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Fuzzy Generalized Fractal Dimensions on Sierpiński and Social Network Graphs

J. Bhuvaneswari, D. Easwaramoorthy*

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India

**Corresponding author:* easandk@gmail.com

Abstract. Fractal theory is the propelled technique to analyze non-linear systems and complex graphs. The quantification of complexity in Sierpiński and social network graphs requires the estimation of Generalized Fractal Dimensions (GFD), where complexity refers to the greater inconsistency and uncertain nature of the systems. This study introduces the fuzzy version of GFD and compares the Fuzzy GFD (FGFD) with the usual GFD for extended Sierpiński and social network graphs. The computational results indicate that the complexity of the graphical structure increases with the number of iterations due to self-similarity, as fractal-based measure values increase with iterations for generalized Sierpiński graphs. The FGFD values are consistently higher than the usual GFD, demonstrating its ability to capture more structural information. Thus, FGFD provides a more effective method for estimating non-linearity and analyzing Sierpiński and real-time graphical networks. The proposed fuzzy-based multifractal measures better quantify complexity levels compared to traditional multifractal measures.

1. Introduction

The classical Euclidean geometry in mathematics concerns sets with integer type measurements, while the fractal geometry deals with entities having non-integer type measurements [1]. A fractal is typically an asymmetrical or fragmented geometric form that can be divided into parts, each of which is (at least approximately) a scaled-down replica of the entire shape, exhibiting a characteristic known as self-similarity [2]. The term “fractal” was coined by Benoit Mandelbrot, a Professor Emeritus, in his seminal work. It is derived from Latin name “fractus” with the meaning broken, cracked, or fractured. This term was used to describe sets that were too irregular to fit into a conventional geometric shape. In his original paper, Mandelbrot formally defined a fractal as a set whose Hausdorff dimensions is strictly greater than its topological dimension [3].

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In general, a fractal set exhibits more complexity, compared to the sets studied in the classical geometry [4]. Karl Weierstrass introduced a function, termed the Weierstrass Function, which possesses the paradoxical characteristic of being continuous at all points but lacking differentiability entirely. The graph of this function, if considered today, would be recognized as a fractal [5]. The complexity and irregularity observed in various generalized Sierpiński and Social network graphs with nonlinear structures and examined and quantified through fractal techniques using the non-integer or fractional measure known as Fractal Dimension [6].

The intricacy of the non-linearity measure has been examined in various contexts using the multifractal measure known as Generalized Fractal Dimensions (GFD). Subsequently, GFD have been estimated to assess the degree of complexity of realistic images, signals, networks or any systems with irregularity [7]. Multifractal dimensional theory has applied in various fields, including signal and image analysis, biomedical signals, medical imaging, and even financial time series, by allowing for a more nuanced understanding of complex and irregular structures and patterns that traditional fractal dimensions might miss [8]. Moreover, the extension of FGFD from the conventional GFD has enabled the assessment of chaotic traits in mathematical waveforms generated by Weierstrass Functions [9]. The Sierpiński type graphical structure appears naturally in many different areas of mathematics and scientific fields Sierpiński gasket graph is one of the most important families of such graphs [10].

The Fuzzy Generalized Fractal Dimensions (FGFD) offers a unique approach to graph complexity by integrating fuzzy set theory with fractal analysis, making it distinct from traditional complexity measures [9]. Graph Entropy quantifies randomness but does not capture fractal properties or hierarchical structures. The fractal dimension measures self-similarity but lacks the ability to handle uncertainty in complex networks. Topological Indices focus on connectivity but do not incorporate fractal or fuzzy characteristics. FGFD bridges these gaps by accounting for uncertainty, self-similarity, and hierarchical complexity, making it more suitable for analyzing real-world networks with irregular and uncertain structures, such as biological networks, social networks, and image processing applications [11]. Scorer et al. studied the Sierpiński gasket graph and applied them to dynamical systems psychology [12].

The investigation into the capacity of wireless networks has garnered considerable attention of scientific researchers in recent years. Despite the recent studies mentioned in [13], and the recognition of the fractal phenomenon as a crucial property in numerous wireless networking scenarios [14], fractal wireless networks have received little attention.

Fuzzy concepts in fractal theory help analyze uncertainty and imprecise structures in generalized Sierpiński graphs and social network graphs. It enhances the study of self-similarity, hierarchical patterns, and structural complexity. In Sierpiński graphs, it refines fractal properties under uncertainty, while in social networks, it improves modelling of dynamic and uncertain relationships. This approach aids in better understanding connectivity, influence, and information flow in complex systems.

