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On (Fuzzy) Weakly Almost Interior Γ-Hyperideals in Ordered Γ-Semihypergroups

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Abstract. In this paper, we concentrate on studying the generalization of almost interior Γ-hyperideals in ordered Γ-semihypergroups. The notion of weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups is introduced. This concept generalizes the notion of almost interior Γ-hyperideals in ordered Γ-semihypergroups. Then, the characterization of ordered Γ-semihypergroups having no proper weakly almost interior Γ-hyperideals is provided. Next, we introduce the concept of fuzzy weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups. Also, some properties of fuzzy weakly almost interior Γ-hyperideals are considered. Moreover, the concepts of weakly almost interior Γ-hyperideals and fuzzy weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups are characterized. The connections between strongly prime (resp., prime, semiprime) weakly almost interior Γ-hyperideals and fuzzy strongly prime (resp., prime, semiprime) weakly almost interior Γ-hyperideals in ordered Γ-semihypergroups are presented.

1. Introduction

When it comes to studying in semigroups, ideal theory is essential. Grošek and Satko [5] extended the concept of ideals in semigroups to the concept of almost ideals in 1980, characterizing the semigroups that have proper almost ideals. Afterwards, Bogdanović [2] introduced the concept almost bi-ideals in semigroups, as a generalization of bi-ideals, by using the concepts of almost ideals and

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bi-ideals of semigroups. Zadeh [22] introduced the concept of fuzzy subsets as a function from a nonempty set X to the unit interval $[0,1]$. Wattanatripop et al. [21] applied the concept of fuzzy subsets to define the notion of fuzzy almost bi-ideals of semigroups in 2018, they examined at some of the connections between almost bi-ideals and fuzzy almost bi-ideals in semigroups. The concepts of (resp., weakly) almost interior ideals and fuzzy (resp., weakly) almost interior ideals in semigroups were introduced and discussed by Kaopusek et al. [8] and Krailoet et al. [9], respectively. In 2022, Chinram and Nakkhasen [3] introduced the concept of almost bi-quasi-interior ideals of semigroups and considered some relationships between almost bi-quasi-interior ideals and their fuzzification in semigroups.

The notion of Γ-semigroups generalized from the classical semigroups, was first introduced by Sen and Saha [15]. Then, Simuen et al. [16] defined the concepts of almost quasi-Γ-ideals and fuzzy almost quasi-Γ-ideals of Γ-semigroups. Later, Jantanan et al. [7] studied the concepts of almost interior Γ-ideals and fuzzy almost interior Γ-ideals in Γ-semigroups. The notion of ordered semigroups is another generalization of the semigroups. In 2022, Suebsung et al. [17] introduced the concepts of (resp., fuzzy) almost bi-ideals and (resp., fuzzy) almost quasi-ideals of ordered semigroups, and they have investigated the characterizations of these concepts.

Since 1934, the research of Marty [10], who developed the notion of hyperstructures, has been studied by many mathematicians. The concept of almost hyperideals in semihypergroups, which is a generalization of hyperideals, was introduced and presented some properties by Suebsung et al. [18]. Then, they have defined the concept of almost quasi-hyperideals in semihypergroups and gave some interesting properties, see [19]. Next, Muangdoo et al. [11] introduced the notions of (resp., fuzzy) almost bi-hyperideals of semihypergroups and discussed some connections between almost bi-hyperideals and their fuzzification in semihypergroups. In 2022, Nakkhasen et al. [12] surveyed some properties of fuzzy almost interior hyperideals in semihypergroups and considered some links between almost interior hyperideals and fuzzy almost interior hyperideals in semihypergroups.

It is known that ordered Γ-semihypergroups are a generalization of semihypergroups. Recently, Rao et al. [14] defined the concept of almost interior Γ-hyperideals of ordered Γ-semihypergroups and provided the relationships between interior Γ-hyperideals and almost interior Γ-hyperideals in ordered Γ-semihypergroups. This article presents the notions of weakly almost interior Γ-hyperideals in ordered Γ-semihypergroups, which extend the idea of almost interior Γ-hyperideals, and provides certain characteristics of these hyperideals. Furthermore, we define the concept of fuzzy weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups, and consider some connections between weakly almost interior Γ-hyperideals and fuzzy weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups.

2. Preliminaries

Firstly, we recall some of the basis definitions and properties, which are necessary for this paper.

A *hypergroupoid* $(H, \circ)$ is a nonempty set H together with a mapping $\circ : H \times H \to \mathcal{P}^*(H)$ called a *hyperoperation*, where $\mathcal{P}^*(H)$ denotes the set of all nonempty set of H (see [4, 10]). We denote by $a \circ b$ the image of the pair (a, b) in $H \times H$. If $x \in H$ and $A, B \in \mathcal{P}^*(H)$, then we denote

$$A \circ B := \bigcup_{a \in A, b \in B} a \circ b, A \circ x := A \circ \{x\} \text{ and } x \circ B := \{x\} \circ B.$$

Definition 2.1. *(see [6]) A hypergroupoid $(S, \circ)$ is called a semihypergroup if $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z \in S$.*

In 2010, Anvariyeh et al. [1] introduced the notion of Γ-semihypergroups, which is a generalization of semihypergroups.

