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Estimation of Finite Population Mean by Utilizing the Auxiliary and Square of the Auxiliary Information

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Abstract. This article fundamentally aims at the proposition of new family of estimators using auxiliary information to assist the estimation of finite population mean of the study variable. The objectives are achieved by devising dual use of supplementary information through straightforward manner. The additional information is injected in mean estimating procedure by considering squared values of auxiliary variable. The utility of the proposed scheme is substantiated by providing rigorous comparative account of the newly materialized structure with the well celebrated existing family of Grover and Kaur (2014). The contemporary advents of the new family are documented throughout the article.

1. Introduction

The utility of auxiliary information to improve the effectiveness of estimation procedures estimating the attributes of population under study is well cherished. The documented realizations of the advents of employing auxiliary information while estimating study parameters can be traced back in eighteenth century France. A keen review of literature substantiate that Pierre-Simon Laplace documented the early advocacy of using supplementary information to assist the estimation of population of country.

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He advised "The register of births, which are kept with care in order to assure the condition of the citizens, can serve to determine the population of great empire without resorting a census of its inhabitants. But for this it is necessary to know the ratio of population to annual the birth." [16] Later on, the applicability of the additional information to enhance the efficiency of underlying estimation procedures was materialized by exploiting the correlation structure existent to govern both study variable and auxiliary variable, [3]. Over the time, streams of propositions devising novel mechanism expounding efficient estimation of study parameter can be in available literature. For a comprehensive review of ongoing research efforts, one may consult to [4], [5], [8], [10], [11], [12], [13], [14], [21], and [23]. It is noteworthy that these efforts comprehends the advances on two fronts, (i) – competent use of auxiliary information and (ii) – introducing novel functional forms incorporating additional information in estimation synergy. In adjacent past, [5] instigated the use of exponent-based formation to inject supplementary information and thus proposed generalized family of estimators encapsulating numerous exiting specifications. The optimality of the proposed scheme was delineated through rigorous empirical evaluation. Motivated by the ongoing proceedings, this research targets the afore-mentioned both fronts, simultaneously. We propose new family of estimators capable of entertaining dual use of auxiliary information by the application of efficient exponent-based functional formation. The comparative performance of the newly developed mechanism is documented with respect to promising novel family of [5] estimators. The empirical evaluations are conducted by the consideration of numerous data sets from statistics and allied research fields. This article is mainly divided into seven major parts. Section 2 briefs about the preliminaries extensively used in this research whereas section 3 summarizes the contemporary methods. Section 4 is dedicated to expound the proposed scheme, whereas section 5 documents the efficiency conditions. Section 6 explore the empirical performance of the competing techniques. Lastly, section 7 comprehends the investigation by offering highlights of the research along with few prospective research venues.

2. Notation and symbols

Let V be a finite population of N units, such as $V = \{V_1, V_2, \dots, V_N\}$. We draw a sample of size n from the population through simple random sampling with out replacement scheme. Let Y_i, X_i, and U_i are study, auxiliary variable, and squared values of auxiliary variable, respectively, for the ith $(i = 1, 2, \dots, N)$ unit of the population.

Let, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, and $\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$ are sample means of the study, auxiliary variable, and squared values of the auxiliary variable respectively.

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$, and $\bar{U} = \frac{1}{N}\sum_{i=1}^{N} U_i$, are population means of the study, auxiliary variable, and squared values of the auxiliary variable respectively. On these grounds sample variances of study, auxiliary variable, and squared values of auxiliary variable are defined as $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$, $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, and $s_u^2 = \frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2$.

Further more, let us define coefficients of variation of X, Y, and U as C_x, C_y, C_u, where $C_y = s_y/\bar{Y}$, $C_x = s_x/\bar{X}$, and $C_u = s_u/\bar{U}$.

