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Numerical Simulation of a Fractional SEIHRD Epidemic Model Using Adams-Bashforth-Moulton Method

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Abstract. Epidemics are infectious diseases that spread rapidly and affect large portions of the population within a specific region and timeframe. Throughout history, such outbreaks have caused devastating impacts on humanity from the Black Death, which eliminated one-third of Europe's population during the Middle Ages, to the Spanish flu, which claimed millions of lives in the early 20th century. While treatment and prevention strategies vary depending on the nature of the disease, common measures often include quarantine, isolation, improved hygiene, and the development of vaccines and medications. In this study, we propose a hypothetical fractional-order epidemic model to investigate the potential spread of the Ebola virus during the Ramadan season of 2025. The model specifically considers the influx of pilgrims into the Kingdom of Saudi Arabia, one of the world's leading destinations for religious tourism during Ramadan. We numerically solve the model using the Adams–Bashforth–Moulton Predictor–Corrector Method, and conduct a detailed analysis of the simulation results to better understand the dynamics of the outbreak and propose effective mitigation strategies.

1. Introduction

Pandemics have played a central role in shaping the course of human civilization throughout history. The earliest recorded epidemic occurred in 430 BCE during the Peloponnesian War, and since then, infectious diseases have repeatedly altered demographic, economic, and political

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structures across societies [1]. Among the earliest major outbreaks was the Plague of Justinian (541–750 CE), caused by Yersinia pestis. It is estimated to have killed between 30 to 50 million people nearly half of the global population at the time leading to the decline of the Byzantine Empire, the disruption of trade, and the loss of vast territories in North Africa and the Middle East [1]. The Black Death (1347–1351), another devastating wave of bubonic plague, swept through Europe and Asia, killing about 25 million people in Europe alone. The consequences extended beyond the death toll: it weakened feudal structures, increased wages for laborers due to workforce scarcity, and altered the social hierarchy of Europe [2]. In the 15th and 16th centuries, smallpox was introduced to the Americas by European colonists. It decimated Indigenous populations, killing an estimated 20 million people, or about 90% of the native population. The epidemic facilitated European conquest by depopulating vast territories and eliminating resistance [3].

The first cholera pandemic (1817–1823), which began in Jessore, India, rapidly spread across South Asia and beyond, claiming millions of lives. Cholera remains a public health threat in developing countries due to contaminated water sources. The World Health Organization (WHO) reports that cholera still affects 1.3 to 4 million people annually [4–6]. In 1918, the world faced one of the deadliest influenza pandemics: the Spanish Flu. It infected over 500 million people and resulted in more than 50 million deaths globally. Occurring at the end of World War I, the pandemic overwhelmed healthcare systems and set the stage for new strategies in public health and disease surveillance [7]. The Hong Kong Flu (1968–1970), caused by the H3N2 influenza virus, led to approximately 1 million deaths worldwide. Although less lethal than previous pandemics, its rapid global spread in just a few weeks after its emergence in Hong Kong highlighted the growing risk posed by global travel and the importance of vaccine development [8]. In 2002, the world witnessed the emergence of Severe Acute Respiratory Syndrome (SARS), caused by a novel coronavirus (SARS-CoV). It originated in Guangdong, China, and spread to 26 countries, infecting more than 8,000 people and killing 774. Although the outbreak was contained through swift international public health measures, it served as a warning about the potential of coronavirus outbreaks [9–11]. The H1N1 influenza pandemic (2009–2010), known as Swine Flu, infected over 60 million individuals in the United States and caused an estimated 151,000 to 575,000 deaths globally. Uniquely, this strain had a higher mortality rate among younger populations compared to seasonal flu [12].

Between 2014 and 2016, the Ebola virus outbreak in West Africa marked one of the deadliest and most challenging epidemics in recent decades. The virus, named after the Ebola River near the original outbreak location, first emerged in a village in Guinea. From there, it spread to neighboring countries, primarily Liberia and Sierra Leone. Over 29,000 people were infected, and more than 11,000 died. The high fatality rate, combined with weak healthcare infrastructure, social stigma, and community mistrust, contributed to the rapid spread and difficulties in containment. The outbreak had significant socio-economic repercussions, including a loss of $ 4.3 billion in GDP across the affected countries and a major decline in international investment. It also exposed global vulnerabilities in epidemic preparedness and led to major reforms in global health security systems [13–15]. Most recently, the COVID-19 pandemic (2019–present), caused by the novel coronavirus SARS-CoV-2, has affected every aspect of life across the globe. As of 2025, over 7 million deaths have been officially recorded. The pandemic has led to unprecedented disruptions in healthcare delivery, global trade, education, and social behavior, while also accelerating innovation in vaccine development and digital health systems [16–18].