Definition 2.2. *(see [1]) Let S and Γ be two nonempty sets. Then, (S, Γ) is called a Γ-semihypergroup if for each $\gamma \in \Gamma$ is a hyperoperation on S, i.e., $x\gamma y \subseteq S$ for all $x, y \in S$, and for any $\alpha, \beta \in \Gamma$ and $x, y, z \in S$, $(x\alpha y)\beta z = x\alpha(y\beta z)$.*

Let A and B be two nonempty subsets of a Γ-semihypergroup (S, Γ). We define

$$A\Gamma B := \bigcup_{\gamma \in \Gamma} A\gamma B = \bigcup_{\gamma \in \Gamma} \{a\gamma b \mid a \in A, b \in B\}.$$

Particularly, if $A = \{a\}$ and $B = \{b\}$, then we define $a\Gamma b := \{a\}\Gamma\{b\}$.

Definition 2.3. *(see [20]) Let S and Γ be two nonempty sets and $\leq$ be an order relation on S. An algebraic hyperstructure $(S, \Gamma, \leq)$ is called an ordered Γ-semihypergroup if the following conditions are satisfied:*

(i) (S, Γ) is a Γ-semihypergroup;

(ii) $(S, \leq)$ is a partially ordered set;

(iii) for every $x, y, z \in S$ and $\gamma \in \Gamma$, $x \leq y$ implies $x\gamma z \leq y\gamma z$ and $z\gamma x \leq z\gamma y$.

Here, $A \leq B$ means that for each $a \in A$, there exists $b \in B$ such that $a \leq b$, for all nonempty subsets A and B of S.

Throughout this paper, we say an ordered Γ-semihypergroup S instead of an ordered Γ-semihypergroup $(S, \Gamma, \leq)$, unless otherwise mentioned.

For any nonempty subset A of an ordered Γ-semihypergroup S, we denote

$$(A] := \{t \in S \mid t \leq a \text{ for some } a \in A\}.$$

For $A = \{a\}$, we write $(a]$ instead of $(\{a\}]$.

Lemma 2.1. [20] *Let A and B be nonempty subsets of an ordered Γ-semihypergroup S. Then, the following statements holds:*

(i) $A \subseteq (A]$;

(*ii*) *if* $A \subseteq B$, *then* $(A] \subseteq (B]$;

(*iii*) $(A]\Gamma(B] \subseteq (A\Gamma B]$ *and* $((A]\Gamma(B]] = (A\Gamma B]$;

(*iv*) $((A]] = (A]$.

The notion of almost interior Γ-hyperideals in ordered Γ-semihypergroups, as a generalization of interior Γ-hyperideals, has been introduced by Rao et al. [14] in 2021 as follows.

Definition 2.4. *[14] Let S be an ordered Γ-semihypergroup. A nonempty subset K of S is called an almost interior Γ-hyperideal of S if*

(*i*) $(x\Gamma K\Gamma y] \cap K \neq \emptyset$ *for every* $x, y \in S$,

(*ii*) $(K] \subseteq K$.

Now, we review the concept of fuzzy subsets, was defined by Zadeh [22]. We say that μ is a *fuzzy subset* [22] of a nonempty set X if $\mu : X \to [0,1]$. For any two fuzzy subsets μ and λ of a nonempty set X, we denote

(*i*) $\mu \subseteq \lambda$ if and only if $\mu(x) \leq \lambda(x)$ for all $x \in X$,

(*ii*) $(\mu \cap \lambda)(x) := \min\{f(x), g(x)\}$ for all $x \in X$,

(*iii*) $(\mu \cup \lambda)(x) := \max\{f(x), g(x)\}$ for all $x \in X$.

For any fuzzy subset μ of a nonempty set X, the *support* of μ is defined by

$$supp(\mu) := \{x \in X \mid \mu(x) \neq 0\}.$$

The *characteristic mapping* C_A of A, where A is a subset of a nonempty set X, is a fuzzy subset of X defined by for every $x \in X$,

$$C_A(x) := \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 2.2. [11] *Let A and B be nonempty subsets of a nonempty set X and let μ and λ be fuzzy subsets of X. Then, the following statements hold:*

(*i*) $C_{A\cap B} = C_A \cap C_B$;

(*ii*) $A \subseteq B$ *if and only if* $C_A \subseteq C_B$;

(*iii*) $supp(C_A) = A$;

(*iv*) *if* $\mu \subseteq \lambda$, *then* $supp(\mu) \subseteq supp(\lambda)$.