This study introduces a Fuzzified Generalized Fractal Dimensions (FGFD) to enhance the analysis of generalized Sierpiński and social network graphs, addressing limitations in traditional fractal measures.

The research provides a detailed comparison between FGFD and classical GFD, demonstrating how FGFD captures finer structural variations and better quantifies graph complexity.

The study incorporates a Gaussian fuzzy membership function to differentiate representative graphs, improving the measurement of non-linearity, uncertainty, and hierarchical structures in real-world networks.

The paper's structure is as follows: Section 2 depicts the Renyi entropy measure and GFD measure for generalized Sierpiński graphs and Social networks. Additionally, it proposed with the Fuzzy Renyi entropy measure and FGFD measure for Sierpiński graphs and Social networks. In Section 3, the generalized Sierpiński graph and the Social network graph are described. The computational results obtained are subsequently scrutinized in detail in Section 4. Finally, concluding comments are presented in Section 5.

2. Mathematical Methods

This section examines the fundamental concepts and the proposed non-linear measures required for this research work in detail.

2.1. **Renyi Entropy.** Renyi entropy, initially proposed by Alfred Renyi [15], stands as a crucial concept in the information theory. It serves as a generalization of Shannon entropy, forming one of the functional groups employed to evaluate the diversity, uncertainty, or randomness within a given system. Renyi entropy, also referred to as the generalized entropy of a specific probability measure, quantifies the information content of a probability distribution. It is defined by the expression:

$$R_q = \frac{1}{1-q}\log_2\left(\sum_{i=1}^{N} p_i^q\right),$$

where $q \geq 0$ & $q \neq 1$, and p_i represents the probability values assigned to each x_{i} for $i = 1, 2, \ldots, N$, taken by the random variable.

2.2. **Fuzzy Renyi Entropy.** Fuzzy Renyi Entropy quantifies uncertainty in a fuzzy system by incorporating the fuzzification process, which assigns membership values between 0 and 1 instead of using crisp classifications. The expression for the Fuzzy Renyi Entropy of order q on a given set V, where $q \geq 0$ & $q \neq 1$, is given below [9].

$$FR_q = \frac{1}{1-q}\log_2\left(\sum_{i=1}^{N}\left(\sum_{x\in V_i}\mu(x)\right)^q\right).$$

Here, the function $\mu : V \to [0,1]$ denotes the fuzzy membership function defined over the underlying space V, which comprises of N subsets denoted as $V_1, V_2, \ldots, V_N$.

Fuzzy Renyi entropy helps measure uncertainty and complexity in generalized Sierpiński graphs and social network graphs. It provides a flexible way to analyze information flow, structural irregularities, and robustness in networks. In Sierpiński graphs, it aids in studying self-similarity and hierarchical structures. In social network graphs, it helps evaluate connectivity, influence dynamics, and information diffusion under uncertainty.

2.3. **Multifractal Analysis for Graphs.** The Renyi entropies are pivotal in non-linear and statistical analyzes due to their association with irregularity and unpredictability. These entropies give rise to a set of fractal measure indices known as Generalized Fractal Dimensions (GFD). The development of Multifractal theory in 1983 was based on the GFD measure [16, 17]. Beyond the conventional fractal dimension, GFD is essential for comprehending and measuring the intricacy of irregular structures in biomedical signals and images along with wavelet theory [18]. This section explores the procedure to compute GFD measure for fractal like graphs.

To define the GFD measure for graphs, we construct the required probability distribution for a given multifaceted graph as follows. Let $G = (V(G), E(G), \varphi_G)$ be a graph with the vertex set $V(G)$, the edge set $E(G)$ and the incidence function φ_G in which the degree of a vertex $(d(v))$ is equal to the number of edges incident on v. Most of the properties of the graph are based on the distribution of degrees of a vertex. Here, the probability distribution of the given graph is defined as follows:

$$p_i = \frac{\sum_{v \in V_i} d(v)}{\sum_{j=1}^{N} d\left(v_j\right)}, \quad i = 1, 2, \ldots, M;$$

where N is the number of vertices in G, M is the number of vertex subsets (copies) partitioned $V(G)$, and p_i is the probability of i^{th} vertex subset. Then, the Renyi Fractal Dimensions or Generalized Fractal Dimensions (GFD) for the given probability measures, with an order $q \in (-\infty, \infty)$ & $q \neq 1$ and denoted by D_q, is given below

$$D_q = \lim_{r \to 0} \frac{1}{q-1} \frac{\log_2 \left(\sum_{i=1}^{M} p_i^q\right)}{\log_2 r}, \tag{2.1}$$

where M is the number of vertex subsets (copies) of $V(G)$.

