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Mathematical Modelling and Nonlinear Analysis of Reaction Diffusion Mechanism of the Volatile Compounds Alongside Spherical Electrodes Embedded within the Chemically Modified Electrodes

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ABSTRACT: The main objective of the present research is to propose a new mathematical formulation for the concentration of volatile substances corresponding to spherical electrodes through applying steady state reaction diffusion equations within the electrode surface in the presence of chemically modified electrodes. This model requires into consideration the diffusion of reactants and charge carriers that occur within the chemically modified layer that is positioned at the electrode surface. All probable experimental responses of the parameter may utilise appropriate mediator, substrate, and current concentrations evaluated analytically through the implementation of the Akbari Ganji Method. A numerical representation of the issue being studied can also be obtained implementing MATLAB software aimed at assisting comprehend the dynamics of the system. An appropriate degree of concurrence is subsequently provided once the ensuring results have been examined using currently accessible numerical data with previously discovered information.

1.Introduction

Over the past two decades, Chemists have made attempts to effectively manipulate an electrode's chemical constituents which has culminated to an enormous surge in pursuit of Chemically modified electrodes (CME). Numerous significant uses for chemically modified electrodes have been discovered in the field of electrocatalysis, corrode detection, molecular electronics, selective electro organic synthesis, solar power generation and electrochromic

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monitor devices ([1], [2], [3], [4], [5]). A relatively modern approach to electrode systems, CME are useful in a variety of fundamental electrochemical investigations for instance the interactions between chemical reactivity and heterogeneous electron transfer to electrode surface chemistry, electrostatic occurrences at electrode surfaces and electron mobility through polymers. They are further valuable in the construction of electrochemical devices and components for utilisation in the molecular electronics, electrochromic displays chemical sensing [6]. The distinctive feature that initiatives a CME detached from other electrode concepts in electrochemistry is that in a reasonable, chemically designed manner a typically exceptionally thin layer of an assigned chemical is stuck to or coated on the electrode surface in order to provide the electrode, the chemical electrochemical, ocular, electrical, transport and other envisioned properties associated with the film. ([7], [8])

Additionally, chemically modified electrodes may contain an assortment of chemical modifiers sometimes embedded in the electrode substrate and possibly these modifiers may have a precisely structured spatial configuration. That is, a CME could contain two electrocatalysis: One that transport energy between the electrode and the first catalyst and the second catalyst performs through accepter or comprising multiple different chemical polymers with the second polymer deposited on the surface of the first to create a bilayer of polymer films might also makeup the chemically modified electrode. A mathematical model explaining transport and electrocatalytic kinetics in surface immobilised modified electrode that function via two different forms of redox interactions – a Michale type addict formation reaction and a simple chemical second order was modelled by Puida et al. [9]. Albery et al. [10] suggested an extensive model for polymer modified electrodes which aims to estimate the continuous momentary reaction of a redox polymer film employing mediated electroanalysis. Vinolyn Syluia et al. [11] presented the analytic result for the transient current of catalytic process at chemically modified electrode using Homotopy Perturbation Method. Romas Baronas Romas et al. established the amperometric biosensors based on chemically modified electrodes [12].

The present research investigation proposes to establish a mathematical formulation concerning the concentration of volatile compounds associated to a spherical electrode exploiting steady state reaction diffusion equations over the surface of electrode amid chemically modified electrode.

2. Mathematical Modelling

We will only provide a brief description, as an exhaustive investigation of the basic assumptions and tangible manifestation of this issue has been done previously by Romas Barnous Romas et al. [12]. This model considers an ordered ping-pong approach for enzyme catalysed substrate conversion with a mediator. The chemically modified electrode consists of a thin layer of low soluble mediator encased over an enzyme membrane. The system comprised as three regions, an enzyme layer for enzymatic reactions and mass transport by diffusion, a diffusion limiting, region for mass transport exclusively and a convective region to maintain analytic concentration constant. An electrode that fact may have been chemically modified acts as where the reaction originates. A comparatively thin layer membrane of the inadequate accessible mediator and an enzyme membrane encapsulate the CME and this is regarded as an electrode.