For any element s of X and $\alpha \in (0,1]$, a *fuzzy point* s_α [13] of X is a fuzzy subset of X defined by for every $x \in X$,

$$s_\alpha(x) := \begin{cases} \alpha & \text{if } x = s, \\ 0 & \text{otherwise.} \end{cases}$$

Let S be an ordered Γ-semihypergroup. For each $x \in S$, we define $H_x := \{(y, z) \in S \times S \mid x \leq y\Gamma z\}$. Then, for any two fuzzy subsets μ and λ of S, the *product* $\mu \circ \lambda$ [20] of μ and λ is defined by

$$(\mu \circ \lambda)(x) = \begin{cases} \sup\limits_{(y,z)\in H_x} [\min\{\mu(y), \lambda(z)\}] & \text{if } H_x \neq \emptyset, \\ 0 & \text{if } H_x = \emptyset, \end{cases}$$

for all $x \in S$.

Let μ be a fuzzy subset of an ordered Γ-semihypergroup S. Then, we define $(\mu] : S \to [0, 1]$ by $(\mu](x) = \sup\limits_{x \leq y} \mu(y)$ for all $x \in S$ (see [20]).

The following results can be verified straightforward.

Lemma 2.3. *Let A and B be subsets of an ordered Γ-semihypergroup S. Then $C_A \circ C_B = C_{(A\Gamma B]}$.*

Proposition 2.1. *Let μ, λ and ν be fuzzy subsets of an ordered Γ-semihypergroup S. Then, the following conditions hold:*

(i) $\mu \subseteq (\mu]$;
(ii) if $\mu \subseteq \lambda$, then $(\mu] \subseteq (\lambda]$;
(iii) if $\mu \subseteq \lambda$, then $(\mu \circ \nu] \subseteq (\lambda \circ \nu]$ and $(\nu \circ \mu] \subseteq (\nu \circ \lambda]$.

Proposition 2.2. *Let μ be a fuzzy subset of an ordered Γ-semihypergroup S. Then, the following statements are equivalent:*

(i) if $x \leq y$, then $\mu(x) \geq \mu(y)$ for all $x, y \in S$;
(ii) $(\mu] = \mu$.

3. Weakly almost interior Γ-hyperideals

In this section, we present and study the notion of weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups as a generalization of almost interior Γ-hyperideals.

Definition 3.1. *Let S be an ordered Γ-semihypergroup. A nonempty subset I of S is called a weakly almost interior Γ-hyperideal of S if it satisfies the following conditions:*

(i) $(x\Gamma I\Gamma x] \cap I \neq \emptyset$ for all $x \in S$;
(ii) $(I] \subseteq I$.

The following proposition obtains direct from the definition of almost interior Γ-hyperideals and weakly almost interior Γ-hyperideals in ordered Γ-semihypergroups.

Proposition 3.1. *Every almost interior Γ-hyperideal of an ordered Γ-semihypergroup S is also a weakly almost interior Γ-hyperideal of S.*

The converse of Proposition 3.1 is not true in general, as shown by the following example below.

Example 3.1. *Let $S = \{a, b, c, d, e, f\}$ and $\Gamma = \{\gamma\}$ with the hyperoperation on S defined by*

γ	a	b	c	d	e	f
a	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{e\}$	$\{f\}$
b	$\{b\}$	$\{c\}$	$\{a\}$	$\{f\}$	$\{d\}$	$\{e\}$
c	$\{c\}$	$\{a\}$	$\{b\}$	$\{e\}$	$\{f\}$	$\{d\}$
d	$\{d\}$	$\{f\}$	$\{e\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
e	$\{e\}$	$\{d\}$	$\{f\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b\}$
f	$\{f\}$	$\{e\}$	$\{d\}$	$\{b, c\}$	$\{a, b\}$	$\{a, c\}$

Then, $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup, where the order relation $\leq$ on S defined by $\leq := \{(x, y) \mid x = y\}$. Let $I = \{a, b\}$. Hence, by routine calculation, we have that I is a weakly almost interior Γ-hyperideal of S. But I is not an almost interior Γ-hyperideal of S, because $(d\Gamma I\Gamma a] \cap I = \emptyset$.

Theorem 3.1. *Let I be a weakly almost interior Γ-hyperideal of ordered Γ-semihypergroup S. If A is any subset of S containing I, then A is also a weakly almost interior Γ-hyperideal of S.*

Proof. Assume that A is a subset of S such that $I \subseteq A$. Let $x \in S$. Then, $(x\Gamma I\Gamma x] \cap I \neq \emptyset$. Thus, $\emptyset \neq (x\Gamma I\Gamma x] \cap I \subseteq (x\Gamma A\Gamma x] \cap A$. It follows that $(x\Gamma A\Gamma x] \cap A \neq \emptyset$. Hence, A is a weakly almost interior Γ-hyperideal of S. □

Corollary 3.1. *Let S be an ordered Γ-semihypergroup. If I_1 and I_2 are weakly almost interior Γ-hyperideals of S, then $I_1 \cup I_2$ is a weakly almost interior Γ-hyperideal of S.*

