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Neutrosophic Semi δ-pre Irresolute Mappings

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Abstract. This study investigates and defined new concept of irresolute mappings called Neutrosophic semi δ-pre irresolute mappings via Neutrosophic topological spaces. Several preservation properties and some characterizations concerning Neutrosophic semi δ-pre irresolute have been obtained.

1. Historical Background

Fuzzy sets were first presented by Zadeh [1]. Chang [2] introduced the concept of fuzzy topological spaces by employing the concept of fuzzy sets. The concept of intuitionistic fuzzy sets proposed by Atanassov [3] in 1986, as a generalization of fuzzy sets. The concept of intuitionistic fuzzy topological spaces presented by Coker [4] in 1997, as a generalization of fuzzy topological spaces. Many mathematicians have expanded the concepts of fuzzy topological to intuitionistic fuzzy topology after Coker presented the concept of intuitionistic fuzzy topology [4] in the his paper an introduction to Intuitionistic Fuzzy Topological Space, which published in the Journal of Fuzzy Sets and Systems.

These mathematicians include Eom and Lee [5], Hanafy [6, 7], Jeon [8], Coker and his associates [9–11], Thakur and his associates [12–14] and Lupianez [21] studied various topological concepts in intuitionistic fuzzy topology. Neutrosophic set and Neutrosophy were first introduced by Smarandache [15, 16] around the start of the twentieth century. Both Alblowi and Salama [17] constructed the first Neutrosophic set in a topological space that is Neutrosophic in 2012. Numerous definitions of Neutrosophic closed sets and Neutrosophic mapping have been provided and investigated by Al-Omeri and Smarandache [18, 19, 37]. Additionally, they have characterized Neutrosophic connectivity and continuity as well as inferred a number of preservation properties. In 2024 Mahima Thakur and others [22] introduce the concept of Neutrosophic semi δ-preopen

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sets and Neutrosophic semi δ-pre continuous mappings in this work and discuss A few of its fundamental characteristics which published in Neutrosophic set and system.

In the Neutrosophic set, truth-membership, indeterminacy-membership, and falsity-membership are independent, and indeterminacy is explicitly quantified. In many circumstances, this supposition is crucial. as when we attempt to merge data from several sensors, or information fusion. This set class is a member of the significant class of Neutrosophic semi-preopen sets and Neutrosophic semi-pre continuity, which is very helpful in Neutrosophic multifunction theory, Neutrosophic the economy, and Neutrosophic theories of control in addition to helping us better understand some unique aspects of the already well-known concepts of Neutrosophic topology. The field's experts are looking into these areas of study because of the wide range of applications.In the future, we hope to extend our work to other fields, such as fuzzy set theory, as in the works of [30–34, 41], and to fixed point theory, as in the studies of [40, 42], also in statistics field as in the study of [35].The neutrosophic set theory as an extension of fuzzy set theory for handling uncertainty. It references recent developments in neutrosophic topology [44, 45] and algebra [38, 39] to highlight the field's active growth. The paper builds upon this foundation by introducing **neutrosophic semi δ-pre irresolute mappings**, with objectives to study their properties and establish characterizations, thereby extending neutrosophic continuous mapping theory.

Now, the intersection of all Neutrosophic closed sets which contains B is said to be closure of B. It is defined by $Cl(B)$. The union of all Neutrosophy open subsets of B is said to be the interior of B. It is defined by $Int(B)$. A Neutrosophy set B of an Neutrosophy topological space $(\mathcal{Z}, \Gamma)$ is said to be Neutrosophy regular open [18] if $B = Int(Cl(B))$. The complement of an Neutrosophy regular open set is said to be Neutrosophy regular closed. Every Neutrosophy regular open (resp. Neutrosophy regular closed) set is Neutrosophy open (resp. Neutrosophy closed) but the converse not need to be true [23, 25]. The δ-closure (denoted by δCl) [23, 26] of an Neutrosophy set B of a Neutrosophy topological space $(\mathcal{Z}, \Gamma)$ is the intersection of all Neutrosophy regular closed sets containing B. The δ-interior (defined by δInt) [23, 43] of a Neutrosophy set B of an Neutrosophy topological space $(\mathcal{Z}, \Gamma)$ is the union of all Neutrosophy regular open sets contained in B. Within Section 1, we will go over various ideas that will be utilized throughout this research. In Section 2, we discuss a few concepts to introduce Neutrosophic semi δ-pre irresolute mappings and study their fundamental characteristics. Finally in section 3 we discus some future work.