We now define error terms as $e_0 = (\bar{y} - \bar{Y})/\bar{Y}$, $e_1 = (\bar{x} - \bar{X})/\bar{X}$, $e_2 = (\bar{u} - \bar{U})/\bar{U}$, such that $E(e_i) = 0$, $i = 0, 1, 2$. $E(e_0^2) = \lambda C_y^2$, $E(e_1^2) = \lambda C_x^2$, $E(e_2^2) = \lambda C_u^2$, where $\lambda = (\frac{1}{n} - \frac{1}{N})$ commonly known as sample fraction.

In procession the error covariances are derived as $E(e_0e_1) = \lambda C_y C_x \rho_{yx}$, $E(e_0e_2) = \lambda C_y C_u \rho_{yu}$, $E(e_1e_2) = \lambda C_x C_u \rho_{xu}$, where ρ_{yx}, ρ_{yu}, and ρ_{xu}, represents sample correlation coefficients defined as $\rho_{yx} = \frac{S_{yx}}{S_y S_x}$, $\rho_{yu} = \frac{S_{yu}}{S_y S_u}$, and $\rho_{xu} = \frac{S_{xu}}{S_x S_u}$.

3. Some Existing Estimators

(i) The typically independent suggest estimator is $\bar{Y}$ with variance

$$\mathrm{Var}(\bar{y}) = \lambda \bar{Y}^2 C_y^2. \tag{3.1}$$

(ii) [3] and [17] proposed traditional ratio type and product type estimators, $\hat{\bar{Y}}_R$ and $\hat{\bar{Y}}_P$, respectively, given by

$$\hat{\bar{Y}}_R = \bar{y}(\frac{\bar{X}}{\bar{x}}), \tag{3.2}$$

$$\hat{\bar{Y}}_P = \bar{y}(\frac{\bar{x}}{\bar{X}}). \tag{3.3}$$

It is a famous truth that, irrespective of the biasedness, the classical ratio and product estimator, $\hat{\bar{Y}}_R$ and $\hat{\bar{Y}}_P$, is more accurate than the mean per unit estimator $\bar{Y}$ if there exist a high positive and negative correlation between y and x, i.e $\rho_{yx} > C_x/2C_y$ and $\rho_{yx} < -C_x/2C_y$.

The MSEs of $\hat{\bar{Y}}_R$ and $\hat{\bar{Y}}_P$, respectively, given by

$$\mathrm{MSE}(\hat{\bar{Y}}_R) \cong \lambda \bar{Y}^2 (C_y^2 + C_x^2 - 2C_y C_x \rho_{yx}) \tag{3.4}$$

$$\mathrm{MSE}(\hat{\bar{Y}}_P) \cong \lambda \bar{Y}^2 (C_y^2 + C_x^2 + 2C_y C_x \rho_{yx}). \tag{3.5}$$

Several authors have proposed a few converted ratio-kind estimators for estimating the finite population mean with the aid of using the use of auxiliary information. Some suitable research on this path include [2], [23], [24], [25], and many others authors. In a latest study, [15] Proposed a standard elegance of estimators $\hat{\bar{Y}}_K$ that consists of a few tailored ratio-kind estimators, given by

$$\hat{\bar{Y}}_K \cong \bar{y}\left\{\frac{a\bar{X} + b}{\alpha(a\bar{X} + b)) + (1 - \alpha)(a\bar{X} + b)}\right\}^g, . \tag{3.6}$$

$$\hat{\hat{Y}}_K \cong \bar{y}\left\{\frac{a\bar{X}+b}{\alpha(a\bar{X}+b))+(1-\alpha)(a\bar{X}+b)}\right\}^g, \tag{3.7}$$

In which a($\neq 0$) and b are known recognized elements of any recognized population parameters, such as C_x the coefficient of variation; ρ_{yx} the correlation between y and x; $\beta_{1(x)}$ the coefficient of skewness and so on.
The minimal MSE of $\hat{\hat{Y}}_K$ on the top of the line cost of ($\alpha\vartheta$g), where $\vartheta = \frac{a\bar{X}}{a\bar{X}+b}$, is given by

$$\text{MSE}_{min}(\hat{\hat{Y}}_K) \cong \lambda\bar{Y}^2C_y^2(1-\rho_{yx}^2) \tag{3.8}$$