From ancient plagues to modern viral pandemics, these historical episodes illustrate humanity's enduring vulnerability to emerging infectious diseases. They emphasize the need for stronger global health infrastructures, early warning systems, equitable access to medical interventions, and interdisciplinary approaches to pandemic response and preparedness. To mitigate the catastrophic consequences of epidemics whether in terms of human loss or economic disruption—it is essential to develop predictive models that can inform public health strategies. In this context, we propose a fractional-order epidemiological model based on the Riemann-Liouville derivative to simulate the potential spread of an epidemic during the 2025 Ramadan Umrah season in the Kingdom of Saudi Arabia. Saudi Arabia is among the top global destinations for mass gatherings, especially during religious periods, receiving millions of pilgrims from around the world. Of particular concern is the possibility of importing Ebola Virus Disease (EVD) cases from African countries with predominantly Muslim populations, many of which have previously experienced Ebola outbreaks or are geographically connected to high-risk zones. These countries include: Algeria, Egypt, Sudan, Morocco, Tunisia, Libya, Mauritania, Somalia, Djibouti, Mali, Niger, Senegal, Chad, Gambia, Guinea, Sierra Leone, Burkina Faso, Comoros, and parts of Nigeria, Ethiopia, and Eritrea. The convergence of pilgrims from these regions into a confined space during Umrah increases the risk of disease transmission, especially in the absence of robust surveillance and preemptive containment strategies.

In recent years, significant progress has been achieved in the development of analytical and numerical techniques for solving classical and fractional differential systems arising in applied sciences. Efficient algorithms have been proposed for linear and nonlinear Volterra integro-differential equations and for fractional differential equations using optimized decomposition-based strategies, highlighting the importance of reliable computational frameworks [19,20]. Moreover, advanced numerical investigations have been employed to explore stochastic and variable-order fractional dynamics, as well as epidemic and biological models, confirming the practical relevance of fractional operators in describing complex phenomena [21, 22]. These efforts have been further supported by recent works on fractional epidemic modeling, synchronization analysis, reaction–diffusion dynamics, and infectious disease transmission, demonstrating that fractional modeling provides flexible and accurate tools for understanding real-world outbreaks and related systems [23–27].

Our objective is to use this fractional model to capture the memory and hereditary properties of the disease dynamics, which classical models often fail to address. The outcomes of this study are intended to support policymakers in evaluating potential risks and implementing evidence-based health interventions for mass gathering scenarios, particularly in preparation for future outbreaks in high-density international events.

2. Fractional Calculus

The concept of fractional derivatives dates back to the early development of calculus. In 1695, L'Hospital posed a question about the interpretation of $\frac{d^n f}{dt^n}$ when $n = \frac{1}{2}$, prompting ongoing efforts by researchers to define derivatives of non-integer order. Among the well-known definitions are the Riemann–Liouville and Caputo fractional derivatives [28, 29], as well as the Caputo–Fabrizio derivative [30]. However, it is important to note that many of these definitions do not fully satisfy fundamental calculus rules such as the chain rule, product rule, and quotient rule. To address these limitations, Khalil et al. introduced the conformable derivative in 2014 [31], a definition that retains many of the key properties of classical integer-order derivatives. For further theoretical insights into the conformable fractional derivative, see references [32–35], and for various applications of this definition, consult [36–39].

Definition 2.1. ***(The Gamma Function)***
The Gamma function, denoted by $\Gamma(.)$, is a key special function in mathematics that extends the concept of the factorial to complex and real number arguments (excluding the non-positive integers). It is defined for complex numbers with a positive real part by the improper integral:

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x}\, dx, \quad \Re(t) > 0.$$

This function satisfies the fundamental recurrence relation:

$$\Gamma(t+1) = t\,\Gamma(t),$$

which closely mirrors the recurrence of the factorial function. In fact, for any natural number n ,

$$\Gamma(n) = (n-1)!.$$

The Gamma function plays a central role in many areas of analysis, complex functions, probability theory, and mathematical physics.