Definition 1.1. *A neutrosophic set B in a Neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is said to be:*

(1) *neutrosophic δ-open (in short, NδO) [36] if $B = Int(\delta Cl(B))$.*
(2) *neutrosophic pre- open (in short, NPO) [18] if $B \subseteq Int(Cl(B))$.*
(3) *neutrosophic semi- open (in short, NSO) [18] if $B \subseteq Cl(Int(B))$.*
(4) *neutrosophic α open (in short, NαO) [18] if $B \subseteq Int(Cl(Int(B)))$.*
(5) *neutrosophic semi-pre open (in short, NSPO) [24] if $B \subseteq Cl(Int(Cl(B)))$.*
(6) *neutrosophic δ-pre open [23] if $B \subseteq Int(\delta Cl(B))$.*

(7) *neutrosophic semi δ-pre open [22] if there exists a neutrosophic δ-preopen set G such that* $G \subseteq B \subseteq \delta Cl(G)$.
(8) *neutrosophic semi δ-pre closed [22] if there exists a neutrosophic δ-preclosed set G such that* $\delta Int(G) \subseteq B \subseteq G$.

The family of all neutrosophic semi δ-pre closed (resp. neutrosophic semi δ-pre open, neutrosophic semi-pre open, neutrosophic δ-pre open) sets of a NTS $(\mathcal{Z}, \Gamma)$ *is denoted by NSδPC(*$\mathcal{Z}$*) (resp. NSδPO(*$\mathcal{Z}$*), NSPO(*$\mathcal{Z}$*), NδPO(*$\mathcal{Z}$*), NPO(*$\mathcal{Z}$*), NαO(*$\mathcal{Z}$*)).*

A neutrosophic set B in a neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is said to be neutrosophic semi-pre closed [24](resp. neutrosophic δ-pre closed [23]) sets of a NTS $(\mathcal{Z}, \Gamma)$ is denoted by NSPC($\mathcal{Z}$) (resp. NδPC($\mathcal{Z}$)).

Definition 1.2. *[22] Let* $(\mathcal{Z}, \Gamma)$ *be a neutrosophic topological space and B be a neutrosophic set of* $\mathcal{Z}$*. Then the neutrosophic semiδ-pre interior (denoted by sδpInt) and neutrosophic semiδ-pre closure (denoted by sδpCl) of B respectively defined as follows: .*

(1) *sδpCl(B)* $= \{G : G \supseteq B,\ G \in NS\delta PC(\mathcal{Z})\}$.
(2) *sδpInt(B)* $= \{G : G \subseteq B,\ G \in NS\delta PC(\mathcal{Z})\}$.

Remark 1.1. *[24] Every neutrosophic semi- preopen (resp. neutrosophic δ-preopen) set is neutrosophic semi-preopen. But the separate converse is not need to be true*

Definition 1.3. *[22] Let* $(\mathcal{Z}, \Gamma)$ *be a neutrosophic topological space and B be a neutrosophic set of* $\mathcal{Z}$*, let* $z_{(\eta,\alpha,\beta)}$ *be a neutrosophic point of* $\mathcal{Z}$*. Then B is said to be: .*

(1) *neutrosophic semi δ-pre neighborhood of* $z_{(\eta,\alpha,\beta)}$ *if there exists a neutrosophic set G NSδPO(*$\mathcal{Z}$*) such that* $z_{(\eta,\alpha,\beta)} \in G \subseteq B$.
(2) *neutrosophic semi δ-pre Q-neighborhood of* $z_{(\eta,\alpha,\beta)}$ *if there exists a neutrosophic set G NSδPO(*$\mathcal{Z}$*) such that* $z_{(\eta,\alpha,\beta)} q G \subseteq B$.