(iii) The usual difference estimator $\hat{\hat{Y}}_D$ is

$$\hat{\hat{Y}}_D = \bar{y} + t_1(\bar{X}-\bar{x}), \tag{3.9}$$

In which t_1 is an unknown elements. It is straightforward to expose that $\hat{\hat{Y}}_D$ is unbiased. The minimal variance of $\hat{\hat{Y}}_D$ on the top of the line fee of t_1, that is, $t_{1(opt)} = \rho_{yx}(S_y/S_x)$, is given by

$$\text{Var}_{min}(\hat{\hat{Y}}_D) \cong \lambda\bar{Y}^2C_y^2(1-\rho_{yx}^2), \tag{3.10}$$

which is same to the variance of the classical regression estimator $\hat{\hat{Y}}_{lr} = \bar{y} + b(\bar{X}-\bar{x})$, where b is the slope estimator of the population regression coefficient $\beta = t_{1(opt)}$. The difference estimator $\hat{\hat{Y}}_D$ is always better perform than the ratio type $\hat{\hat{Y}}_R$ and product type $\hat{\hat{Y}}_P$ estimators when estimating $\bar{Y}$.
(iv) [19] proposed an improved difference type estimatorof $\hat{\hat{Y}}_D$, given by

$$\hat{\hat{Y}}_{R,D} = t_2\bar{y} + t_3(\bar{X}-\bar{x}), \tag{3.11}$$

where t_2 and t_3 are selected quantities. The minimum MSE of $\hat{\hat{Y}}_{R,D}$ at the optimum values,

$$t_{2(opt)} = \frac{1}{1+\lambda C_y^2(1-\rho_{yx}^2)}, \tag{3.12}$$

and

$$t_{3(opt)} = \frac{\bar{Y}C_y\rho_{yx}}{\bar{X}C_x[1+\lambda C_y^2(1-\rho_{yx}^2)]}, \tag{3.13}$$

is given by

$$\text{MSE}_{min}(\hat{\hat{Y}}_{R,D}) \cong \frac{\lambda\bar{Y}^2C_y^2(1-\rho_{yx}^2)}{1+\lambda C_y^2(1-\rho_{yx}^2)}. \tag{3.14}$$

the above (11), it is shown that the $\hat{\hat{Y}}_{R,D}$ is better perform than $\hat{\hat{Y}}_D$, i.e,

$$\text{MSE}_{min}(\hat{\hat{Y}}_{R,D}) \cong \text{Var}_{min}(\hat{\hat{Y}}_D) - \frac{\lambda\bar{Y}^2C_y^4(1-\rho_{yx}^2)^2}{1+\lambda C_y^2(1-\rho_{yx}^2)}. \tag{3.15}$$

(v) [1] proposed a ratio and product type exponential estimators, given by

$$\hat{\hat{Y}}_{BT,R} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right), \tag{3.16}$$

$$\hat{\hat{Y}}_{BT,P} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right). \tag{3.17}$$

The MSEs of $\hat{\hat{Y}}_{BT,R}$ and $\hat{\hat{Y}}_{BT,P}$, respectively, are given by

$$\text{MSE}_{min}(\hat{\hat{Y}}_{BT,R}) \cong \frac{\lambda\bar{Y}^2}{4}\Big(4C_y^2 + C_x^2 - 4\rho_{yx}C_yC_x\Big), \tag{3.18}$$

$$\text{MSE}_{min}(\hat{\hat{Y}}_{BT,P}) \cong \frac{\lambda\bar{Y}^2}{4}\Big(4C_y^2 + C_x^2 + 4\rho_{yx}C_yC_x\Big). \tag{3.19}$$

Following the work in [1], [22] proposed a generalized ratio type exponential estimator,

$$\hat{\hat{Y}}_S = \bar{y}\exp(\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}). \tag{3.20}$$