Here the Generalized Fractal Dimensional measure is defined by means of generalized Renyi Entropy.

2.4. **Fuzziness of Multifractal Analysis for Graphs.** Here, FGFD is proposed in this section and used to measure the degree of fuzziness.

The definition of FGFD for a given fractal graph is as follows.

Let $G = (V(G), E(G), \varphi_G)$ be a graph with N vertices. The vertex set $V(G)$ is partitioned into M vertex subsets(copies) say copy $V_1, V_2, \ldots, V_M$ according to the problem proceeded.

Then the fuzzy membership function of i^{th} vertex subset is defined as

$$\mu_i = \frac{\sum_{v \in V_i} \mu(v)}{\sum_{u \in V(G)} \mu(u)}, \quad i = 1, 2, \ldots, M,$$

where μ is the fuzzy membership function on the vertex set $V(G)$, (i.e). $\mu : V(G) \to [0, 1]$.

A fuzzy membership function on the set V_i divided into V vertices as subset of the i^{th} copy $V_1, V_2, \ldots, V_i$ is characterized as a function $\mu : V_i \to [0, 1]$.

The FGFD with the order $q \in (-\infty, \infty)$, $q \neq 1$ for the provided fuzzy membership function of the given graph is denoted and defined as

$$FD_q = \lim_{r \to 0} \frac{1}{q-1} \frac{\log_2 \left(\sum_{i=1}^{M} (\mu_i)^q \right)}{\log_2 r}, \tag{2.2}$$

where M is the number of vertex subsets of $V(G)$, and μ_i represents the fuzzy membership function, assigning values in $[0, 1]$. The parameter r serves as the scale factor, approaching zero for fine grained analysis, while q is the entropy order, which controls the sensitivity to high or low membership values, influencing the overall complexity measurement of the graph.

Here, the FGFD is established by utilizing generalized Fuzzy Renyi Entropy, alternatively referred to as Fuzzy Renyi Fractal Dimensions for the given graph.

2.5. **Gaussian Fuzzy Membership Function.** The general Gaussian fuzzy membership function, denoted as $g : V \to [0, 1]$, is a mathematical expression used to determine the membership degree of an element x in a fuzzy set defined over a set V. The function is parametrized by a mean value $(\bar{x})$ and the standard deviation (σ) is defined below.

$$g(x; \bar{x}, \sigma) = e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}.$$

3. Experimental Fractal Type Graphs

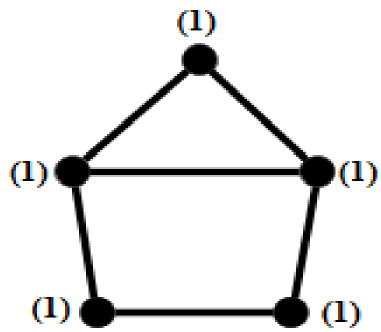
A fractal is an entity that exhibits self-similarity when viewed at different levels of magnification. Fractal also possess the irregularity nature some times. Hence, We are experimenting self-similar fractal graphs and irregular fractal graphs in this paper.

3.1. **Generalized Sierpiński Graph.** A graph $G = (V(G), E(G), \varphi_G)$ with an order of $N \geq 2$, and let t be a positive integer. The set of words of length t on the alphabet V is defined as V^t. In this set, each word $u \in V^t$ of length t is represented by its individual letters $u_1 u_2 \ldots u_t$ [19]. The generalized Sierpiński graph of G with dimension t represented as $S(G, t)$ is a graph having the vertex set V^t. An edge (u, v) exists in this graph if and only if there is an index $l \in \{1, \ldots, t\}$ such that:

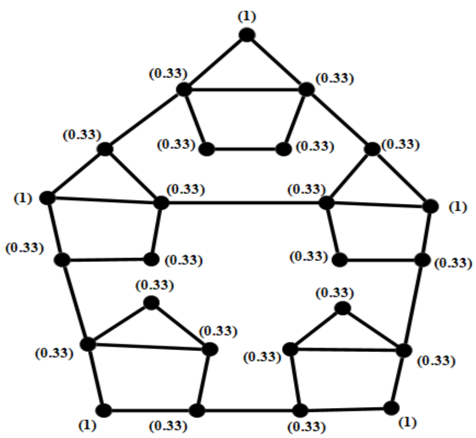
(i). $u_m = v_m$ if $m < l$,

(ii). $u_l \neq v_l$ and $(u_l, v_l) \in E(G)$,

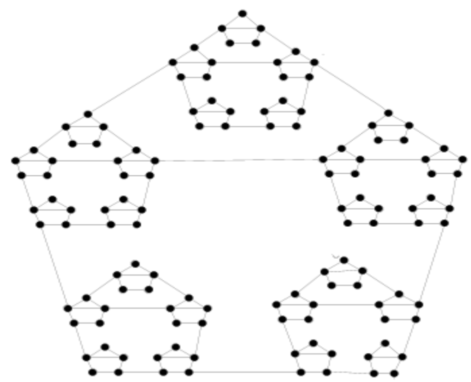
(iii). $u_m = v_l$ and $v_m = u_l$ if $m > l$.