Definition 2.2. ***(Riemann-Liouville definition)***
For $\alpha \in [n-1, n)$, the α derivative of f is

$$_{RL}D_a^\alpha(f)(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(x)}{(t-x)^{\alpha-n+1}} dx.$$

Theorem 2.1. ***(Properties of the Riemann–Liouville Fractional Derivative)*** *Let $\alpha > 0$, $n = \lceil \alpha \rceil$, and suppose that $f \in C^n[a, b]$. The left-sided Riemann–Liouville fractional derivative of order α, defined by*

$$_{RL}D_{a+}^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-x)^{n-\alpha-1} f(x)\, dx,$$

satisfies the following properties:

(1) ***Linearity:*** *For any constants* $c_1, c_2 \in \mathbb{R}$*,*

$$_{RL}D^{\alpha}_{a+}(c_1 f(t) + c_2 g(t)) = c_1 D^{\alpha}_{a+} f(t) + c_2 D^{\alpha}_{a+} g(t).$$

(2) ***Action on Power Functions:*** *For* $\lambda > -1$*,*

$$_{RL}D^{\alpha}_{0+} t^{\lambda} = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\alpha+1)} t^{\lambda-\alpha}, \quad t > 0.$$

(3) ***Composition with Integer Derivatives:*** *For* $m \in \mathbb{N}$*,*

$$_{RL}D^{\alpha}_{a+} D^{m}_{a+} f(t) =_{RL} D^{\alpha+m}_{a+} f(t),$$

under suitable smoothness and integrability conditions.

(4) ***Semigroup Property:*** *For* $0 < \alpha, \beta < 1$*, and suitable* f*,*

$$_{RL}D^{\alpha}_{a+} D^{\beta}_{a+} f(t) =_{RL} D^{\alpha+\beta}_{a+} f(t).$$

(5) ***Derivative of a Constant:***

$$_{RL}D^{\alpha}_{a+} c = \frac{c}{\Gamma(1-\alpha)} (t-a)^{-\alpha}, \quad \text{for } \alpha \notin \mathbb{N}.$$

(6) ***Non-locality:*** *The operator is non-local, meaning that*

$$_{RL}D^{\alpha}_{a+} f(t)$$

depends on the entire history of $f(\tau)$ *for* $\tau \in [a, t]$*.*

(7) ***Relation to Fractional Integral:***

$$_{RL}D^{\alpha}_{a+} f(t) = \frac{d^n}{dt^n} \left[I^{n-\alpha}_{a+} f(t) \right],$$

where I^{β}_{a+} *is the left-sided Riemann–Liouville fractional integral of order* $\beta > 0$*,*

$$I^{\beta}_{a+} f(t) = \frac{1}{\Gamma(\beta)} \int_a^t (t-x)^{\beta-1} f(x)\, dx.$$

Proof. We refer the reader to [40, 41]. □

3. Initialization of the Fractional SEIHRD Model with the Riemann–Liouville Derivative

For a fixed $a = 0$ and $\alpha \in [0, 1)$ the fractional SEIHRD model for a hypothetical Ebola outbreak in Saudi Arabia can be expressed as a system of fractional nonlinear ordinary differential equations, as shown below:

$$\begin{aligned}
{}_{RL}D_0^{\alpha}S &= -\beta\frac{S(I+\theta H+\omega D)}{N}\\
{}_{RL}D_0^{\alpha}E &= \beta\frac{S(I+\theta H+\omega D)}{N}-\sigma E\\
{}_{RL}D_0^{\alpha}I &= \sigma E-(\gamma+\delta)I\\
{}_{RL}D_0^{\alpha}H &= \delta I-(\eta+\mu)H\\
{}_{RL}D_0^{\alpha}R &= \gamma I+\eta H\\
{}_{RL}D_0^{\alpha}D &= \mu H,
\end{aligned}$$

The components of the aforementioned fractional nonlinear system are defined as follows:

(1) $S(t)$: Number of susceptible individuals.
(2) $E(t)$: Number of exposed individuals (infected but not yet infectious).
(3) $I(t)$: Number of infectious individuals exhibiting symptoms.
(4) $H(t)$: Number of hospitalized individuals.
(5) $R(t)$: Number of recovered individuals.
(6) $D(t)$: Number of deceased individuals (who may still be infectious in the case of Ebola).

The model parameters are interpreted as:

(1) β: Baseline transmission rate.
(2) θ: Modification factor for the infectiousness of hospitalized individuals.
(3) ω: Modification factor for the infectiousness of deceased individuals.
(4) σ: Rate at which exposed individuals become infectious (incubation rate).
(5) γ: Recovery rate of infectious individuals.
(6) δ: Rate at which infectious individuals are hospitalized.
(7) η: Recovery rate of hospitalized individuals.
(8) μ: Mortality rate among hospitalized individuals.

To study this epidemiological model, we propose two sets of initial conditions and consider three scenarios: the first represents a baseline (normal) situation, the second assumes an effective public health intervention, and the third reflects a deterioration in the healthcare system.

