, Y)Z + \frac{1}{n-1}[S(Y, Z)X - S(X, Z)Y] = 0. \tag{7.3}$$

Replacing X by ξ in above equation then using (2.13) and (2.15), we get

$$g(Y, Z)\xi - \eta(Z)Y + \frac{1}{n-1}[S(Y, Z)\xi - (n-1)\eta(Z)Y] = 0,$$

which can be written as

$$S(Y,Z)\xi = (n-1)[2\eta(Z)Y - g(Y,Z)\xi]. \tag{7.4}$$

Taking inner product of (7.4) with ξ, we find

$$S(Y,Z) = -2(n-1)\eta(Z)\eta(Y) - (n-1)g(Y,Z). \tag{7.5}$$

Hence from the above discussion, we state the following theorem:

Theorem 7.1. *An n-dimensional W_1-flat LP-Sasakian manifold is an η-Einstein manifold of the form* (7.5).

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