Definition 1.4. *[22] if* $f^{-1}(B) \in$ *NSδPO(*$\mathcal{Z}$*) for any Neutrosophic open set B of* $\mathcal{Z}_2$*, then a mapping* $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ *is considered Neutrosophic semi δ-pre continuous*

Definition 1.5. *[22] A mapping* $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ *is called:*

(1) *Neutrosophic continuous (briefly, NC) if* $g^{-1}(B)$ *is neutrosophic open set in* $\mathcal{Z}_1$ *for every neutrosophic open set B of* $\mathcal{Z}_2$
(2) *Neutrosophic semi δ-pre continuous (briefly, NSδPC) if* $g^{-1}(B)$ *NSδPO(*$\mathcal{Z}$*) for each neutrosophic open set B of* $\mathcal{Z}_2$*.*

Definition 1.6. *A mapping g from a neutrosophic topological space* $(\mathcal{Z}_1, \Gamma_1)$ *to another neutrosophic topological space* $(\mathcal{Z}_2, \Gamma_2)$ *is said to be:*

(1) *Neutrosophic α-irresolute [27] if* $g^{-1}(B)$ *is NαO(*$\mathcal{Z}_1$*) for every neutrosophic set NSαO(*$\mathcal{Z}_2$*).*
(2) *Neutrosophic semi-pre-irresolute [27] if* $g^{-1}(B)$ *is NβO(*$\mathcal{Z}_1$*) for every neutrosophic set NSPO(*$\mathcal{Z}_2$*).*
(3) *Neutrosophic pre-irresolute [28] if* $g^{-1}(B)$ *is NPO(*$\mathcal{Z}_1$*) for every neutrosophic set NPO(*$\mathcal{Z}_2$*).*

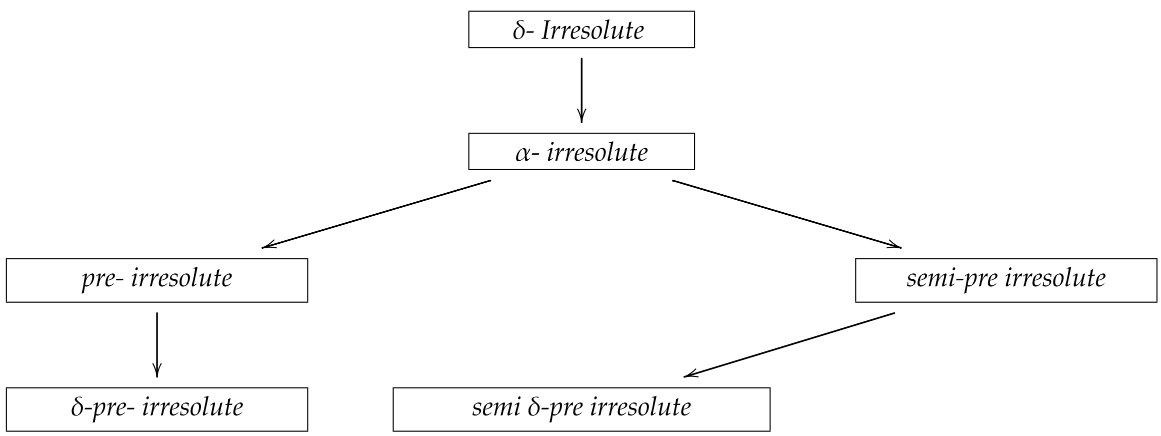
(4) *Neutrosophic δ-irresolute [29] if $g^{-1}(B)$ is NPO($\mathcal{Z}_1$) for every neutrosophic set NPO($\mathcal{Z}_2$).*
(5) *neutrosophic δ-pre irresolute [23] if if $g^{-1}(B)$ is NδPO($\mathcal{Z}_1$) for every neutrosophic set NδPO($\mathcal{Z}_2$).*

2. Neutrosophic Semi δ-Pre Irresolute Mappings

In the second section, we studied and obtained new concept in irresolute called concept of neutrosophic semi δ-pre irresolute mappings and study several of their properties in neutrosophic topological spaces