First set of initial conditions:

$$\begin{aligned}
S(0) &= N-1\\
E(0) &= 1\\
I(0) &= 0\\
H(0) &= 0\\
R(0) &= 0\\
D(0) &= 0,
\end{aligned}$$

Second set of initial conditions:

$$\begin{aligned} S(0) &= N-2 \\ E(0) &= 1 \\ I(0) &= 1 \\ H(0) &= 0 \\ R(0) &= 0 \\ D(0) &= 0, \end{aligned}$$

According to the Saudi General Authority for Statistics, a record number of Umrah pilgrims arriving from outside the Kingdom was registered during the month of Ramadan in the year 1446 AH (2025), with the total exceeding 10 million pilgrims. Accordingly, we will assume that the average population under study in this case is 5 million people ($N = 5,000,000$). In the context of studying a potential Ebola outbreak during the Umrah season in Saudi Arabia, the epidemiological time frame has been extended to five consecutive months (150 days). This extension allows for a comprehensive analysis that covers all stages of human movement and population dynamics during this religious season.

The extended study period begins in Jumada al-Thani 1446 AH (approximately January 4 to February 1, 2025), representing a pre-epidemic phase in the hypothetical scenario. This phase assumes that the infection may begin to appear or spread silently before the major influx of international pilgrims. In Rajab 1446 AH (February 2 to March 2, 2025), international pilgrims start to arrive gradually in Saudi Arabia as Umrah visas become available. This phase marks the beginning of pilgrim influx, especially from distant countries, where travel is planned weeks in advance. By Sha'ban 1446 AH (March 3 to March 31, 2025), the number of pilgrims increases steadily. This phase involves intense logistical preparations and crowd management by Saudi authorities. It is characterized by a gradual rise in population density, which could enhance the risk of infectious disease transmission. The peak period occurs in Ramadan 1446 AH (April 1 to April 30, 2025), where the number of pilgrims reaches its maximum, particularly during the last ten days of the month and on the 27th night of Ramadan. This period is of high epidemiological importance due to the extreme crowding and increased contact rates. Finally, Shawwal 1446 AH (May 1 to May 29, 2025) represents the post-peak phase, where most pilgrims depart after the Eid celebration. However, some may remain for a short period in Makkah or Madinah. This phase is important for monitoring any residual transmission and for implementing containment strategies.

4. Why the Fractional Adams–Bashforth–Moulton Method?

To solve the fractional SEIHRD model involving the Riemann–Liouville derivative of order $\alpha \in (0,1)$, choosing an appropriate numerical method is crucial for ensuring accuracy, numerical stability and computational efficiency. Hence the best suited method is the fractional Adams–Bashforth–Moulton method because it is well-adapted to Riemann–Liouville derivatives, as it uses numerical quadrature consistent with the RL definition. Also, it is a multi step method, offering higher accuracy than one-step methods like Euler. Furthermore it handles nonlinear systems effectively and it has been widely used in the literature for similar fractional epidemiological models.

Basic Idea of the Method:

We convert the fractional differential equation using the Riemann–Liouville integral form:

$$ {}_{RL}D_0^{\alpha} y(t) = f\left(t, y(t)\right) \Rightarrow y(t) = y(0) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-x)^{\alpha-1} f\left(x, y(x)\right) dx. $$

This integral is then numerically approximated using quadrature rules (e.g., trapezoidal rule, Simpson's rule, or Adams–Bashforth–Moulton).

Numerical Implementation Steps:

1) Discretize the time domain using steps $t_n = nh$, where h is the time step size.

2) Use the predictor–corrector scheme of the fractional Adams–Bashforth–Moulton method to approximate the solution.

3) Approximate the fractional integral at each step using a memory-dependent recurrence.

4) Compute the solution for all compartments $S(t), E(t), I(t)$,... at each time step.

Finally, we conclude that the most appropriate numerical method for solving the fractional SEIHRD model with the Riemann–Liouville derivative is the fractional Adams–Bashforth–Moulton method, as it offers a reliable balance of accuracy, stability, and efficiency, and is well-suited for nonlinear systems involving Riemann–Liouville operators. Because, recent studies, such as those [42, 43] highlight the efficiency of the ABM approach over classical methods, especially for fractional models in epidemiology and control systems. Therefore, the fractional ABM method is adopted in this study as the preferred tool for accurately simulating the proposed SEIHRD model under realistic and sensitive conditions.