We anticipate a sequential Ping-Pong approach of stimulated Enzyme(A), substrate(B), transformation within the midst of mediator(C).

$$A_1 + B \leftrightharpoons AB \rightarrow A_2 + X$$

$$A_2 + C \rightarrow A_1 + Y$$

The intermediate complex concentration is predicted that it stays consistent in the steady state estimation. A series of reaction diffusion equations may be utilised as well to provide insight in the distribution of mass and reactions in the enzyme layer, provided that the electrode has spherical geometry and confined enzyme is distributed correspondingly throughout the uniformly thick film. A system of reaction diffusion equations could possibly be used to represent it as

$$\frac{2\mathfrak{D}_{\mathcal{S}}}{u}\frac{dB}{du} = \mathfrak{D}_B\frac{d^2B}{du^2} - \mu_B\,(B(u)C(u)) \tag{2.1}$$

$$\frac{2\mathfrak{D}_{\mathcal{M}}}{u}\frac{dC}{du} = \mathfrak{D}_C\frac{d^2C}{du^2} - \mu_C(\,B(u)C(u)) \tag{2.2}$$

Following are the boundary conditions that describe the problem

$$\text{At } u = 0, = \mathcal{C}_T\,;\frac{dB}{du} = 0. \tag{2.3}$$

$$\text{At } u = 0,\, s = \mathbb{K}s^{\infty}\,;\frac{dC}{du} = 0. \tag{2.4}$$

The Net flux is represented as

$$\mathcal{C}_{\mathfrak{J}} = \mathfrak{D}_B(\frac{dB}{du})_{u=r} = -\mathfrak{D}_C(\frac{dC}{du})_{u=0} \tag{2.5}$$

The results of the final evaluation should be presented in dimensionless parameters before we progress on to a complete mathematical review of the boundary value problem described in the equations (2.1) -(2.5)

We introduce the Non-Dimensional Parameters as,

$$x = \frac{B}{\Delta_2 s^{\infty}}\,;y = \frac{C}{\mathcal{C}_T}\,;\vartheta = \frac{u}{r}\,;\xi_B = \frac{\Delta_2\mathcal{C}_T r^2}{\mathfrak{D}_B} = \frac{\phi_R}{\phi_{\mathcal{S}}} = \frac{\Delta_1}{{\Delta_3}^2}\,;\xi_C = \frac{\mu\mathbb{K}s^{\infty}r^2}{\mathfrak{D}_C} = \frac{\phi_R}{\phi_C} = \frac{\Delta_1}{\Delta_2{\Delta_3}^2}\,; \tag{2.6}$$

The reaction diffusion parameters ξ_B and ξ_C are used to measure the correlation between the chemical reaction rate and the amount of charge percolation or substrate diffusion. Three independent dimensionless parameters $\Delta_1, \Delta_2, \Delta_3$ have been utilized to simulate the current.

$$\Delta_1 = \frac{\phi_B}{\phi_D}\,;\quad \Delta_2 = \frac{\phi_C}{\phi_B}\,;\quad \Delta_3 = \frac{\phi_R\phi_B}{{\phi_D}^2}\,; \tag{2.7}$$

The system nonlinear reaction-diffusion equation can be written as

$$\frac{d^2x(\vartheta)}{d\vartheta^2} + \frac{2}{t}\frac{dx(\vartheta)}{d\vartheta} - \xi_B x(\vartheta)y(\vartheta) = 0 \tag{2.8}$$

$$\frac{d^2y(\vartheta)}{d\vartheta^2} + \frac{2}{t}\frac{dy(\vartheta)}{d\vartheta} - \xi_C x(\vartheta)y(\vartheta) = 0 \tag{2.9}$$

The appropriate boundaries were expressed as

$$\frac{dx(0)}{d\vartheta} = 0; x(0) = 1. \tag{2.10}$$

$$\frac{dy(1)}{d\vartheta} = 0\ ;\ y(1) = 1. \tag{2.11}$$

The following equation represents the normalized steady-state current response

$$\Omega = (\frac{dx}{dt})_{t=1}\ or\ \Omega = -(\frac{dy}{dt})_{t=0} \tag{2.12}$$

3. Analytical Solution of The Concentrations Using Akbari Ganji Mathod

Over the last few decades, a number of authors have concentrated on studying the solution of nonlinear equations using a range of methods, such as the Homotopy perturbation Method [13-16], Taylor Series Method [17-19], Akbari Ganji Method [20-23] and Variational Iteration Method [24-25]. Akbari Ganji's Method is highly precise, effective, and effective in a conservative approach. An AGM solution function exhibiting indeterminate constant coefficients solves its inception and boundary requirements. The above procedure can be implemented for resolving nonlinear equations (2.8) through (2.11).