Definition 2.1. *A mapping g from a neutrosophic topological space $(\mathcal{Z}_1, \Gamma_1)$ to another neutrosophic topological space $(\mathcal{Z}_2, \Gamma_2)$ is said to be neutrosophic semi δ-pre irresolute if $g^{-1}(B)$ is NSδPO($\mathcal{Z}_1$) for every neutrosophic set NSδPO($\mathcal{Z}_2$).*

Remark 2.1. *From some types of neutrosophic irresolute mappings and the above definition, we have the following diagram:*



Remark 2.2. *Every neutrosophic semi δ-pre irresolute mappings is neutrosophic semi δ-pre continuous but via versa is not need to be true. see the following examples.*

Example 2.1. *Let $\mathcal{Z}_1 = \{x, y\}$, $\mathcal{Z}_2 = \{a, b\}$ and neutrosophic sets V defined as follows:*

$V = \{\langle x, 0.6, 0.4, 0.5\rangle, \langle y, 0.5, 0.5, 0.5\rangle\}$. *Suppose that $\Gamma_1 = \{\tilde{0}, \tilde{1}\}$ and $\Gamma_2 = \{\tilde{0}, \tilde{1}\}$ be neutrosophic topologies on $\mathcal{Z}_1$ and $\mathcal{Z}_2$ respectively. Then the mapping $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ defined by $g(x) = a$ and $g(y) = b$ is neutrosophic semi δ-pre continuous and hence neutrosophic continuous but not neutrosophic semi δ-pre irresolute.*

Example 2.2. *$\mathcal{Z}_1 = \{x, y\}$, $\mathcal{Z}_2 = \{a, b\}$ and neutrosophic sets U defined as follows:*
$U = \{\langle x, 0.5, 0.5, 0.5\rangle, \langle y, 0.6, 0.4, 0.5\rangle\}$. *Suppose that $\Gamma_1 = \{\tilde{0}, \tilde{1}\}$ and $\Gamma_2 = \{\tilde{0}, U, \tilde{1}\}$ be neutrosophic topologies on $\mathcal{Z}_1$ and $\mathcal{Z}_2$ respectively. Then the mapping $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ defined by $g(x) = a$ and $g(y) = b$ is neutrosophic semi δ-pre irresolute but not neutrosophic continuous.*

Remark 2.3. *From Example 2.1 and Example 2.2 we shows that the concepts of neutrosophic continuous and neutrosophic semi δ-pre irresolute mappings are independent.*

Theorem 2.1. *Suppose that $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ be a mapping then the following statements are equivalent:*

(1) *g is neutrosophic semi δ-pre irresolute.*
(2) *If $g^{-1}(B)$ NSδPC($\mathcal{Z}_1$) for each neutrosophic set B NSδPC($\mathcal{Z}_2$).*
(3) *For each neutrosophic point $z_{(\eta,\alpha,\beta)} \in \mathcal{Z}_1$ and each neutrosophic set B NSδPO($\mathcal{Z}_2$) such that $g(z_{(\eta,\alpha,\beta)}) \in B$ there is aneutrosophic set G NSδPC($\mathcal{Z}_1$). such that $z_{(\eta,\alpha,\beta)}$ G and $g(G) \subseteq B$.*
(4) *For each neutrosophic point $z_{(\eta,\alpha,\beta)}$ of $\mathcal{Z}_1$ and each neutrosophic semi δ-pre neighborhood B of $g(z_{(\eta,\alpha,\beta)})$, $g^{-1}(B)$ is an neutrosophic semi δ-pre neighborhood of $z_{(\eta,\alpha,\beta)}$.*
(5) *For each neutrosophic point $z_{(\eta,\alpha,\beta)}$ of $\mathcal{Z}_1$ and each neutrosophic semi δ-pre neighborhood B of $g(z_{(\eta,\alpha,\beta)})$,, there is a neutrosophic semi δ-pre neighborhood V of $z_{(\eta,\alpha,\beta)}$ such that $g(V) \subseteq B$.*
(6) *For each neutrosophic point $z_{(\eta,\alpha,\beta)}$ of $\mathcal{Z}_1$ and each neutrosophic set B NSδPO($\mathcal{Z}_2$) such that $g(z_{(\eta,\alpha,\beta)})qB$, there is anneutrosophic set G NSδPO($\mathcal{Z}_1$) such that $z_{(\eta,\alpha,\beta)}qG$ and $g(G) \subseteq B$.*
(7) *for each neutrosophic point $z_{(\eta,\alpha,\beta)}$ of $\mathcal{Z}_1$ and each neutrosophic semi δ-pre Q_n neighborhood B of $g(z_{(\eta,\alpha,\beta)})$, $g^{-1}(B)$ is a neutrosophic semi δ-pre Q_n-neighborhood of $z_{(\eta,\alpha,\beta)}$.*
(8) *for each neutrosophic point $z_{(\eta,\alpha,\beta)}$ of $\mathcal{Z}_1$ and every neutrosophic semi δ-pre Q_n neighborhood B of $g(z_{(\eta,\alpha,\beta)})$, there is a neutrosophic semi δ-pre Q_n-neighborhood V of $z_{(\eta,\alpha,\beta)}$ such that $g(V) \subseteq B$.*
(9) *$g(s\delta pCl(B)) \subseteq s\delta pCl(g(B))$, for each neutrosophic set B of $\mathcal{Z}_1$.*
(10) *$s\delta pCl(g^{-1}(G)) \subseteq g^{-1}(s\delta pCl(G))$, for each neutrosophic set G of $\mathcal{Z}_2$.*
(11) *$g^{-1}(s\delta pInt(G)) \subseteq s\delta pInt(g^{-1}(G))$, for each neutrosophic set G of $\mathcal{Z}_2$.*