5. Baseline Scenario

Table 1 shows the values of parameters for the fractional SEIHRD model (hypothetical Ebola case without intervention:

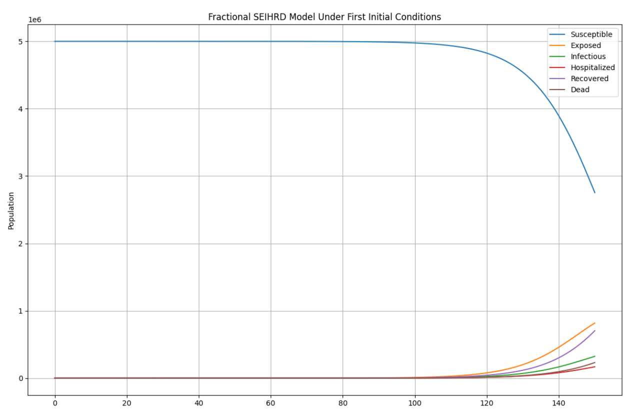
Figure 1 represent the solution of the fractional SEIHRD model for Ebola virus under the baseline scenario in the two proposed initial conditions with the fractional order $\alpha = 0.9$ in Saudi Arabia during Ramadan Omrah season of 2025.

6. Effective Public Health Intervention Scenario

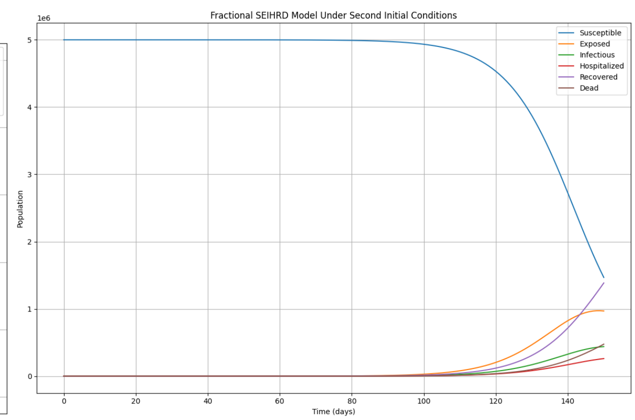
Table 2 shows the values of parameters for the fractional SEIHRD model (hypothetical Ebola case with effective public health intervention:

Table 1. Baseline scenario parameters for the fractional SEIHRD model (hypothetical Ebola case without intervention)

Parameter (Symbol)	Value / Description
β (Transmission rate)	0.5
θ (Hospital infection modifier)	0.5
ω (Cadaver infection modifier)	1.1
σ (Incubation rate)	$\frac{1}{6}$
γ (Recovery rate from infectious)	0.15
δ (Hospitalization rate)	$\frac{1}{5}$
η (Recovery rate from hospital)	0.15
μ (Death rate in hospital)	0.15
Note: This baseline scenario assumes a hypothetical Ebola outbreak in the absence of any public health intervention.	



(A) The baseline scenario under first initial conditions



(B) The baseline scenario under second initial conditions

Figure 1. Simulation of the fractional SEIHRD model for Ebola virus under the baseline scenario with two initial conditions and fractional order $\alpha = 0.9$

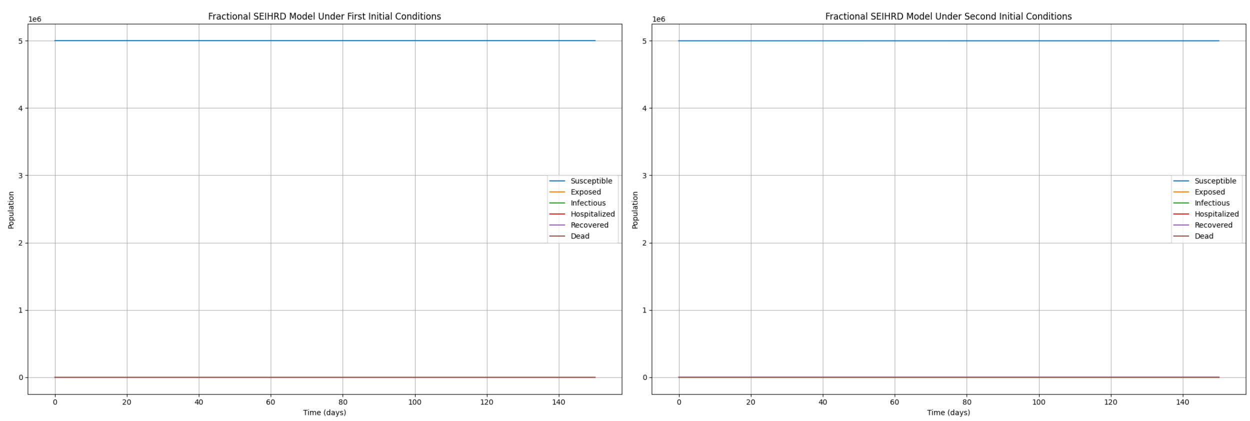
Figure 2 show the solution of the fractional SEIHRD model for Ebola virus under the effective public health intervention scenario in the two proposed initial conditions with the fractional order $\alpha = 0.9$ in Saudi Arabia during Ramadan Omrah season of 2025