Consider the trial solution for Equations (2.8) and (2.9) as

$$x(\vartheta) = \mathrm{A}\cosh(\alpha\vartheta) + \mathrm{B}\sinh(\alpha\vartheta) \tag{3.1}$$

$$y(\vartheta) = \mathrm{C}\cosh(\beta\vartheta) + \mathrm{D}\cosh(\beta\vartheta) \tag{3.2}$$

Using the boundary conditions (2.10) and (2.11), we get

$$\mathrm{A} = \frac{1}{\cosh\alpha},\ \mathrm{B} = 0,\ \mathrm{C} = 1,\ \mathrm{D} = -\tanh\beta \tag{3.3}$$

Substituting (3.3) in (3.1) and (3.2) we have

$$x(\vartheta) = \frac{cosh(\alpha\vartheta)}{\cosh(\alpha)} \tag{3.4}$$

$$y(\vartheta) = \frac{cosh((1-\vartheta)\beta)}{cosh(\beta)} \tag{3.5}$$

Eqns. (2.8) and (2.9) becomes,

$$\frac{d^2}{d\vartheta^2}[\frac{cosh(\alpha\vartheta)}{\cosh(\alpha)}] + \frac{2}{\vartheta}\frac{d}{d\vartheta}[\frac{cosh(\alpha\vartheta)}{\cosh(\alpha)}] - \xi_B\left[\frac{cosh(\alpha\vartheta)}{\cosh(\alpha)}\right][\frac{cosh((\vartheta+1)\beta)}{cosh(\beta)}] = 0 \tag{3.6}$$

$$\frac{d^2}{d\vartheta^2}[\frac{cosh((\vartheta+1)\beta)}{cosh(\beta)}] + \frac{2}{\vartheta}\frac{d}{dt}[\frac{cosh((\vartheta+1)\beta)}{cosh(\beta)}] - \xi_C\left[\frac{cosh(\alpha\vartheta)}{\cosh(\alpha)}\right][\frac{cosh((\vartheta+1)\beta)}{cosh(\beta)}] = 0 \tag{3.7}$$

By the application of L-Hospitals' rule in (18) and (19) and $\vartheta \to 0$, we get

$$\alpha = \sqrt{\frac{\xi_B}{3}}\ \text{and}\ \beta = \sqrt{\frac{\xi_C}{3\cosh\alpha}} \tag{3.8}$$

Substituting the values of α and β in equations (3.4) and (3.5), we obtain the analytical formulation for the substrate and mediator concentrations, which is stated as,

$$x(\vartheta) = \frac{cosh\left(\vartheta(\frac{\xi_B}{3})^{\frac{1}{2}}\right)}{cosh\left((\frac{\xi_B}{3})^{\frac{1}{2}}\right)} \tag{3.9}$$

$$y(\vartheta)= \frac{cosh\left((1-\vartheta)\left(\frac{\xi_C}{3\cosh\left(\frac{\xi_B}{3}\right)^{\frac{1}{2}}}\right)^{\frac{1}{2}}\right)}{cosh\left(\left(\frac{\xi_C}{3\cosh\left(\frac{\xi_B}{3}\right)^{\frac{1}{2}}}\right)^{\frac{1}{2}}\right)} \tag{3.10}$$

When the current is normalized, it becomes,

$$\Omega = \left(\frac{\xi_B}{3}\right)^{\frac{1}{2}} \tanh\left(\left(\frac{\xi_B}{3}\right)^{\frac{1}{2}}\right) \qquad \text{(or)} \tag{3.11}$$

$$\Omega = \left(\frac{\xi_C}{3\cosh\left(\frac{\xi_B}{3}\right)^{\frac{1}{2}}}\right)^{\frac{1}{2}} tanh\left(\left(\frac{\xi_C}{3\cosh\left(\frac{\xi_B}{3}\right)^{\frac{1}{2}}}\right)^{\frac{1}{2}}\right) \tag{3.12}$$

4. Specifying Cases

By contrasting the diffusion coefficients and kinetic properties, we have deduced the substrate and mediator's concentration equations here.

4.1 Electron absorption over the chemical layer occurs at a faster pace than substrate transport

In consideration of the circumstances $\frac{\xi_C}{\xi_B} \ll 1$ or $\Delta_2 \rightarrow \infty$. Although it is expected that electron charge penetration around the chemical chain will occur quickly and that the chemical interaction between the mediator and substrate will occur too quickly, relatively many mediator species will be ingested throughout the chemical layer, and it is assumed that $\xi_C = 1$.