Proof. $(1) \Rightarrow (2)$: Straightforward.
$(1) \Rightarrow (3)$: Suppose that $z_{(\eta,\alpha,\beta)}$ be a neutrosophic point of $\mathcal{Z}_1$ and $B \in$ NSδPO($\mathcal{Z}_2$) such that $g(z_{(\eta,\alpha,\beta)}) \in B$. Let $G = g^{-1}(B)$, then by (1), $G \in$ NSδPO($\mathcal{Z}_1$) such that $z_{(\eta,\alpha,\beta)} \in G$ and $g(G) \subseteq B$.
$(3) \Rightarrow (1)$: Let $B \in$ NSδPO($\mathcal{Z}_2$) and $z_{(\eta,\alpha,\beta)} \in g^{-1}(B)$. Therefore $g(z_{(\eta,\alpha,\beta)}) \in B$. By using (3) there is a neutrosophic set $G \in$ NSδPO($\mathcal{Z}_1$) such that $z_{(\eta,\alpha,\beta)} \in G$ and $g(G) \subseteq B$. Hence $z_{(\eta,\alpha,\beta)} \in G \subseteq g^{-1}(B)$. This implies $g^{-1}(B) \in$ NSδPO($\mathcal{Z}_1$).

$(1) \Rightarrow (4)$: Suppose that $z_{(\eta,\alpha,\beta)}$ is a neutrosophic point of $\mathcal{Z}_1$ and B be a semi δ-pre neighborhood of $g(z_{(\eta,\alpha,\beta)})$. Hence there is a neutrosophic set V NSδPO($\mathcal{Z}_1$ such that $g(z_{(\eta,\alpha,\beta)}) \in V \subseteq B$. Let that $F^{-1}(V) \in$ NSδPO($\mathcal{Z}_1$) and $g^{-1}(V)$, $g^{-1}(B)$. Therefore $g^{-1}(B)$ is a neutrosophic semi δ-pre neighborhood of $z_{(\eta,\alpha,\beta)}$ in $\mathcal{Z}_1$.

$(4) \Rightarrow (5)$: Suppose that $z_{(\eta,\alpha,\beta)}$ is a neutrosophic point of $\mathcal{Z}_1$ and B be a semi δ-pre neighborhood of $g(z_{(\eta,\alpha,\beta)})$. Hence $V = g^{-1}(B)$ is a neutrosophic semi δ-pre neighborhood of $z_{(\eta,\alpha,\beta)}$ and $g(V) = g(g^{-1}(B)) \subseteq B$.