7. Deterioration in the Healthcare System Scenario

Table 3 shows the values of parameters for the fractional SEIHRD model (hypothetical Ebola case with deterioration in the healthcare system:

Table 2. Effective Public Health Intervention Scenario

Parameter (Symbol)	**Value / Description**
β (Transmission rate)	0.2
θ (Hospital infection modifier)	0.3
ω (Cadaver infection modifier)	0.5
σ (Incubation rate)	$\frac{1}{6}$
γ (Recovery rate from infectious individuals)	$\frac{1}{7}$
δ (Hospitalization rate)	0.25
η (Recovery rate from hospital)	$\frac{1}{7}$
μ (Death rate in hospital)	$\frac{1}{12}$
Note: This scenario represents a hypothetical Ebola outbreak under the assumption of effective public health interventions, including early detection, hospital isolation, safe burial practices, and reduced community transmission.	



(a) The effective public health intervention scenario under first initial conditions

(b) The effective public health intervention scenario under second initial conditions

Figure 2. Simulation of the fractional SEIHRD model for Ebola virus under the effective public health intervention scenario with two initial conditions and fractional order $\alpha = 0.9$

Figure 3 show the solution of the fractional SEIHRD model for Ebola virus under the deterioration in the healthcare system scenario in the two proposed initial conditions with the fractional order $\alpha = 0.9$ in Saudi Arabia during Ramadan Omrah season of 2025.

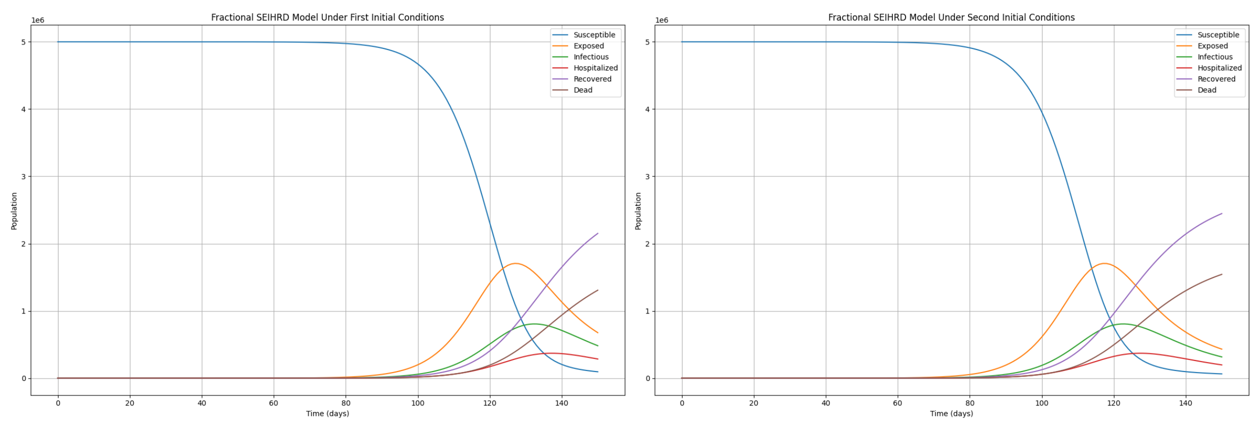
8. Comparison of the Three Proposed Scenarios

1) Baseline scenario:

a) The graphical solution under the first initial condition (Figure 1 (a)) shows that the impact of

TABLE 3. Deterioration in the Healthcare System Scenario

Parameter (Symbol)	Value / Description
β (Transmission rate)	0.6
θ (Hospital infection modifier)	0.8
ω (Cadaver infection modifier)	1.5
σ (Incubation rate)	$\frac{1}{9}$
γ (Recovery rate from infectious individuals)	$\frac{1}{12}$
δ (Hospitalization rate)	0.125
η (Recovery rate from hospital)	$\frac{1}{12}$
μ (Death rate in hospital)	$\frac{1}{6}$
Note: This scenario represents a hypothetical Ebola outbreak under the assumption of a deteriorated healthcare system, with reduced recovery and hospitalization capacity, and increased risk of infection from hospitals and cadavers.	



(A) Deterioration in the healthcare system scenario under the first initial conditions

(B) Deterioration in the healthcare system scenario under the second initial conditions

FIGURE 3. Simulation of the fractional SEIHRD model for Ebola virus under deterioration in the healthcare system scenario with two initial conditions and fractional order $\alpha = 0.9$

the virus on the population begins around day 90. This delay is due to the large population size of 5,000,000, where the number of susceptible individuals starts to decline reaching 2,750,000 on the last day, while the others exposed, infected, hospitalized, recovered and deceased begin to rise reaching 900,000; 250,000; 200,000; 800,000; 225,000 respectively.
b) The graphical solution under the second initial condition (Figure 1 (b)) shows that the impact of the virus on the population begins around day 85. This delay is due to the large population size of 5,000,000, where the number of susceptible individuals starts to decline reaching 1,500,000 on

the last day, while the others exposed, infected, hospitalized, recovered and deceased begin to rise reaching 999,000; 390,000; 250,000; 1,400,000; 500,000 respectively.