Example 3.2. *Let $S = \{a, b, c, d, e\}$ and $\Gamma = \{\alpha\}$ be the nonempty sets. Define the hyperoperation as:*

α	a	b	c	d	e
a	$\{d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{d\}$	$\{a, b, d, e\}$
b	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d, e\}$
c	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d, e\}$
d	$\{d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{d\}$	$\{a, b, d, e\}$
e	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d, e\}$	$\{a, b, d\}$	$\{a, b, d, e\}$

Next, we define an order relation $\leq$ on S as:

$$\leq := \{(a, a), (b, b), (c, c), (d, d), (e, e), (a, b), (a, c), (a, e), (b, c), (b, e), (d, b), (d, c), (d, e)\}.$$

Then, $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup. Let $I_1 = \{a, b\}$ and $I_2 = \{d\}$. Verifying that I_1 and I_2 are weakly almost interior Γ-hyperideals of S is a routine process. However, $I_1 \cap I_2$ is not a weakly almost interior Γ-hyperideal of S.

The intersection of any two weakly almost interior Γ-hyperideals of an ordered Γ-semihypergroup S does not necessarily have to be a weakly almost interior Γ-hyperideal of S, as shown by Example 3.2.

Theorem 3.2. *Let S be an ordered Γ-semihypergroup and $|S| > 1$. Then, the following statements are equivalent:*

(i) S has no proper weakly almost interior Γ-hyperideal;

(ii) for every $x \in S$, there exists $a_x \in S$ such that $(a_x\Gamma(S \setminus \{x\})\Gamma a_x] = \{x\}$.

Proof. $(i) \Rightarrow (ii)$ Assume that (i) holds. For any $x \in S$, we have that $S \setminus \{x\}$ is not a weakly almost interior Γ-hyperideal of S. So, there exists $a_x \in S$ such that $(a_x\Gamma(S \setminus \{x\})\Gamma a_x] \cap (S \setminus \{x\}) = \emptyset$. We obtain that

$$(a_x\Gamma(S \setminus \{x\}\Gamma a_x)] \subseteq S \setminus (S \setminus \{x\}) = \{x\}.$$

It turns out that $(a_x\Gamma(S \setminus \{x\})\Gamma a_x] = \{x\}$.

$(ii) \Rightarrow (i)$ Assume that (ii) holds. Let A be any a proper subset of S. Then, $A \subseteq S \setminus \{x\}$ for some $x \in S$. By assumption, there exists $a_x \in S$ such that $(a_x\Gamma(S \setminus \{x\})\Gamma a_x] = \{x\}$. Thus,

$$\begin{aligned}(a_x\Gamma A\Gamma a_x] \cap A &\subseteq (a_x\Gamma(S \setminus \{x\})\Gamma a_x] \cap (S \setminus \{x\})\\ &= \{x\} \cap (S \setminus \{x\}) = \emptyset.\end{aligned}$$

Hence, A is not a weakly almost interior Γ-hyperideal of S. This shows that S has no proper weakly almost interior Γ-hyperideal of S. □

4. Fuzzy weakly almost interior Γ-hyperideals

The concept of fuzzy weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups and some of the relationships between them are discussed in this section.

Definition 4.1. *Let μ be a nonzero fuzzy subset of an ordered Γ-semihypergroup S. Then, μ is called a fuzzy weakly almost interior Γ-hyperideal of S if for every fuzzy point s_α of S, $(s_\alpha \circ \mu \circ s_\alpha] \cap \mu \neq 0$.*

From the Definition 4.1, we obtain that the following remark holds.

Remark 4.1. *Let s_α be any fuzzy point of an ordered Γ-semihypergroup S. Then, $(s_\alpha \circ \mu \circ s_\alpha] \cap \mu \neq 0$ if and only if there exist $x, a \in S$ such that $x \leq s\Gamma a\Gamma s$ and $\mu(x), \mu(a) \neq 0$.*

Theorem 4.1. *Let μ be a fuzzy weakly almost interior Γ-hyperideal of an ordered Γ-semihypergroup S. If λ is a fuzzy subset of S such that $\mu \subseteq \lambda$, then λ is also a fuzzy weakly almost interior Γ-hyperideal of S.*

Proof. Assume that λ is a fuzzy subset of S such that $\mu \subseteq \lambda$. Let s_α be a fuzzy point of S. Then, $(s_\alpha \circ \mu \circ s_\alpha] \cap \mu \neq 0$. Since $\mu \subseteq \lambda$, $0 \neq (s_\alpha \circ \mu \circ s_\alpha] \cap \mu \subseteq (s_\alpha \circ \lambda \circ s_\alpha] \cap \lambda$. Also, $(s_\alpha \circ \lambda \circ s_\alpha] \cap \lambda \neq 0$. Hence, λ is a fuzzy weakly almost interior Γ-hyperideal of S. □

Corollary 4.1. *Let μ and λ be fuzzy weakly almost interior Γ-hyperideals of an ordered Γ-semihypergroup S. Then, $\mu \cup \lambda$ is a fuzzy weakly almost interior Γ-hyperideal of S.*