(A) First iteration



(B) Second iteration



(C) Third iteration

FIGURE 1. Iterations of generalized Sierpiński graph

3.2. **Social Network Graph.** In the context of a social media platform, where the network is represented as a graph with 16 platforms (depicted in red) serving as nodes, and the links between them representing connections forged by internet celebrities (depicted in blue), network graphs offer a powerful tool to analyze the dynamics of interactions and relationships within this digital ecosystem. Each edge (connection) between two vertices (platforms or nodes) signifies some form of interaction, collaboration, or association facilitated by the internet celebrities.

A notable aspect of this network is the presence of extreme, anti-establishment actors who, at one point, were propelled into the spotlight as internet celebrities through the very same social media platforms. However, these individuals and sometimes groups are now being characterized as dangerous individuals by the leading social media companies, including Facebook, Instagram, Twitter, and YouTube. This characterization has led to a phenomenon known as de-platforming, where in these individuals are removed from or denied access to these major social media platforms due to offenses such as organized hate.

The act of de-platforming has ignited a significant debate, raising questions about the role of liberal big tech in potentially stifling free speech. The controversy revolves around whether these major technology companies are assuming editorial roles by deciding who can or cannot participate in the digital public sphere. Furthermore, the discussions have emerged regarding the effectiveness of de-platforming as a strategy to address issues such as organized hate and whether it achieves its intended goals [20].

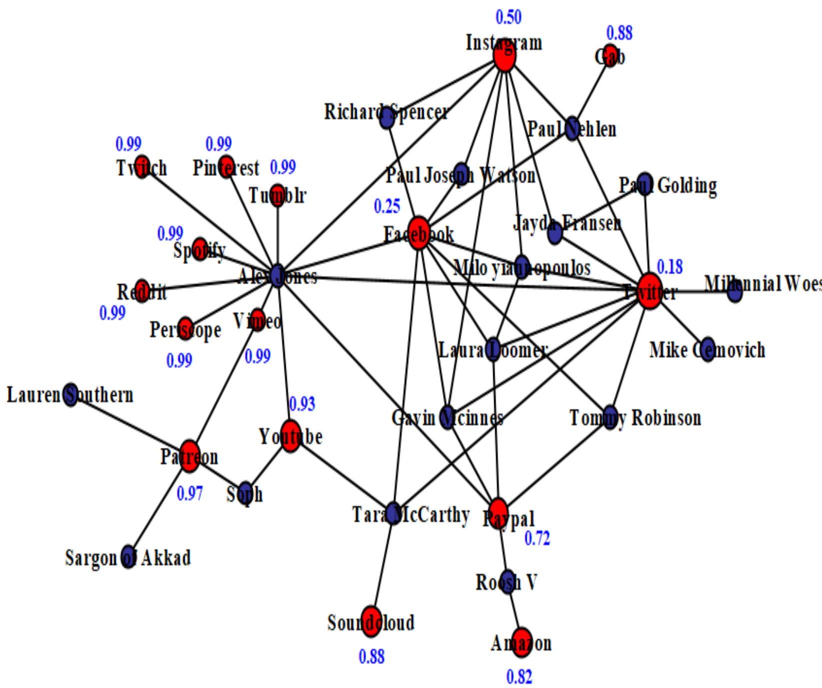


FIGURE 2. Social network graph

4. RESULTS AND DISCUSSION

The effectiveness of the developed fuzzy multifractal measure is evaluated by conducting simulations using the representative generalized Sierpiński and Social network graphs described through the MATLAB software.