2) Effective public health intervention scenario:

The graphical solution under the first and second initial condition (Figure 2 (a) (b)) shows that the Ebola virus does not affect people, and this is due to the effective health intervention.

3) Deterioration in the healthcare system scenario:

a) The graphical solution under the first initial condition (Figure 3 (a)) shows that the impact of the virus on the population begins around day 70. This delay is due to the large population size of 5,000,000, where the number of susceptible individuals starts to decline reaching 150,000 on the last day, while the recovered and deceased begin to rise reaching 2,200,000 and 1,350,000 respectively. But the exposed increase to reach its peak of 1,700,000 on day 127, then begins to decline to reach 700,000 on the last day. Also, the infected increases to reach its peak of 800,000 on day 133, then decreases to 500,000 on the last day. Similarly, for the hospitalized increases to reach its peak of 400,000 on day 135, then decreases to 300,000 on the last day.

b) The graphical solution under the second initial condition (Figure 3 (b)) shows that the virus has the same behavior under the second initial conditions, but its impact on the studied community is more deadly.

Closing remark:

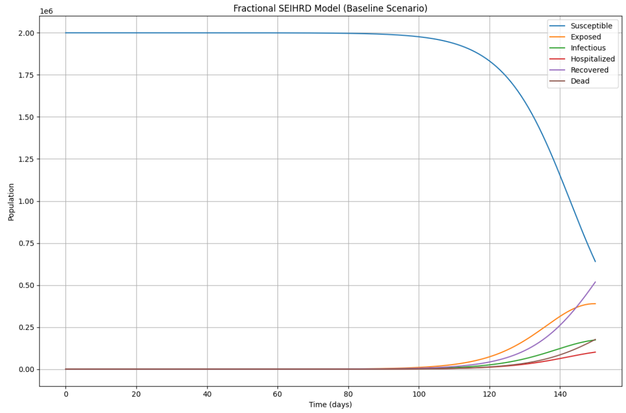
The comparative analysis of the three proposed scenarios clearly demonstrates the critical role of public health interventions in controlling the spread of the Ebola virus. The effective public health intervention scenario successfully prevents significant outbreaks, maintaining the susceptible population at safe levels. In contrast, the deterioration in the healthcare system scenario leads to a rapid and severe escalation in infections, hospitalizations, and deaths, with earlier outbreak onset compared to the baseline scenario. The baseline scenario shows moderate progression of the epidemic, but still results in considerable health impacts over time. Overall, the findings highlight that proactive and robust healthcare measures are essential to mitigating the impact of Ebola, especially in densely populated communities.

9. Comparative Study on the Impact of Fractional Order and Population Size

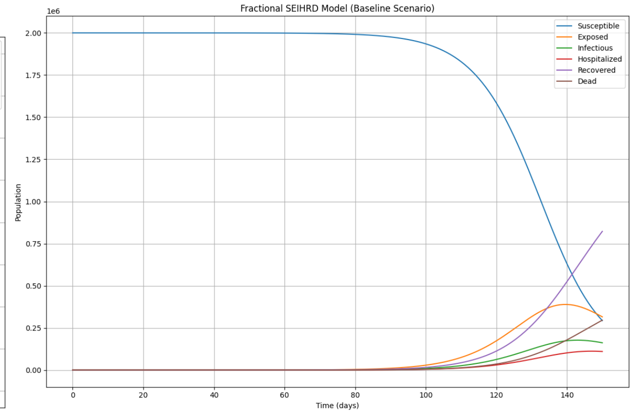
- Figure 4 show that the number of population (susceptible individuals) does't not affect the behavior of solution, just different of values, the larger the population size, the greater the values of the proposed epidemic model in the solution.
- After changing the fractional order α (see Figure 5 and Figure 6), we observe that the greater the value of α (the closer it is to one), the clearer the solution becomes. Therefore, we proposed $\alpha = 0.9$ in solving all the scenarios numerically.