Equations (2.8) and (2.9) were reduced to

$$\frac{d^2x(\vartheta)}{d\vartheta^2} + \frac{2}{t}\frac{dx(\vartheta)}{d\vartheta} - \xi_B x(\vartheta) = 0 \tag{4.1}$$

$$\frac{d^2y(\vartheta)}{d\vartheta^2} + \frac{2}{t}\frac{dy(\vartheta)}{d\vartheta} = 0 \tag{4.2}$$

We found the analytical statement for the mediator and substrate as, utilizing boundary conditions (2.10) – (2.11),

$$x(\vartheta) = \frac{cosh\left(v(\frac{\xi_B}{3})^{\frac{1}{2}}\right)}{cosh\left((\frac{\xi_B}{3})^{\frac{1}{2}}\right)} \tag{4.3}$$

$$y(\vartheta) = 1 \tag{4.4}$$

The current transforms $\Omega = \left(\frac{\xi_B}{3}\right)^{\frac{1}{2}} \tanh\left(\left(\frac{\xi_B}{3}\right)^{\frac{1}{2}}\right)$ (4.5) *(ii)Substrate diffusion flows across the chemical layer greater than electron diffusion*

Considering the circumstances into account $\frac{\xi_C}{\xi_B} \gg 1$ or $\Delta_2 \rightarrow 0$. Due to the quick substrate diffusion within the layer and the assumption that minimal substrate is wasted during transit through the layer, take the following into consideration ξ_B =1.

Equations (2.8) and (2.9) simplified to

$$\frac{d^2x(\vartheta)}{d\vartheta^2} + \frac{2}{\vartheta}\frac{dx(\vartheta)}{d\vartheta} = 0 \tag{4.6}$$

$$\frac{d^2y(\vartheta)}{d\vartheta^2} + \frac{2}{\vartheta}\frac{dy(\vartheta)}{d\vartheta} - \xi_c y(\vartheta) = 0 \tag{4.7}$$

With boundary conditions (2.10) – (2.11) applied, we were able to get the analytical equation for the substrate and mediator as,

$$x(\vartheta) = 1 \tag{4.8}$$

$$y(\vartheta) = \frac{cosh\left((1-\vartheta)(\frac{\xi_C}{3})^{\frac{1}{2}}\right)}{cosh\left((\frac{\xi_C}{3})^{\frac{1}{2}}\right)} \tag{4.9}$$

The current changes to $\Omega = -\left[\left(\frac{\xi_C}{3}\right)^{\frac{1}{2}} \tanh\left(\left(\frac{\xi_C}{3}\right)^{\frac{1}{2}}\right)\right]$ (4.10)

5. Previous Analytical Result

The variation iteration method found analytical solutions to the nonlinear equations involving polymer films. Equations (2.8) and (2.9) with associated boundary conditions were resolved by Rebouillat et al. [26] employing the variation iteration approach. Following is how they arrived at the analytical formulation for the concentration of mediator and substrate in polymer modified ultramicroelectrodes:

$$x(\vartheta) = \frac{sinh\left(\vartheta(\xi_B)^{\frac{1}{2}}\right)}{\vartheta sinh\left((\xi_B)^{\frac{1}{2}}\right)} \tag{5.1}$$

$$y(\vartheta) = \frac{exp\left(-(\xi_C)^{\frac{1}{2}}\right)}{(\xi_C)^{\frac{1}{2}} f\left((\xi_B)^{\frac{1}{2}}\right) t}\{f(\xi_C)\ \exp\left[-(\xi_C)^{\frac{1}{2}}(1-\vartheta)\right] - \exp\left[(\xi_C)^{\frac{1}{2}}(1-\vartheta)\right]\} \tag{5.2}$$

Where $f(\xi_C) = \frac{1+(\xi_C)^{\frac{1}{2}}}{1-(\xi_C)^{\frac{1}{2}}}$ (5.3)

6. Numerical Simulation

The accuracy of the AGM solution was tested by numerically solving the nondimension al form of equations (2.8)-(2.9) related to boundary conditions (2.10)-(2.11). The efficiency of the current methodology is illustrated by graphical correlations between our analytical results and numerical outcomes. Tables 1 and 2 compare preceding analytical results with dimensionless substrate and mediator concentrations based on the analytical representation. For every parameter setting under comparison, it provides a satisfactory level of agreement. The most recent analytical result (AGM Method) is separated from the previously analysed result by the highest typical error of 0.02% in the mediator and 0.05% in the substrate.