Example 4.1. *Consider the ordered Γ-semihypergroup $(S, \Gamma, \leq)$ in Example 3.2, we define two fuzzy subsets μ and λ of S by for every $x \in S$,*

$$\mu(x) = \begin{cases} 0.8 & \text{if } x \in \{a, b\}, \\ 0 & \text{otherwise} \end{cases} \quad \text{and } \lambda(x) = \begin{cases} 0.5 & \text{if } x = d, \\ 0 & \text{otherwise.} \end{cases}$$

By routine computations, we find out that μ and λ are fuzzy weakly almost interior Γ-hyperideals of S. However, $\mu \cap \lambda$ is not a fuzzy weakly almost interior Γ-hyperideal of S, because $\mu \cap \lambda = 0$.

From Example 4.1, we know that the intersection of two fuzzy weakly almost interior Γ-hyperideals of an ordered Γ-semihypergroup S need not be a fuzzy weakly almost interior Γ-hyperideal of S.

Theorem 4.2. *Let I be a nonempty subset of an ordered Γ-semihypergroup S. Then, I is a weakly almost interior Γ-hyperideal of S if and only if C_I is a fuzzy weakly almost interior Γ-hyperideal of S.*

Proof. Assume that I is a weakly almost interior Γ-hyperideal of S. Let s_α be any fuzzy point of S. Then, $(s\Gamma I\Gamma s] \cap I \neq \emptyset$. Thus, there exists $a \in S$ such that $a \in (s\Gamma I\Gamma s]$ and $a \in I$. So, $C_I(a) = 1$ and $a \leq s\Gamma x\Gamma s$ for some $x \in I$. Since $x \in I$, $C_I(x) = 1$. It follows that

$$(s_\alpha \circ C_I \circ s_\alpha](a) \geq \min\{s_\alpha(s), C_I(x), s_\alpha(s)\} \neq 0.$$

We obtain that $[(s_\alpha \circ C_I \circ s_\alpha] \cap C_I](a) \neq 0$. Thus, C_I is a fuzzy weakly almost interior Γ-hyperideal of S.

Conversely, assume that C_I is a fuzzy weakly almost interior Γ-hyperideal of S. Let $s \in S$. Choose $t = 1$. Then, $(s_1 \circ C_I \circ s_1] \cap C_I \neq 0$. So, there exist $x, a \in S$ such that $x \leq s\Gamma a\Gamma s$ and $C_I(x), C_I(a) \neq 0$. This implies that $x, a \in I$. Also, $x \in (s\Gamma I\Gamma s]$. Thus, $x \in (s\Gamma I\Gamma s] \cap I$, and then $(s\Gamma I\Gamma s] \cap I \neq \emptyset$. Therefore, I is a weakly almost interior Γ-hyperideal of S. □

Theorem 4.3. *Let μ be a fuzzy subset of an ordered Γ-semihypergroup S. Then, μ is a fuzzy weakly almost interior Γ-hyperideal of S if and only if $supp(\mu)$ is a weakly almost interior Γ-hyperideal of S.*

Proof. Assume that μ is a fuzzy weakly almost interior Γ-hyperideal of S. Let $s \in S$. Choose $t = 1$. So, $(s_1 \circ \mu \circ s_1] \cap \mu \neq 0$. So, there exist $x, a \in S$ such that $x \leq s\Gamma a\Gamma s$ and $\mu(x), \mu(a) \neq 0$. Also, $x, a \in supp(\mu)$. Since $x \leq s\Gamma a\Gamma s$, $x \in (s\Gamma(supp(\mu))\Gamma s]$. It turns out that $x \in (s\Gamma(supp(\mu))\Gamma s] \cap supp(\mu)$, that is, $(s\Gamma(supp(\mu))\Gamma s] \cap supp(\mu) \neq \emptyset$. Hence, $supp(\mu)$ is a weakly almost interior Γ-hyperideal of S.

Conversely, assume that $supp(\mu)$ is a weakly almost interior Γ-hyperideal of S. Let s_α be any fuzzy point of S. Then, $(s\Gamma(supp(\mu))\Gamma s] \cap supp(\mu) \neq \emptyset$. Thus, there exists $x \in S$ such that $x \in (s\Gamma(supp(\mu))\Gamma s]$ and $x \in supp(\mu)$. So, $x \leq s\Gamma a\Gamma s$ for some $a \in supp(\mu)$. This means that $\mu(x), \mu(a) \neq 0$. We have that $(s_\alpha \circ \mu \circ s_\alpha] \cap \mu \neq 0$. Therefore, μ is a fuzzy weakly almost interior Γ-hyperideal of S. □

Let S be an ordered Γ-semihypergroup. A weakly almost interior Γ-hyperideal I of S is called *minimal* if for any weakly almost interior Γ-hyperideal A of S such that $A \subseteq I$ implies that $A = I$.