10. Conclusion

In this work, a fractional-order SEIHRD epidemic model for a hypothetical Ebola outbreak during the Ramadan season of 2025 in Saudi Arabia had been formulated to describe the disease

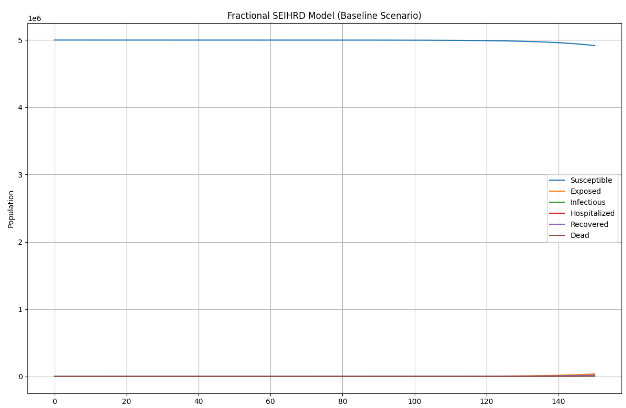


(A) The baseline scenario under first initial conditions

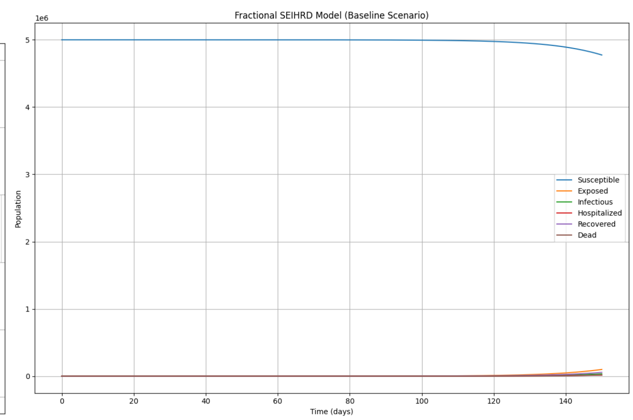


(B) The baseline scenario under second initial conditions

Figure 4. Simulation of the fractional SEIHRD model for Ebola virus under the baseline scenario with two initial conditions and fractional order $\alpha = 0.9$ and population size 2,000,000



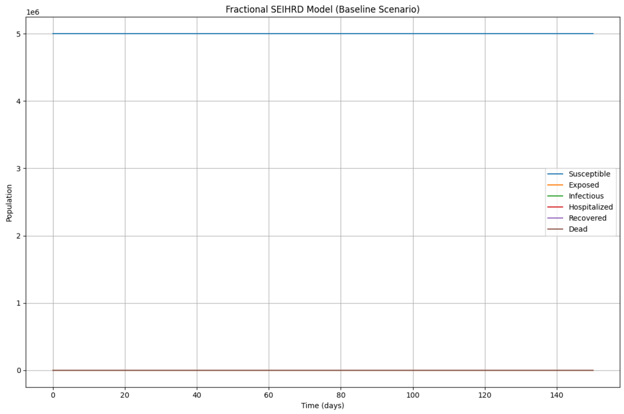
(A) The baseline scenario under first initial conditions



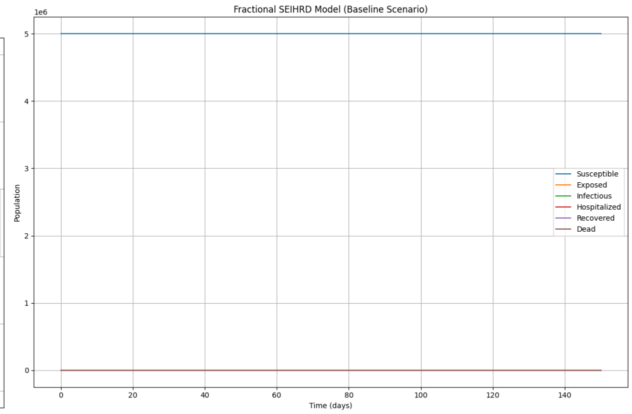
(B) The baseline scenario under second initial conditions

Figure 5. Simulation of the fractional SEIHRD model for Ebola virus under the baseline scenario with two initial conditions and fractional order $\alpha = 0.8$ and population size 5,000,000

transmission dynamics under increased population mobility. The model had been numerically solved using the Adams–Bashforth–Moulton predictor–corrector method, and the impacts of key epidemiological parameters on the evolution of the compartments had been investigated through simulation. The obtained results had confirmed that fractional-order effects had provided a more flexible and realistic description of memory and hereditary properties in epidemic spread. Moreover, the numerical outcomes had highlighted that early control measures and effective intervention strategies had significantly reduced the peak of infection and hospitalization. Finally, the



(A) The baseline scenario under first initial conditions



(B) The baseline scenario under second initial conditions

Figure 6. Simulation of the fractional SEIHRD model for Ebola virus under the baseline scenario with two initial conditions and fractional order $\alpha = 0.5$ and population size 5,000,000

proposed framework had been shown to be useful for supporting public health planning and mitigation decisions in highly crowded seasonal events.

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