Table 1. Comparison among the new (3.9) and previous analytical results (5.1) for the substrate concentration for different reaction diffusion parameter values.

Substrate Concentration									
ξ_C= 0.1 and ξ_B =0.1				ξ_C= 1 and ξ_B = 1			ξ_C= 10 and ξ_B = 10		
ϑ	**Previous work Eqn. (5.1)**	**This work Eqn. (3.9)**	**% of variation between (5.1) and (3.9)**	**Previous work Eqn. (5.1)**	**This work Eqn. (3.9)**	**% of variation between (5.1) and (3.9)**	**Previous work Eqn. (5.1)**	**This work Eqn. (3.9)**	**% of variation between (5.1) and (3.9)**
0.1	0.9837	0.9837	0.0000	0.8523	0.8551	0.0028	0.2727	0.3192	0.0465
0.3	0.9850	0.9850	0.0000	0.8637	0.8665	0.0028	0.3103	0.3623	0.0520
0.5	0.9876	0.9876	0.0000	0.8868	0.8895	0.0027	0.3948	0.4542	0.0594
0.7	0.9916	0.9916	0.0000	0.9221	0.9243	0.0022	0.5476	0.6073	0.0597
0.9	0.9969	0.9969	0.0000	0.9705	0.9715	0.0010	0.8086	0.8424	0.0338
1.0	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
Average Error %			**0.0000**			**0.0023**			**0.0502**

Table 2. Deviation of New analytical solution (3.10) with Previous Analytical findings (5.2) of concentration of mediator for various values of reaction diffusion parameters

Mediator Concentration									
ξ_C= 0.1 and ξ_B =0.1				ξ_C= 1 and ξ_B = 1			ξ_C= 10 and ξ_B = 10		
ϑ	**Previous work Eqn. (5.2)**	**This work Eqn. (3.10)**	**% of the variation between (5.2) and (3.10)**	**Previous work Eqn. (5.2)**	**This work Eqn. (3.10)**	**% of the variation between (5.2) and (3.10)**	**Previous work Eqn. (5.2)**	**This work Eqn. (3.10)**	**% of the variation between (5.2) and (3.10)**
0.1	0.9837	0.9837	0.0000	0.8523	0.8551	0.0028	0.2727	0.3192	0.0165
0.3	0.9850	0.9850	0.0000	0.8637	0.8665	0.0028	0.3103	0.3623	0.0020
0.5	0.9876	0.9876	0.0000	0.8868	0.8895	0.0024	0.3948	0.4542	0.0194
0.7	0.9916	0.9916	0.0000	0.9221	0.9243	0.0010	0.5476	0.6073	0.0197
0.9	0.9969	0.9969	0.0000	0.9705	0.9715	0.0010	0.8086	0.8424	0.0238
1.0	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
Average Error %			0.0000			0.0020			0.0220

7. Results and Discussion

The equations (3.9) and (3.10) provides a newly developed analytical formulation of the concentration of substrate and mediator in simple closed form. Simple new analytical formulae (3.11) and (3.12) describe how much normalized current is present.

Fig. 1

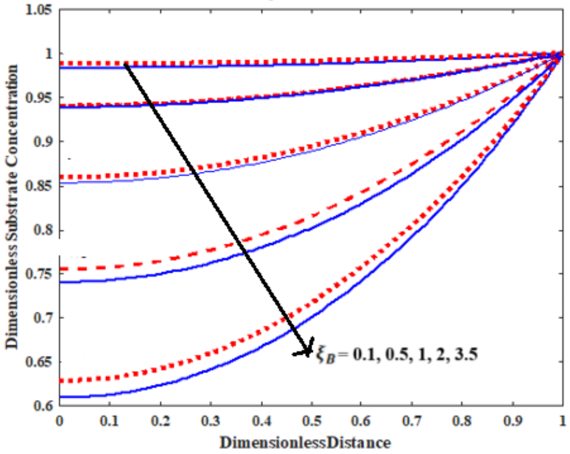

Figure 1. Analytical and numerical evaluations of the solutions for various values of reaction diffusion parameter ξ_B for the substrate concentration.