The minimum MSE of $\hat{\hat{Y}}_S$ turns out to equivalent to $\text{Var}_{min}(\hat{\hat{Y}}_D)$, i.e, $\text{MSE}_{min}(\hat{\hat{Y}}_S) \cong \lambda\bar{Y}C_y^2(1-\rho_{yx}^2)$.
(vi) Based on the estimator [1], [19], [20], [22] proposed a estimator, given by

$$\hat{\hat{Y}}_{SG} = \{t_4\bar{y} + t_5(\bar{X}-\bar{x})\}\exp\Big(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2N\bar{X}}\Big), \tag{3.21}$$

where t_4 and t_5 are suitably chosen constant.
Following these work, [4] proposed related estimator by combining [1] and [19]

$$\hat{\hat{Y}}_{SG} = \{t_6\bar{y} + t_7(\bar{X}-\bar{x})\}\exp(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}), \tag{3.22}$$

in which t_6 and t_7 are elements.
In a recant study, proposed a estimators, given by

$$\hat{\hat{Y}}_{GK,G} = \{t_8\bar{y} + t_9(\bar{X}-\bar{x})\}\exp(\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}), \tag{3.23}$$

in which t_8 and t_9 are elements. Note that $\hat{\hat{Y}}_{GK,G}$ contains the estimators given known in [4] and [9].
The minimum MSE of $\hat{\hat{Y}}_{GK,G}$ at the optimum values,

$$t_{8(opt)} = \frac{8-\lambda\vartheta^2C_x^2}{8\{1+\lambda C_y^2(1-\rho_{yx}^2)\}},$$

and

$$t_{9(opt)} = \frac{\bar{Y}[\lambda\vartheta^3C_x^3 + 8C_y\rho_{yx} - \lambda\vartheta^2C_x^2C_y^2\rho_{yx} - 4\vartheta C_x\{1-\lambda C_y^2(1-\rho_{yx}^2)\}]}{8\bar{X}C_x\{1+\lambda C_y^2(1-\rho_{yx}^2)\}}.$$

is given by

$$\text{MSE}_{min}(\hat{\hat{Y}}_{GK,G}) \cong \frac{\lambda\bar{Y}^2\Big\{64C_y^2(1-\rho_{yx}^2) - \lambda\vartheta^4C_x^4 - 16\lambda\vartheta^2C_x^2C_y^2(1-\rho_{yx}^2)\Big\}}{64\Big\{1+\lambda C_y^2(1-\rho_{yx}^2)\Big\}}. \tag{3.24}$$

The above equation can be written as, given by

$$\text{MSE}_{min}(\hat{\hat{Y}}_{GK,G}) \cong \text{Var}_{min}(\hat{\hat{Y}}_D) - V_1. \tag{3.25}$$

Where $V_1 = \frac{\lambda^2\bar{Y}^2\{\vartheta^2C_x^2+8C_y^2(1-\rho_{yx}^2)\}^2}{64\{1+\lambda C_y^2(1-\rho_{yx}^2)\}}$

4. Proposed Estimator

It is Well-recognized that the usage of auxiliary variables increase the precision of an estimator each the estimation level and at the designing level. In many surveys, the auxiliary records is broadly speaking to be had text-color the sampling design or frame. The concept is that if there exists the ideal amount of correlation among the examine and auxiliary variables, the squared values of the auxiliary variables also are correlated with the values of the examine variable. Thus, the squared auxiliary variable (this is the squared values of auxiliary variable) may be taken into consideration a brand new auxiliary variable, and this greater records may also assist us to growth the performance of an estimator. By those notions, we endorse an advanced estimator of the finite populace mean. The proposed estimator consists of the greater records withinside the shape of an auxiliary variable and withinside the shape of the squared cost of the auxiliary variable.
Following [4], [5] and [20], we suggest a exponential type estimator $\hat{\hat{Y}}_{pr}$, given by

$$\hat{\hat{Y}}_{Pr} = \{t_{10}\bar{y} + t_{11}(\bar{X} - \bar{x}) + t_{12}(\bar{U} - \bar{u})\} \exp\left(\frac{a(\bar{X} - \bar{x})}{a(\bar{X} + \bar{x}) + 2b}\right), \tag{4.1}$$

where t_{10}, t_{11}, and t_{12} are suitably constants, which will be determined later. Where a and b are explained in Table1.
$\hat{\hat{Y}}_{Pr}$ also rewriting as

$$\hat{\hat{Y}}_{Pr} = \{t_{10}\bar{Y}(1 + e_0) - t_{11}\bar{X}e_1 - t_{12}\bar{U}e_2\}\Big\{1 - \frac{\vartheta e_1}{2} + \frac{3\vartheta^2 e_1^2}{8} + \dots\Big\}. \tag{4.2}$$