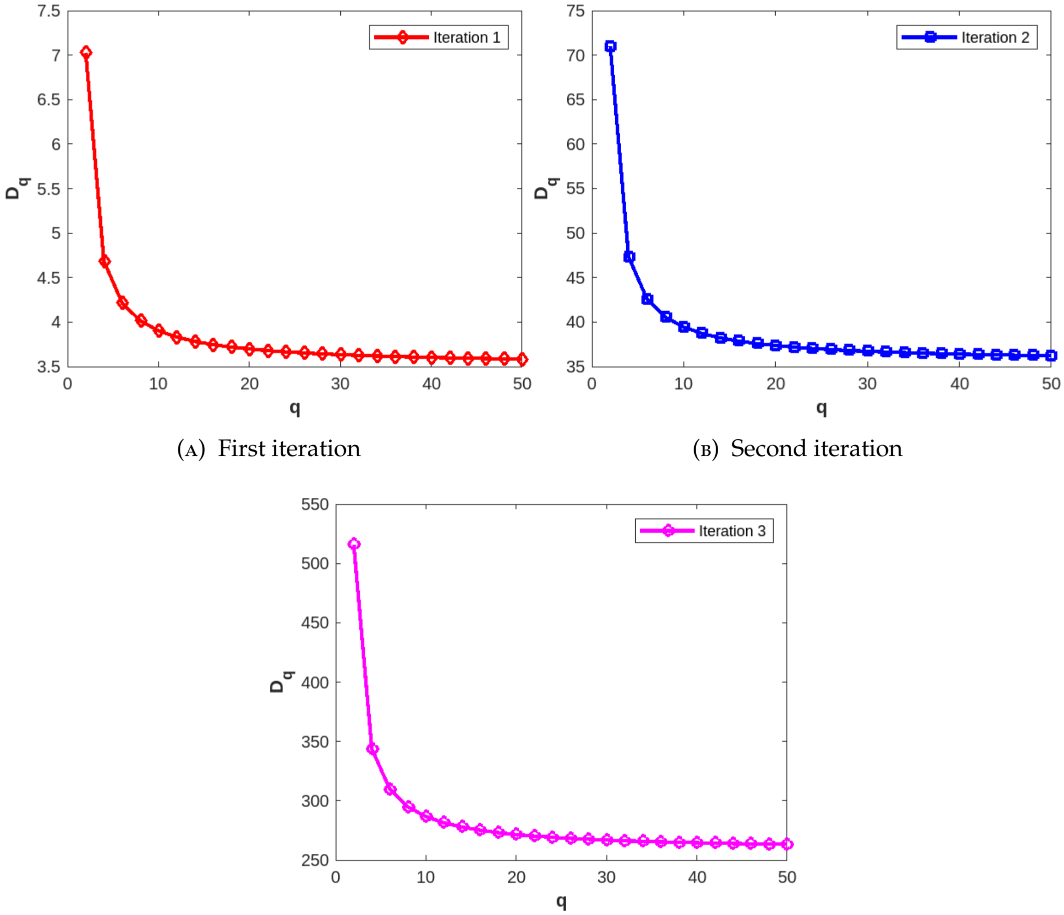
The probability distributions for each vertex of the generalized Sierpiński graph were acquired, along with the corresponding GFD values. Additionally, the fuzzy membership function values for each vertex were determined using the Gaussian fuzzy membership function (g). Subsequently, the FGFD for the entire representative generalized Sierpiński graph was derived from the computed Gaussian membership values.

Table 1. GFD values for three iterations of generalized Sierpiński graph

q	Iteration 1	Iteration 2	Iteration 3
2	7.0255	70.983	515.97
4	4.6837	47.322	343.98
6	4.2153	42.590	309.58
8	4.0146	40.562	294.84
10	3.9031	39.435	286.65
12	3.8321	38.718	281.44
14	3.7830	38.222	277.83
16	3.7470	37.858	275.18
18	3.7194	37.579	273.16
20	3.6977	37.579	271.56
22	3.6800	37.182	270.27
24	3.6655	37.035	269.20
26	3.6533	36.911	268.30
28	3.6429	36.806	267.54
30	3.6339	36.715	266.88
32	3.6261	36.636	266.31
34	3.6192	36.567	265.80
36	3.6131	36.506	265.35
38	3.6077	36.451	264.96
40	3.6028	36.402	264.60
42	3.5985	36.357	264.28
44	3.5945	36.317	263.98
46	3.5908	36.280	263.72
48	3.5875	36.247	263.47
50	3.5845	36.216	263.25

Table 1 appears to represent the results of an iterative process over different values of q for multiple iterations. The GFD values have been calculated for q values ranges from 2 to 50 for three iteration. The results suggest that as q increases, the GFD values in Table 1 decrease over successive iterations.

The decreasing trend seems to stabilize as q increases, indicating that the iterative process converges to a certain value for each q in the later iterations. The values seem to be decreasing at a diminishing rate, and as q approaches higher values, the rate of decrease slows down.