Figure (1) depicts the conventional substrate concentration for various amounts of the diffusion parameter ξ_B. The data presented here reveals that x is substantially comparable to 1 for every value of ξ_C and ξ_B that is less than or equal to 1. As the concentration of substrate goes down, ξ_B increases. The value of $\xi_B < 0.1$, the concentration reaches the constant value and the curve turns into straight line. As the concentration rate is high for the lowest value of ξ_B, the Concentration of substrate is inversely proportional to the reaction rate constant ξ_B.

Fig. 2

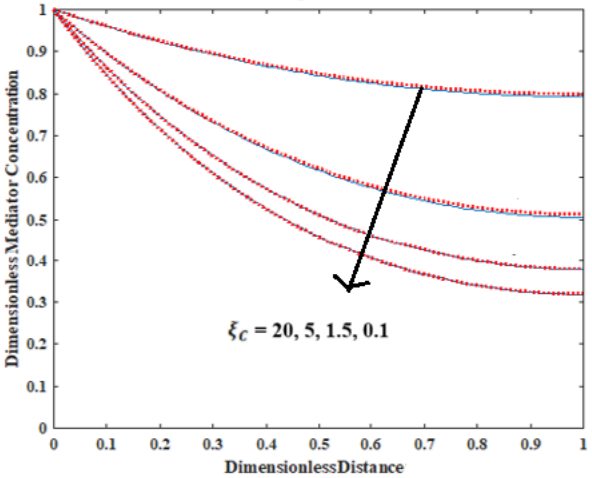

Figure 2. Comparison of the solutions both analytically and numerically for different values of ξ_C for fixed ξ_B = 0.1 for the concentration of mediator.

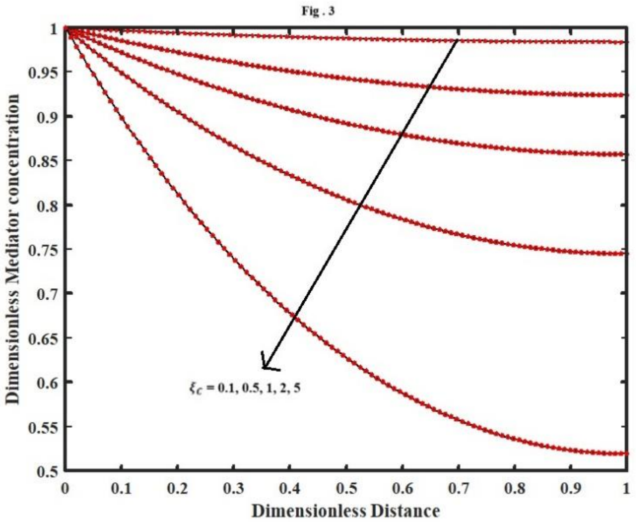

Figure 3. Analytical and numerical comparison of the solutions for varying values of ξ_C for fixed ξ_B = 1 for mediator concentration.

The series of normalized concentration profiles for a mediator is shown in Figure 2 and Figure 3 for various values of the ξ_C and for fixed value of ξ_B is 0.1 and 1 respectively. All values of ξ_C and ξ_B that are both less than or equal to 1 can be deduced that it is substantially equal to 1. As y increases either ξ_C increases or ξ_B decreases. Since the mediator concentration slowly increases for all the highest value of ξ_C, the concentration of mediator is directly correlated to the reaction diffusion parameter ξ_C. The solid line represents the numerical solution, and the dotted line represents the analytical result.

By the comparison of Figure 2 and Figure 3 depicts that the increasing value of the reaction parameter ξ_B rises the concentration of mediator corresponding the lowest value of ξ_C. In Figure 2, the reaction rate constant $\xi_C \geq 0.1$ yields the concentration of mediator reaches the highest stable value as well as the curve turned into straight line. But in Figure 3 $\xi_C \geq 0.1$, the Mediator concentration falls down and slowly reaches the steady state. That is, the mediator concentration directly correlated to the reaction rate constant ξ_B but inversely proportional to ξ_C.

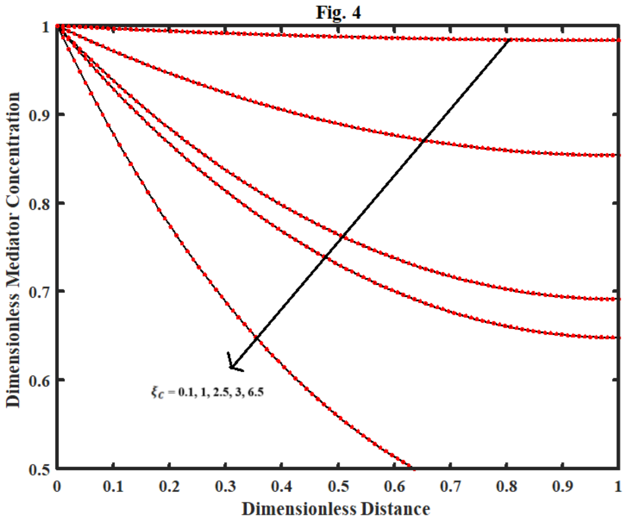

Figure 4. The concentration of Mediator depends on the reaction rate constant ξ_C as the flow of substrate diffusion across the chemical layer greater than electron diffusion.