By expending (25) and upto two degree of approximation in e_i, we can write

$$(\hat{\hat{Y}}_{Pr} - \bar{Y}) = -\bar{Y} + t_{10}\bar{Y} + t_{10}\bar{Y}e_0 - \frac{1}{2}t_{10}\vartheta\bar{Y}e_1 - t_{11}\bar{X}e_1 - t_{12}\bar{U}e_2 - -\frac{1}{2}t_{10}\vartheta\bar{Y}e_0e_1 + \frac{3}{8}t_{10}\vartheta^2\bar{Y}e_1^2 + \frac{1}{2}t_{11}\vartheta\bar{X}e_1^2 + \frac{1}{2}t_{12}\vartheta\bar{U}e_2.$$

From (26), the bias and MSE of $\hat{\hat{Y}}_{Pr}$ up to first degree of approximation are, respectively, given by

$$\text{Bias}(\hat{\hat{Y}}_{Pr}) = \frac{1}{8}\left[-8\bar{Y} + 4\lambda\vartheta C_x(t_{11}\bar{X}C_x + t_{12}\bar{U}C_u\rho_{xu}) + t_{10}\bar{Y}\{8 + \lambda\vartheta C_x(3\vartheta C_x - 4C_y\rho_{yx})\}\right], \tag{4.3}$$

$$\begin{aligned}\text{MSE}_{min}(\hat{\hat{Y}}_{Pr}) \cong{}& \bar{Y}^2 + t_{11}\lambda\bar{X}C_x^2(-\bar{Y}\vartheta + t_{11}\bar{X}) + t_{12}^2\lambda\bar{U}^2C_u^2 + t_{12}\lambda\bar{U}C_xC_u\rho_{xu}(-\bar{Y}\vartheta + 2t_{11}\bar{X}) \\ &+ \bar{Y}^2t_{10}^2\left[1 + \lambda\left\{C_y^2 + \vartheta C_x(\vartheta C_x - 2C_y\rho_{yx})\right\}\right] + \frac{1}{4}t_{10}\bar{Y}\Big\{-8\bar{Y} + \lambda C_x\Big\{\vartheta C_x(-3\vartheta\bar{Y} \\ &+ 8t_{11}\bar{X}) + 8t_{12}\bar{U}C_u\rho_{xu} + 4C_y\rho_{yx}(\bar{Y}\vartheta - 2t_{11}\bar{X})\Big\} - 8t_{12}\lambda\bar{U}C_yC_u\rho_{yu}\Big\}.\end{aligned}$$

The values of t_{10}, t_{11} and t_{12} obtained from (28) are, respectively, given by

$$t_{10(0pt)} = \frac{8 - \lambda\vartheta^2C_x^2}{8\{1 + \lambda C_y^2(1 - \varphi_{yxu}^2)\}} t_{11(opt)} = \frac{\bar{Y}\begin{bmatrix}\lambda\vartheta C_x^3(-1 + \rho_{xu}^2) + (-8C_y + \lambda\vartheta^2C_x^2C_y)(\rho_{yx} - \rho_{yu}\rho_{xu}) \\ +4\vartheta C_x(-1 + \rho_{xu}^2) - 1 + \lambda C_y^2(1 - \varphi_{yxu}^2)\end{bmatrix}}{8\bar{X}C_x(-1 + \rho_{xu}^2)\{1 + \lambda C_y^2(1 - \varphi_{yxu}^2)\}}$$

and

$$t_{12(0pt)} = \frac{\bar{Y}(8-\lambda\vartheta^2 C_x^2)C_y(\rho_{yx}\rho_{xu}-\rho_{yu})}{8\bar{U}C_u(-1+\rho_{xu}^2)\{1+\lambda C_y^2(1-\varphi_{yxu}^2)\}},$$

where $\varphi_{yxu}^2 = \frac{\rho_{yx}^2+\rho_{yu}^2-2\rho_{yx}\rho_{yu}\rho_{xu}}{1-\rho_{xu}^2}$.