Definition 4.2. *Let S be an ordered Γ-semihypergroup. A fuzzy weakly almost interior Γ-hyperideal μ of S is called minimal if for any fuzzy weakly almost interior Γ-hyperideal λ of S such that $\lambda \subseteq \mu$ implies that $supp(\lambda) = supp(\mu)$.*

Now, the relationship between minimal weakly almost interior Γ-hyperideals and minimal fuzzy weakly almost interior Γ-hyperideals in ordered Γ-semihypergroups is then briefly examined.

Theorem 4.4. *Let S be an ordered Γ-semihypergroup, and I be a nonempty subset of S. Then, I is a minimal weakly almost almost interior Γ-hyperideal of S if and only if C_I is a minimal fuzzy weakly almost interior Γ-hyperideal of S.*

Proof. Assume that I is a minimal weakly almost interior Γ-hyperideal of S. By Theorem 4.2, C_I is a fuzzy weakly almost interior Γ-hyperideal of S. Let λ be any fuzzy weakly almost interior Γ-hyperideal of S such that $\lambda \subseteq C_I$. By Lemma 2.2 and Theorem 4.3, we have that $supp(\lambda)$ is a weakly almost interior Γ-hyperideal of S such that $supp(\lambda) \subseteq supp(C_I)$. Since I is minimal, $supp(\lambda) = I = supp(C_I)$. Hence, C_I is a minimal fuzzy weakly almost interior Γ-hyperideal of S.

Conversely, assume that C_I is a minimal fuzzy weakly almost interior Γ-hyperideal of S. Thus, I is a weakly almost Γ-hyperideal of S by Theorem 4.2. Now, let A be any weakly almost interior Γ-hyperideal of S such that $A \subseteq I$. Then, C_A is a fuzzy weakly almost interior Γ-hyperideal of S such that $C_A \subseteq C_I$. Since C_I is minimal and by Lemma 2.2, we have that $A = supp(C_A) = supp(C_I) = I$. Therefore, I is a minimal weakly almost interior Γ-hyperideal of S. $\square$

The following corollary can be achieved by Theorem 4.2 and Theorem 4.3.

Corollary 4.2. *Let S be an ordered Γ-semihypergroup. Then, S has no proper weakly almost interior Γ-hyperideal if and only if for every fuzzy weakly almost interior Γ-hyperideal μ of S, $supp(\mu) = S$.*

Let S be an ordered Γ-semihypergroup and P be a weakly almost interior Γ-hyperideal of S. Then: (i) P is said to be *prime* if for any weakly almost interior Γ-hyperideals A and B of S such that $(A\Gamma B] \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$; (ii) P is said to be *semiprime* if for any weakly almost interior Γ-hyperideal A of S such that $(A\Gamma A] \subseteq P$ implies that $A \subseteq P$; (iii) P is said to be *strongly prime* if for any weakly almost interior Γ-hyperideals A and B of S such that $(A\Gamma B] \cap (B\Gamma A] \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 4.3. *Let μ be a fuzzy weakly almost interior Γ-hyperideal of an ordered Γ-semihypergroup S. Then, μ is said to be a fuzzy prime weakly almost interior Γ-hyperideal of S if for any fuzzy weakly almost interior Γ-hyperideals λ and ν of S such that $\lambda \circ \nu \subseteq \mu$ implies that $\lambda \subseteq \mu$ or $\nu \subseteq \mu$.*

Definition 4.4. *Let μ be a fuzzy weakly almost interior Γ-hyperideal of an ordered Γ-semihypergroup S. Then, μ is said to be a fuzzy semiprime weakly almost interior Γ-hyperideal of S if for any fuzzy weakly almost interior Γ-hyperideal λ of S such that $\lambda \circ \lambda \subseteq \mu$ implies that $\lambda \subseteq \mu$.*

Definition 4.5. *Let μ be a fuzzy weakly almost interior Γ-hyperideal of an ordered Γ-semihypergroup S. Then, μ is said to be a fuzzy strongly prime weakly almost interior Γ-hyperideal of S if for any fuzzy weakly almost interior Γ-hyperideals λ and ν of S such that $(\lambda \circ \nu) \cap (\nu \circ \lambda) \subseteq \mu$ implies that $\lambda \subseteq \mu$ or $\nu \subseteq \mu$.*

It is obvious that every fuzzy strongly prime weakly almost interior Γ-hyperideal of an ordered Γ-semihypergroup is a fuzzy prime weakly almost interior Γ-hyperideal, and every fuzzy prime weakly almost interior Γ-hyperideal of an ordered Γ-semihypergroup is a fuzzy semiprime weakly almost interior Γ-hyperideal.

Finally, we consider the connections between strongly prime (resp., prime, semiprime) weakly almost interior Γ-hyperideals and their fuzzifications in ordered Γ-semihypergroups.