(A) First iteration (B) Second iteration

(C) Third iteration

Figure 3. Graphical representation of GFD for three iterations of generalized Sierpiński graph

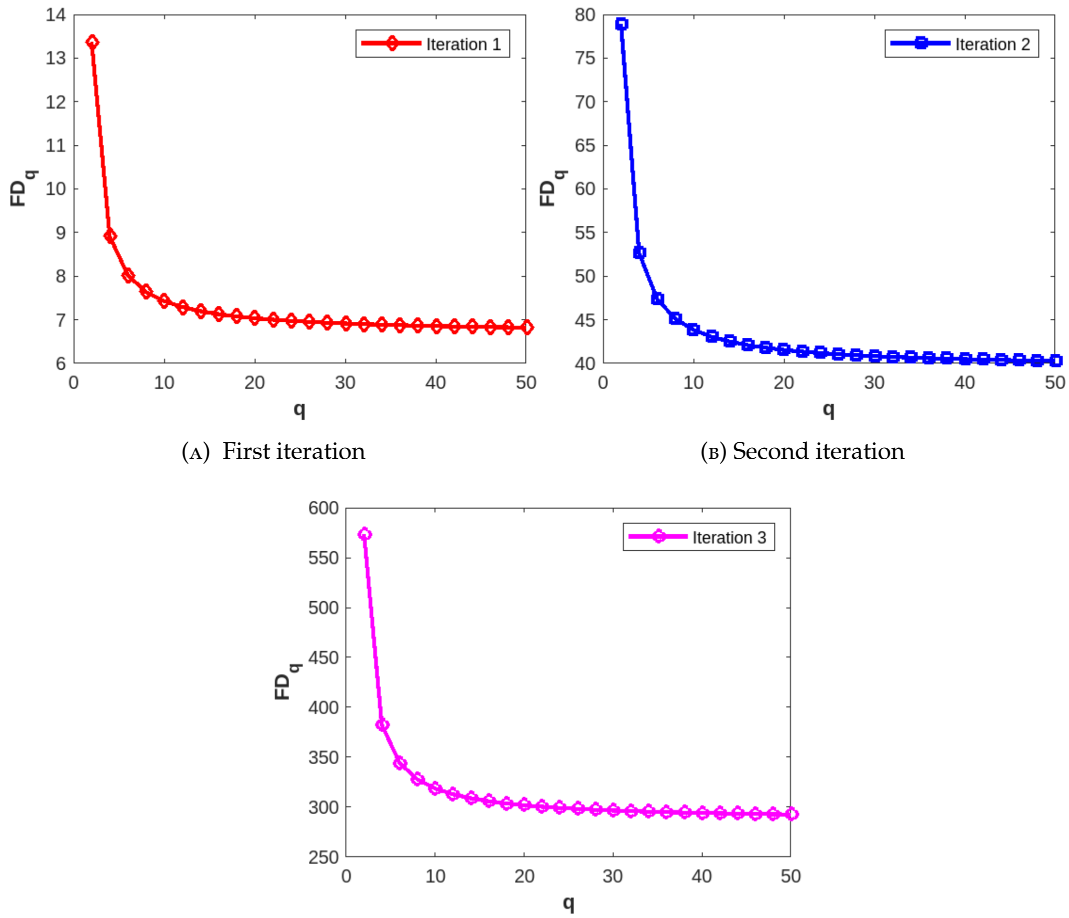
In Figure 3 the increasing GFD values with successive iterations indicate that the self-similar structure undergoes a significant transformation as it evolves. This suggests that with each iteration, the geometric framework becomes more refined, either by attaining a higher level of order or by developing greater complexity. The growth in GFD values signifies that additional structural details emerge at finer scales, enhancing the overall intricacy of the pattern. This evolution follows the principles of fractal geometry, where self-replicating patterns create increasingly elaborate formations. Depending on the nature of the iterative process, the structure may become more systematically organized, reinforcing its inherent self-similarity, or it may exhibit heightened complexity through the introduction of new geometric variations. In either case, the increase in GFD values serves as a quantitative measure of the structure's evolving characteristics, providing insight into the interplay between order and complexity in self-similar formations.