Figure 4 demonstrates that the concentration of mediator for the various values of reaction rate constant ξ_C whenever the electron diffusion on the chemical layer smaller than the substrate diffusion as $\Delta_2 \rightarrow 0$. Considering the aforementioned along with $\xi_B = 1$ to account for the rapid diffusion of substrate within the layer and the expectation that a fraction of substrate gets eliminated during transit through the layer. Whenever the reaction rate constant ξ_C escalates, the mediator concentration reduces. The mediator concentration drops towards the steady state at the absolute highest significance of the reaction rate constant.

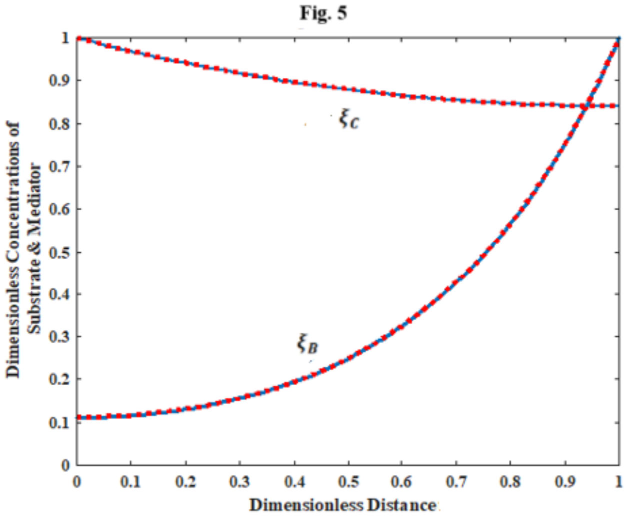

Figure 5. Indeterminate substrate and mediator concentration versus normalized distance over enormous parameter values.

Figure 5 illustrates the concentration of the unknown substrate and mediator as a function of normalised distance over a wide range of parameter values. According to the substrate concentration decreases, ξ_B grows, and the concentration of mediator that corresponds to the lowest value of ξ_C also increases as the reaction parameter ξ_B grows. This means that the concentration of the mediator impacts the reaction rate constant ξ_B, which is reversely correlated with ξ_C.