Theorem 4.5. *Let S be an ordered Γ-semihypergroup and P be a nonempty subset of S. Then, P is a strongly prime weakly almost interior Γ-hyperideal of S if and only if C_P is a fuzzy strongly prime weakly almost interior Γ-hyperideal of S.*

Proof. Assume that P is a strongly prime weakly almost interior Γ-hyperideal of S. Also, C_P is a fuzzy weakly almost interior Γ-hyperideal of S by Theorem 4.2. Let λ and ν be any two fuzzy weakly almost interior Γ-hyperideals of S such that $(\lambda \circ \nu) \cap (\nu \circ \lambda) \subseteq C_P$. Suppose that $\lambda \nsubseteq C_P$ and $\nu \nsubseteq C_P$. Thus, there exist $x, y \in S$ such that $\lambda(x) \neq 0$ and $\nu(y) \neq 0$, but $C_P(x) = 0$ and $C_P(y) = 0$. So, $x, y \notin P$. By using Theorem 4.3, we have that $supp(\lambda)$ and $supp(\nu)$ are weakly almost interior Γ-hyperideals of S such that $x \in supp(\lambda)$ and $y \in supp(\nu)$. We obtain that, $supp(\lambda) \nsubseteq P$ and $supp(\nu) \nsubseteq P$. By assumption, $((supp(\lambda))\Gamma(supp(\nu))] \cap ((supp(\nu))\Gamma(supp(\lambda))] \nsubseteq P$. Then, there exists $t \in ((supp(\lambda))\Gamma(supp(\nu))] \cap ((supp(\nu))\Gamma(supp(\lambda))]$, but $t \notin P$. It follows that $C_P(t) = 0$, and then $[(\lambda \circ \nu) \cap (\nu \circ \lambda)](t) = 0$. Since $t \in ((supp(\lambda))\Gamma(supp(\nu))]$ and $t \in ((supp(\nu))\Gamma(supp(\lambda))]$, we have that $t \leq a_1\Gamma b_1$ and $t \leq b_2\Gamma a_2$ for some $a_1, a_2 \in supp(\lambda)$ and $b_1, b_2 \in supp(\nu)$. It turns out that

$$(\lambda \circ \nu)(t) = \sup_{t \leq a_1\Gamma b_1} [\min\{\lambda(a_1), \nu(b_1)\}] \neq 0 \text{ and } (\nu \circ \lambda)(t) = \sup_{t \leq b_2\Gamma a_2} [\min\{\nu(b_2), \lambda(a_2)\}] \neq 0.$$

This implies that $[(\lambda \circ \nu) \cap (\nu \circ \lambda)](t) \neq 0$, as a contradiction. So, $\lambda \subseteq C_P$ or $\nu \subseteq C_P$. This shows that C_P is a fuzzy strongly prime weakly almost interior Γ-hyperideal of S.

Conversely, assume that C_P is a fuzzy strongly prime weakly almost interior Γ-hyperideal of S. Then, P is a weakly almost interior Γ-hyperideal of S by Theorem 4.2. Let A and B be any two weakly almost interior Γ-hyperideals of S such that $(A\Gamma B] \cap (B\Gamma A] \subseteq P$. By using Lemma 2.2 and

Lemma 2.3, it follows that

$$(C_A \circ C_B) \cap (C_B \circ C_A) = C_{(A\Gamma B]} \cap C_{(B\Gamma A]} = C_{(A\Gamma B]\cap(B\Gamma A]} \subseteq C_P.$$

By the hypothesis, $C_A \subseteq C_P$ or $C_B \subseteq C_P$. It follows that $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime weakly almost interior Γ-hyperideal of S. □

Theorem 4.6. *Let P be a nonempty subset of an ordered Γ-semihypergroup S. Then, P is a prime weakly almost interior Γ-hyperideal of S if and only if C_P is a fuzzy prime weakly almost interior Γ-hyperideal of S.*

Proof. Assume that P is a prime weakly almost interior Γ-hyperideal of S. By using Theorem 4.2, we obtain that C_P is a fuzzy weakly almost interior Γ-hyperideal of S. Let λ and ν be any two fuzzy weakly almost interior Γ-hyperideals of S such that $\lambda\circ\nu \subseteq C_P$. Suppose that $\lambda \nsubseteq C_P$ and $\nu \nsubseteq C_P$. Then, there exist $x, y \in S$ such that $\lambda(x) \neq 0$ and $\nu(y) \neq 0$, while $C_P(x) = 0$ and $C_P(y) = 0$. So, $x \in supp(\lambda)$, $y \in supp(\nu)$ with $x, y \notin P$. By Theorem 4.3, we have that $supp(\lambda)$ and $supp(\nu)$ are weakly almost interior Γ-hyperideals of S. This implies that $supp(\lambda) \nsubseteq P$ and $supp(\nu) \nsubseteq P$. By assumption, it follows that $((supp(\lambda)\Gamma(supp(\nu)))] \nsubseteq P$. Also, there exists $t \in ((supp(\lambda)\Gamma(supp(\nu)))]$ such that $t \notin P$. This means that $C_P(t) = 0$. It turns out that $(\lambda \circ \nu)(t) = 0$, because $\lambda \circ \nu \subseteq C_P$. Since $t \in ((supp(\lambda)\Gamma(supp(\nu)))]$, $t \leq a\Gamma b$ for some $a \in supp(\lambda)$ and $b \in supp(\nu)$. Thus,

$$(\lambda \circ \nu)(t) = \sup_{t\leq a\Gamma b} [\min\{\lambda(a), \nu(b)\}] \neq 0.$$