$(5) \Rightarrow (3)$: Let $z_{(\eta,\alpha,\beta)}$ be a neutrosophic point of $\mathcal{Z}_1$ and $B \in$ NSδPO($\mathcal{Z}_2$) such that $g(z_{(\eta,\alpha,\beta)}) \in B$. Hence there exist neutrosophic semi δ-pre neighborhoodU of $z_{(\eta,\alpha,\beta)}$ in $\mathcal{Z}_1$ such that $z_{(\eta,\alpha,\beta)} \in V$ and $g(V) \subseteq B$. This implies there is a neutrosophic set $G \in$ NSδPO($\mathcal{Z}_1$) such that $z_{(\eta,\alpha,\beta)} \in G \subseteq V$ and so $g(G) \subseteq g(V) \subseteq B$.

(1) $\Rightarrow$ (6) Suppose that $z_{(\eta,\alpha,\beta)}$ be a neutrosophic point of $\mathcal{Z}_1$ and $B \in$ NSδPO($\mathcal{Z}_2$) such that $g(z_{(\eta,\alpha,\beta)})q \in B$. Now put $G = g^{-1}(B)$. Hence $G \in$ NSδPO($\mathcal{Z}_1$), $z_{(\eta,\alpha,\beta)}qG$ and $f(G) = g(g^{-1}(B)) \subseteq B$.

(6) $\Rightarrow$ (1) Suppose that $B \in$ NSδPO($\mathcal{Z}_2$) and $z_{(\eta,\alpha,\beta)}$ in $g^{-1}(B)$ clearly $g(z_{(\eta,\alpha,\beta)}) \in B$, now let the neutrosophic point $z^c_{(\eta,\alpha,\beta)}$ denoted by

$$z^c_{(\eta,\alpha,\beta)}(x) = \begin{cases} (\eta,\alpha,\beta), & \text{if } x = z, \\ (1,0,0), & \text{if } x \neq z, \end{cases}$$

This implies $g(z^c_{(\eta,\alpha,\beta)})qB$ and then by (6), there exists a neutrosophic set $G \in$ NSδPO($\mathcal{Z}_1$), such that $z^c_{(\eta,\alpha,\beta)q}G$ and $g(G) \subseteq B$. Now $z^c_{(\eta,\alpha,\beta)q}G$ then $z^c_{(\eta,\alpha,\beta)} \in G$. Therefore $z^c_{(\eta,\alpha,\beta)} \subseteq g^{-1}(B)$. Then $g^{-1}(B) \in$ NSδPO($\mathcal{Z}_1$).

(6) $\Rightarrow$ (7) : Suppose that $z_{(\eta,\alpha,\beta)}$ is a neutrosophic point of $\mathcal{Z}_1$ and B be a semi δ-pre Q_n-neighborhood of $g(z_{(\eta,\alpha,\beta)})$. Then there is a neutrosophic open set $B_1 \in$ NSδPO($\mathcal{Z}_2$) such that $g(z_{(\eta,\alpha,\beta)})_q \subseteq B_1 \subseteq B$. By hypothesis there is a neutrosophic set $G \in$ NSδPO($\mathcal{Z}_1$) such that $z_{(\eta,\alpha,\beta)q}G$ and $g(G) \subseteq B_1$. So that $z_{(\eta,\alpha,\beta)q}G \subseteq g^{-1}(B_1) \subseteq g^{-1}(B)$. Then $g^{-1}(B)$ is a neutrosophic semi δ-pre Q_n-neighborhood of $z_{(\eta,\alpha,\beta)}$.

(7) $\Rightarrow$ (8) : Suppose that $z_{(\eta,\alpha,\beta)}$ is a neutrosophic point of $\mathcal{Z}_1$ and B be a semi δ-pre Q_n-neighborhood of $g(z_{(\eta,\alpha,\beta)})$. Therefore $V = g^{-1}(B)$ is a neutrosophic semi δ-pre Q_n-neighborhood of $z_{(\eta,\alpha,\beta)}$ and $g(V) = g(g^{-1}(B)) \subseteq B$.