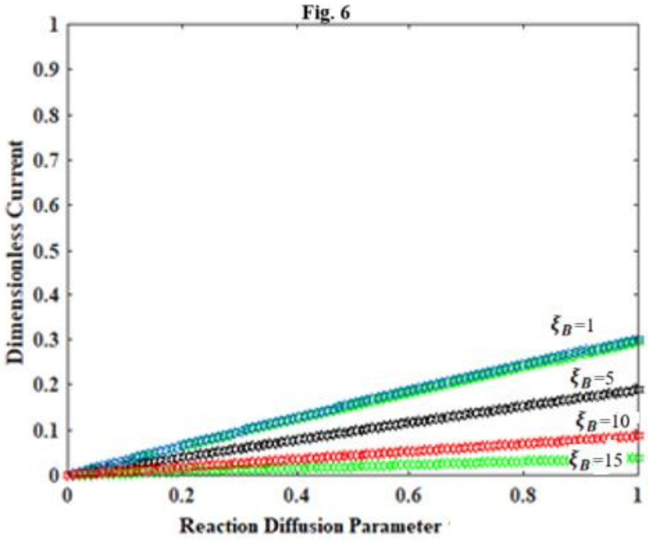

Figure 6. Representation of normalized current for the various values of ξ_B

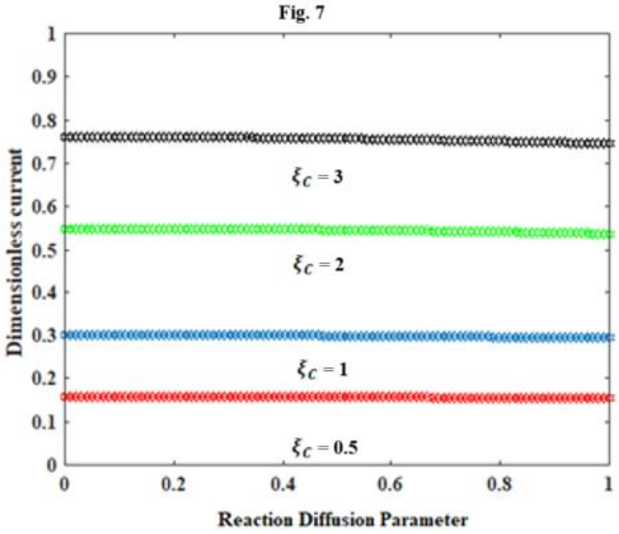

Figure 7. Plot of normalized current for different values of ξ_C.

Figure 6 represent the concentration of normalized current for various values of reaction rate constant ξ_B. The figure suggests that an increase in ξ_B leads to a corresponding decreasing in the concentration of current. The concentration of normalised current for a range of response rate constant ξ_C values is shown in Figure 7. The figure implies that a escalates in the current concentration corresponds to an increase in ξ_C.

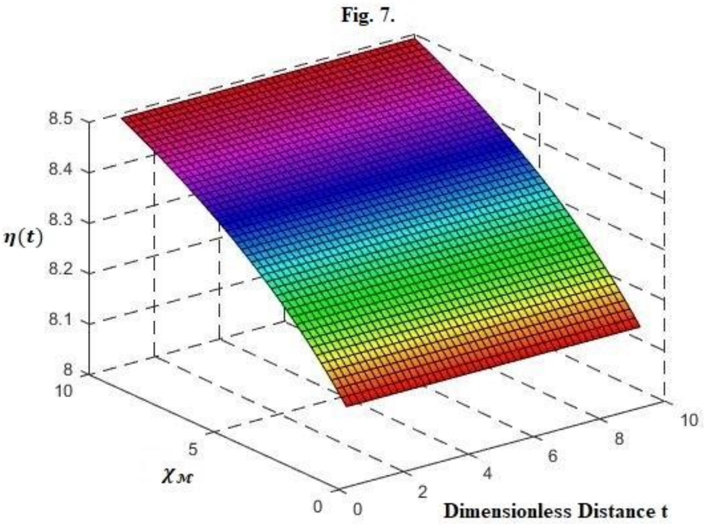

Figure 7. Three-dimensional Mediator concentration versus dimensionless distance and the reaction rate constant ξ_C.

8. Conclusion

The system of nonlinear reaction diffusion equations in chemically modified electrodes at a state of stability have been determined analytically in the present study. The novel analytical formulations for the concentration of volatile chemicals corresponding to spherical electrodes was obtained using the Akbari Ganji Method, which applies steady state reaction diffusion equations throughout the electrode surface considering chemically modified electrodes remain afford. A straightforward new computational equation is provided for the steady state current response. Comparing the final results to preceding restricted case results, it indicates more quantitative consistency. These outstanding results are used to model the biosensor based on the system of nonlinear reaction diffusion equations. In contrast to other analytical techniques, this approach is uncomplicated, yields accurate data, and has a straightforward solution. Other boundary value problems in the chemical and physical sciences can be easily resolved with this method. In addition to mixed complicated boundary conditions, it seems as though the process may be extended to additional two- and three-dimensional geometries and interacting steps of first-order chemical reactions.

Nomenclature

$\mathfrak{D}_B$	Diffusion coefficient of substrate $\mu m^2/s$	ξ_B	Dimensionless diffusion parameter
$\mathfrak{D}_C$	Diffusion coefficient of mediator $\mu m^2/s$	ξ_C	Dimensionless reaction parameter
B	Concentration of substrate μm	x	Dimensionless Concentration substrate
C	Concentration of oxidized mediator μm	y	Dimensionless concentration of mediator
μ	Biomolecular rate constants ms	ϑ	Dimensionless distance
u	Distance from the electrode μm	Ω	Dimensionless normalized current
$\mathcal{C}_T$	Total concentration of mediator & Substrate	ϕ_i	Relevant flux factor
s^{∞}	Bulk concentration of substrate μm	A_1	Oxidized enzyme
$\mathcal{C}_{\mathfrak{J}}$	Net flux	A_2	Reduced enzyme
$\mathbb{K}$	Partition coefficient	X, Y	Reaction Products

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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