This is a contradiction to the fact that $(\lambda \circ \nu)(t) = 0$. This shows that $\lambda \subseteq C_P$ or $\nu \subseteq C_P$. Hence, C_P is a fuzzy prime weakly almost interior Γ-hyperideal of S.

Conversely, assume that C_P is a fuzzy prime weakly almost interior Γ-hyperideal of S. By Theorem 4.2, P is a weakly almost Γ-hyperideal of S. Let A and B be any weakly almost interior Γ-hyperideals of S such that $(A\Gamma B] \subseteq P$. By Lemma 2.2 and Lemma 2.3, it follows that $C_A \circ C_B = C_{(A\Gamma B]} \subseteq C_P$. By the given assumption, $C_A \subseteq C_P$ or $C_B \subseteq C_P$. This implies that, $A \subseteq P$ or $B \subseteq P$. Therefore, P is a prime weakly almost interior Γ-hyperideal of S. □

Theorem 4.7. *Let S be an ordered Γ-semihypergroup and P be a nonempty subset of S. Then, P is a semiprime weakly almost interior Γ-hyperideal of S if and only if C_P is a fuzzy semiprime weakly almost interior Γ-hyperideal of S.*

Proof. Assume that P is a semiprime weakly almost interior Γ-hyperideal of S. By Theorem 4.2, we obtain that C_P is a fuzzy weakly almost Γ-hyperideal of S. Let λ be any fuzzy weakly almost interior Γ-hyperideal of S such that $\lambda \circ \lambda \subseteq C_P$. Suppose that $\lambda \nsubseteq C_P$. So, there exists $x \in S$ such that $\lambda(x) \neq 0$ and $C_P(x) = 0$. Also, $x \in supp(\lambda)$ and $x \notin P$. By Theorem 4.3, $supp(\lambda)$ is a weakly almost interior Γ-hyperideal of S where $supp(\lambda) \nsubseteq P$. By assumption, $((supp(\lambda)\Gamma(supp(\lambda)))] \nsubseteq P$. Thus, there exists $t \in S$ such that $t \in ((supp(\lambda)\Gamma(supp(\lambda)))]$, but $t \notin P$. This implies that $C_P(t) = 0$. It follows that $(\lambda\circ\lambda)(t)=0$, because $\lambda\circ\lambda\subseteq C_P$. Since $t\in((supp(\lambda)\Gamma(supp(\lambda)))]$, $t\leq a\Gamma b$ for some $a,b\in supp(\lambda)$. It turns out that $(\lambda\circ\lambda)(t)=\sup\limits_{t\leq a\Gamma b}[\min\{\lambda(a),\lambda(b)\}]\neq 0$, which is a contradiction. Hence, $\lambda\subseteq C_P$. Therefore, C_P is a fuzzy semiprime weakly almost Γ-hyperideal of S.

Conversely, assume that C_P is a fuzzy semiprime weakly almost Γ-hyperideal of S. It follows that P is a weakly almost interior Γ-hyperideal of S by Theorem 4.2. Let A be a weakly almost interior Γ-hyperideal of S such that $(A\Gamma A]\subseteq P$. By using Lemma 2.2 and Lemma 2.3, we have that $C_A\circ C_A=C_{(A\Gamma A]}\subseteq C_P$. Since C_P is semiprime, $C_A\subseteq C_P$. It follows that $A\subseteq P$. This shows that P is a semiprime weakly interior Γ-hyperideal of S. □

5. Conclusions

In 2021, Rao et al. [14] introduced the concept of almost interior Γ-hyperideals as a generalization of interior Γ-hyperideals of ordered Γ-semihypergroups. In this paper, we introduced the notion of weakly almost interior Γ-hyperideals of ordered Γ-semihypergroups which is a generalization of almost interior Γ-hyperideals. Next, we shown that the union of (fuzzy) weakly almost interior Γ-hyperideals is also a (fuzzy) weakly almost interior Γ-hyperideal, but the intersection of them need not to be a (fuzzy) weakly almost interior Γ-hyperideal in ordered Γ-semihypergroups. Then, we characterized the ordered Γ-semihypergroups having no proper weakly almost interior Γ-hyperideal. Finally, we discussed the connections between weakly almost interior Γ-hyperideals and their fuzzification in ordered Γ-semihypergroups. In our future study, we plan to investigate other kinds of almost Γ-hyperideals and their fuzzifications in ordered Γ-semihypergroups or other algebraic structures.

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