(7) $\Rightarrow$ (8) : Suppose that $z_{(\eta,\alpha,\beta)}$ is a neutrosophic point of $\mathcal{Z}_1$ and $B \in$ NSδPO($\mathcal{Z}_1$) such that $g(z_{(\eta,\alpha,\beta)})_qB$. Hence B is neutrosophic semi δ-pre Q_n-neighborhood of $g(z_{(\eta,\alpha,\beta)})$. So there is a neutrosophic semi δ-pre Q_n-neighborhood V of $z_{(\eta,\alpha,\beta)}$ such that $g(V) \subseteq B$. Let V being a neutrosophic semi δ-pre Q_n-neighborhood of $z_{(\eta,\alpha,\beta)}$. This implies there exists a neutrosophic set $G \in$ NSδPO($\mathcal{Z}_1$) such that $z_{(\eta,\alpha,\beta)q}G \subseteq V$. Then$z_{(\eta,\alpha,\beta)q}$Gand $g(G) \subseteq g(V) \subseteq B$.

(2) $\Rightarrow$ (9) : Suppose that B be a neutrosophic set of $\mathcal{Z}_1$. Put $B = g^{-1}(g(B))$, this show $B \subseteq g^{-1}(s\delta pCl(g^{-1}(B)))$. Let $s\delta pCl(g(B)) \in$ NSδPC($\mathcal{Z}_2$) and this implies $g^{-1}(s\delta pCl(g(B))) \in$ NSδPC($\mathcal{Z}_1$). Then $s\delta pCl(B) \subseteq g^{-1}(s\delta pCl(g(B)))$ and $g(s\delta pCl(B)) \subseteq g(g^{-1}(s\delta pCl(B))) \subseteq s\delta pCl(g(B))$.

(9) $\Rightarrow$ (2) : Suppose that $B \in$ NSδPC($\mathcal{Z}_2$) hence $g(s\delta pCl(g^{-1}(B))) \subseteq s\delta pCl(g(^{-1}(B))) \subseteq s\delta pCl(B) = B$. Therefore $s\delta pCl(g^{-1}(B)) \subseteq g^{-1}(B)$ and so $g^{-1}(B) \in$ NSδPC($\mathcal{Z}_1$).

(9) $\Rightarrow$ (10) : Let G be any neutrosophic set of $\mathcal{Z}_2$, then $g^{-1}(G)$ is a neutrosophic set of $\mathcal{Z}_1$. Therefore by hypothesis (1), $g(s\delta pCl(g^{-1}(G))) \subseteq s\delta pCl(g(g^{-1}(G))) \subseteq s\delta pCl(G)$. Then $s\delta pCl(g^{-1}(G)) \subseteq g^{-1}(s\delta pCl(G))$.

(10) $\Rightarrow$ (9) : Suppose that G be any neutrosophic set of $\mathcal{Z}_2$, then $g^{-1}(G)$ is a neutrosophic set of $\mathcal{Z}_2$. Now by (10), $s\delta pCl(g^{-1}(g(B))) \subseteq g^{-1}(s\delta pCl(g(B)))$. Therefore $g(s\delta pCl(B)) \subseteq s\delta pCl(g(B))$.

(1) $\Rightarrow$ (11) : Suppose that G be any neutrosophic set of $\mathcal{Z}_2$, this implies $s\delta pInt(G) \in$ NSδPO($\mathcal{Z}_2$) and $g^{-1}(s\delta pInt(G)) \in$ NSδPC($\mathcal{Z}_1$). Since $g^{-1}(s\delta pInt(G)) \subseteq g^{-1}(G)$, hence $g^{-1}(s\delta pInt(G)) \subseteq s\delta pInt(g^{-1}(G))$.