Table 2. FGFD values for three iterations of generalized Sierpiński graph

q	Iteration 1	Iteration 2	Iteration 3
2	13.368	150.95	1096.9
4	8.9118	100.63	731.29
6	8.0206	90.569	658.16
8	7.6387	86.256	626.82
10	7.4265	83.860	609.41
12	7.2915	82.336	598.33
14	7.1980	81.280	590.66
16	7.1295	80.506	585.04
18	7.0770	79.914	580.73
20	7.0356	79.447	577.34
22	7.0021	79.068	574.59
24	6.9745	78.756	572.32
26	6.9512	78.493	570.41
28	6.9314	78.270	568.78
30	6.9143	78.077	567.38
32	6.8995	77.909	566.16
34	6.8864	77.761	565.09
36	6.8748	77.631	564.14
38	6.8645	77.514	563.29
40	6.8552	77.410	562.53
42	6.8469	77.315	561.85
44	6.8393	77.230	561.23
46	6.8324	77.152	560.66
48	6.8261	77.080	560.14
50	6.8203	77.015	559.66

Table 2 represents the values of the FGFD for different q values across multiple iterations. The FGFD values in Table 2 seem to represent a fuzzified a measure of the generalized Sierpiński graph. The multi level fractal dimensions often quantify the complexity or self-similarity of geometric patterns. In Table 2 there is a consistent decreasing trend in the FGFD values as q increases for each iteration. The FGFD values appear to converge as q increases, and the rate of decrease slows down in later iterations. It is suggested that the FGFD stabilizes for higher values of q.

Generally, a higher fractal dimension is often associated with more complex and intricate patterns. The increasing FGFD values might indicate a tendency toward more similarities and complex patterns as the number of iterations increases.



(A) First iteration (B) Second iteration

(C) Third iteration

Figure 4. Graphical representation of FGFD for three iterations of generalized Sierpiński Graph

Then the values of the FGFD are plotted against the corresponding q values from 2 to 50 for the generalized Sierpiński graph. The graphs for the FGFD method are depicted in Figure 4 for three iterations. It is also noticed from Tables 1 - 2 and Figures 3 - 4 GFD and FGFD values are differed significantly for each iteration.

When comparing the FGFD to the GFD, FGFD values are consistently higher, suggesting that the FGFD methodology provides a more comprehensive evaluation of complexity and similarity patterns, particularly in the presence of uncertainties and variations. This indicates that FGFD is more effective than classical GFD in capturing the intricate structural details of self-similar formations. By incorporating fuzziness into its computational framework, FGFD allows for a more flexible assessment of patterns, making it better suited for analyzing structures where deterministic methods may fail to account for subtle variations. Unlike GFD, which relies on precise and rigid measurements of geometric similarity, FGFD introduces a degree of adaptability that enhances its ability to detect complex levels of self-similarity. This suggests that FGFD not only refines the measurement process but also provides a more nuanced representation of structural complexity, making it a valuable tool in analyzing patterns that exhibit both ordered and uncertain characteristics.

Table 3. GFD and FGFD values for social network graph

q	GFD (D_q)	FGFD (FD_q)
2	28.572	4.2574
4	19.048	2.8383
6	17.143	2.5545
8	16.327	2.4328
10	15.873	2.3652
12	15.585	2.3222
14	15.385	2.2925
16	15.238	2.2706
18	15.126	2.2539
20	15.038	2.2408
22	14.966	2.2301
24	14.966	2.2213
26	14.857	2.2139
28	14.815	2.2076
30	14.779	2.2021
32	14.747	2.1974
34	14.719	2.1932
36	14.694	2.1895
38	14.672	2.1863
40	14.652	2.1833
42	14.634	2.1806
44	14.618	2.1782
46	14.603	2.1760
48	14.590	2.1740
50	14.578	2.1722

In Table 3, the GFD values are notably higher compared to the FGFD values indicating a substantial scale differences between the two measures. The GFD and FGFD Values of the given Social network are calculated for the order q ranges from 2 to 50 as shown in Table 3. Both GFD and FGFD exhibit decreasing trends as q increases. This quantitative contrast highlights the different magnitudes of multi level fractal dimensions captured by the two methods.