putting the above obtained values of t_{10}, t_{11}, and t_{12} in (28), and after a few simplifications, we get the minimal MSE of $\hat{\bar{Y}}_{Pr}$, given by

$$\mathrm{MSE}_{min}(\hat{\bar{Y}}_{Pr}) \cong \frac{\lambda\bar{Y}^2\left\{64C_y^2(1-\varphi_{yxu}^2)-\lambda\vartheta^4C_x^4-16\lambda\vartheta C_y^2C_x^2(1-\varphi_{yxu}^2)\right\}}{64\{1+\lambda C_y^2(1-\varphi_{yxu}^2)\}}. \tag{4.4}$$

It can be shown that proposed estimator $\mathrm{MSE}_{min}(\hat{\bar{Y}}_{Pr})$ is always better perform then the difference estimator $\mathrm{Var}_{min}(\hat{\bar{Y}}_D)$, i.e.,

$$\mathrm{MSE}_{min}(\hat{\bar{Y}}_{Pr}) \cong \mathrm{Var}_{min}(\hat{\bar{Y}}_D) - \{V_1+V_2\}, \tag{4.5}$$

where V_1 is defined as before and V_2, is given by

$$V_2 = \frac{\lambda\bar{Y}^2C_y^2(\rho_{yu}-\rho_{yx}\rho_{xu})^2(-8+\lambda\vartheta^2C_x^2)^2}{64(1-\rho_{xu}^2)\{1+\lambda C_y^2(1-\rho_{yx}^2)\}\{1+\lambda C_y^2(1-\varphi_{yxu}^2)\}}.$$

Table 1. Some possible members of the suggested family of estimators

S.No	a	b	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
1	1	C_x	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
2	1	$\beta_{2(x)}$	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
3	$\beta_{2(x)}$	C_x	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
4	C_x	$\beta_{2(x)}$	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
5	1	ρ_{yx}	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
6	C_x	ρ_{yx}	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
7	ρ_{yx}	C_x	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
8	$\beta_{2(x)}$	ρ_{yx}	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
9	ρ_{yx}	$\beta_{2(x)}$	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$
10	1	$N\bar{X}$	$\hat{\bar{Y}}_{GK,G}$	$\hat{\bar{Y}}_{Pr}$

Table 2. Some possible members of the suggested family of estimators

S.No	a	b	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
1	1	C_x	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
2	1	$\beta_{2(x)}$	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
3	$\beta_{2(x)}$	C_x	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
4	C_x	$\beta_{2(x)}$	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
5	1	ρ_{yx}	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
6	C_x	ρ_{yx}	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
7	ρ_{yx}	C_x	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
8	$\beta_{2(x)}$	ρ_{yx}	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
9	ρ_{yx}	$\beta_{2(x)}$	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$
10	1	$N\bar{X}$	$\hat{\hat{Y}}_{GK,G}$	$\hat{\hat{Y}}_{Pr}$

5. Conclusion

This research persuaded the enhancement of efficiency of estimation procedures involving the estimation of finite population mean. The objectives are aimed by devising new class of estimators facilitating the launch of dual use of auxiliary information through exponent-based formation. The duality of the supplementary information is executed by considering the squares of information available on supplementary variable. The comparative performance of the proposed approach is enumerated by the application of well acknowledged data sets from multi-disciplinary research literature. Furthermore, rigorous evaluation analysis is conducted in comparison to the leading family of Grover and Kaur (2014). The tedious comparative performance evaluation of contemporary methods reveals that every instant of newly proposed family out performs every member of existing family. Moreover, this distinction is witnessed with respect to all considered data sets. In future, it will be interesting to extend the proposed scheme for more complicated study designs.

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