(11) $\Rightarrow$ (1) : Suppose that $G \in$ NSδPO($\mathcal{Z}_2$), this implies $s\delta pInt(G) = G$ and $g^{-1}(G) \subseteq$

$s\delta pInt(g^{-1}(G))$. Hence $g^{-1}(G) = s\delta pInt(g^{-1}(G))$ and $g^{-1}(G) \in$ NSδPO($\mathcal{Z}_1$). Then g is neutrosophic semi δ-pre irresolute. □

Definition 2.2. *Suppose that $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ be a mapping then g is called neutrosophic R-open if the image of each neutrosophic open set of $\mathcal{Z}_1$ is neutrosophic δ-open in $\mathcal{Z}_2$.*

Theorem 2.2. *If $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ is neutrosophic-almost continuous and neutrosophic R-open mapping, therefore g is neutrosophic semi δ-pre irresolute.*

Proof. Let $B \in$ NSδPO($\mathcal{Z}_2$). Hence there exist a neutrosophic set $G \in$ NSδPO($\mathcal{Z}_1$) such that $G \subseteq B \subseteq \delta Cl(G)$, this implies $g^{-1}(G) \subseteq g^{-1}(B) \subseteq g^{-1}(\delta ClG)) \subseteq \delta Cl(g^{-1}(G))$ because g is neutrosophic R-open. Hence g is neutrosophic δ-almost continuous and neutrosophic R-open, $g^{-1}(G) \in$ NSδPO($\mathcal{Z}_1$). Then $g^{-1}(B) \in$ NSδPO($\mathcal{Z}_1$). □

Theorem 2.3. *Suppose $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ and $f : (\mathcal{Z}_2, \Gamma_2) \longrightarrow (\mathcal{Z}_3, \Gamma_3)$ be neutrosophic semi δ-pre irresolute mappings then $f \circ g$ is neutrosophic semi δ-pre irresolute.*

Proof. Let $B \in$ NSδPO($\mathcal{Z}_3$). We have f is neutrosophic semi δ-pre irresolute, $f^{-1}(B) \in$ NSδPO($\mathcal{Z}_2$). This implies $(f \circ g)^{-1}(B) = g^{-1}(f^{-1}(B)) \in$ NSδPO($\mathcal{Z}_1$), because g isneutrosophic semi δ-pre irresolute. Then $f \circ g$ is neutrosophic semi δ-pre irresolute. □

Theorem 2.4. *Suppose $g : (\mathcal{Z}_1, \Gamma_1) \longrightarrow (\mathcal{Z}_2, \Gamma_2)$ is neutrosophic semi δ-pre irresolute and $f : (\mathcal{Z}_2, \Gamma_2) \longrightarrow (\mathcal{Z}_3, \Gamma_3)$ is neutrosophic semi δ-pre continuous mapping, then $f \circ g$ is neutrosophic semi δ-pre continuous.*

Proof. Let G be any neutrosophic fuzzy open set of $\mathcal{Z}_3$. We have f is neutrosophic semi δ-pre continuous $f^{-1}(G) \in$ NSδPO($\mathcal{Z}_2$). Hence '$(f \circ g)^{-1}(G) = g^{-1}(f^{-1}(G)) \in$ NSδPO($\mathcal{Z}_1$) since g is neutrosophic semi δ-pre irresolute. Then $f \circ g$ is neutrosophic semi δ-pre continuous. □

3. Conclusion

After giving the fundamental concepts of neutrosophic sets and neutrosophic topological spaces, we present a new concept of mappings called Neutrosophic semi δ-pre irresolute mappings is introduced in this paper, it is shown by some examples that the concepts of Neutrosophic semi δ-pre irresolute mappings is more stronger than the Neutrosophic semi δ-pre continuous mappings moreover it is independent to the Neutrosophic continuous mappings. We present many properties and characteri zations of these class of Neutrosophic mappings is studied and obtained in this article. This present paper contains the next steps of Neutrosophic Seperation Axioms

4. Further Work

An important outcome of this study is the emergence of new questions that may stimulate further research, as the development of generalized mathematical frameworks often leads to deeper and broader investigations. As future work, we aim to integrate neutrosophic topological concepts with operator theory. Recent advances in operator theory and mathematical modeling have provided powerful analytical tools for the study of generalized mappings and structural properties [46,48,49]. In addition, results concerning nonstationary fields and correlation structures have enriched the theoretical foundation of such analyses [47]. These developments strongly motivate further exploration of neutrosophic semi δ-pre irresolute mappings within a unified and generalized mathematical framework.

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