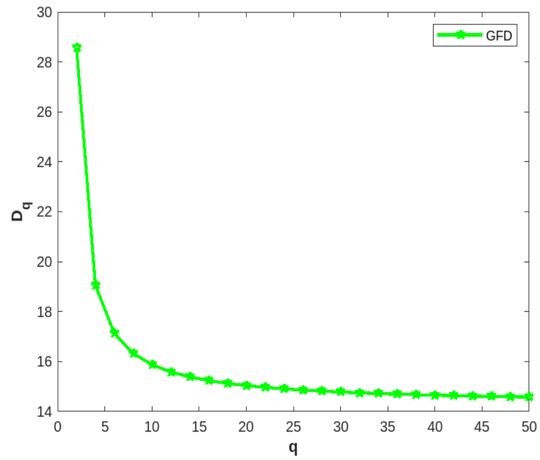


Figure 5. GFD for social network graph

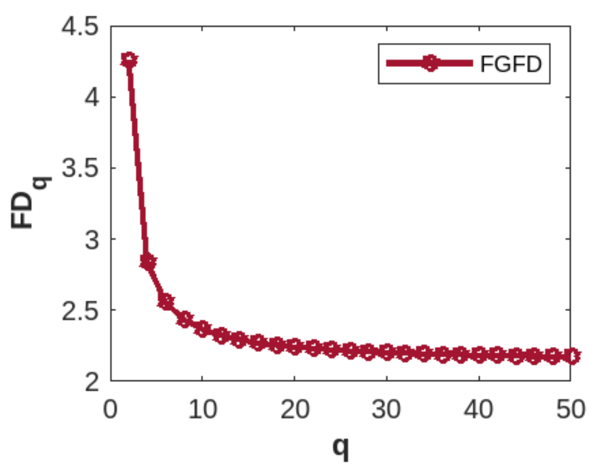


Figure 6. FGFD for Social network graph

The GFD and FGFD values are plotted graphically of the network graph and illustrated in Figures 5–6. Figures 5–6 reveal a difference in specific manner, in the curves plotted for GFD and FGFD, as q increases. Table 3 and Figures 5–6 demonstrate that the GFD values are consistently higher than the FGFD values, suggesting a fundamental difference in how these two methodologies interpret self-similar structures.

Sierpiński graphs, a self-similar fractal structure, which have deterministic self-similarity at each iteration, whereas social network graphs, which have community-based small-world properties, possess more irregularity rather than self-similarity. Specifically, this implies that the level of self-similarity in the social network graph is generally lower than that of the generalized Sierpiński graph. The lower FGFD values, compared to GFD, indicate that the fuzziness incorporated in FGFD captures subtle uncertainties and variations that are not accounted for in the more deterministic GFD approach. This suggests that FGFD, by integrating fuzzy logic, provides a more flexible and adaptive assessment of self-similarity, especially in complex structures where variations and irregularities are present. Unlike GFD, which strictly adheres to precise geometric similarity, FGFD can recognize nuanced patterns within self-similar structures, making it more effective in evaluating systems characterized by inherent vagueness. Consequently, the difference in values highlights the advantages of FGFD in detecting deeper and more complex self-similarity levels, particularly in networks and structures that exhibit non-uniform or evolving patterns.

The higher FGFD values compared to GFD arise because FGFD accounts for uncertainty in graph topology, effectively capturing the complexity of structural variations. Unlike GFD, which assumes a fixed structure, FGFD incorporates fuzzy logic, making it more sensitive to irregularities and dynamic changes within the network. In discussions on de-platforming, the dynamic behaviour and morphological properties of the social network has been investigated and estimated evidently by the mathematical analysis using fractal and multifractal measures. Explicitly linking fractal measures to real-world network behaviors such as resilience, fragmentation, and influence diffusion helps readers better to understand how structural complexity affects social interactions and information flow.

In this context, the Fuzzy GFD is compared with the traditional GFD for self-similar and social networks and concluded that the Fuzzy GFD is numerically higher than the GFD at each level of q. Beyond the GFD, the Fuzzy based GFD can be compared further with fuzzified nonlinear measures and entropies for the real time graphical structures.

5. Conclusion

In this study, generalized Sierpiński and social network graphs were analyzed using Fuzzy Multifractal Theory. The Fuzzy Generalized Fractal Dimensions (FGFD), incorporating the Gaussian fuzzy membership function, was introduced and applied to these graphical structures. Simulation results demonstrated the advantages of fuzzy multifractal analysis over traditional multifractal analysis, particularly in the context of Sierpiński graphs and social networks. The key findings of this proposed research work indicate that self-similar structures, such as the generalized Sierpiński graph, exhibit higher FGFD values than GFD, whereas social network graphs, which lack strong self-similarity, display higher GFD values than FGFD. This suggests that fuzzy multifractal dimensions provide a more effective measure of complexity for self-similar graphs while offering valuable insights into the structural intricacies of social networks. In future, this framework can be extended to analyze dynamic social networks, real-time chemical structures, and large-scale network models, enhancing its applicability. Additionally, FGFD can be applied to other types of graphs, such as biological networks, transportation systems, and financial networks, broadening its scope. Furthermore, exploring machine learning applications in graph classification using FGFD could open new possibilities for automated network analysis. These extensions will further demonstrate the versatility and robustness of the proposed fuzzy multifractal approach in diverse scientific and technological domains.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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