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GRAPH QUASICONTINUOUS FUNCTIONS AND DENSELY CONTINUOUS FORMS

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Abstract. Let X, Y be topological spaces. A function $f : X \to Y$ is said to be graph quasicontinuous if there is a quasicontinuous function $g : X \to Y$ with the graph of g contained in the closure of the graph of f. There is a close relation between the notions of graph quasicontinuous functions and minimal usco maps as well as the notions of graph quasicontinuous functions and densely continuous forms. Every function with values in a compact Hausdorff space is graph quasicontinuous; more generally every locally compact function is graph quasicontinuous.

1. Definitions and preliminaries

In what follows let X, Y be topological spaces and $\mathbb{R}$ be the space of real numbers with the usual metric. In the paper [16] Kempisty introduced a notion of quasicontinuity for real-valued functions defined in $\mathbb{R}$. For general topological spaces this notion can be given the following equivalent formulation [22].

A function $f : X \to Y$ is called quasicontinuous at $x \in X$ if for every open set $V \subset Y$, $f(x) \in V$ and open set $U \subset X$, $x \in U$ there is a nonempty open set $W \subset U$ such that $f(W) \subset V$. If f is quasicontinuous at every point of X, we say that f is quasicontinuous.

Quasicontinuous functions found applications in the study of minimal usco and minimal cusco maps [12, 13], in the study of densely continuous forms [12], in the study of topological groups [4, 20, 21], in the study of dynamical systems [6], in proofs of some generalizations of Michael's selection theorem [10] and in other areas.

The notion of graph quasicontinuity was introduced in [19].

A function $f : X \to Y$ is said to be graph quasicontinuous if there is a quasicontinuous function $g : X \to Y$ with the graph g contained in the closure $\overline{f}$ of the graph f.

In sections 2 and 3 we will show that there is a close relation between notions of graph quasicontinuous functions and minimal usco maps as well as between notions of graph quasicontinuous functions and densely continuous forms. In section 4 we will prove that the uniform limit of graph quasicontinuous functions is graph quasicontinuous if the range space is a boundedly compact metric space.

A set-valued map, or a multifunction, from X to Y is a function that assigns to each element of X a subset of Y. If F is a set-valued map from X to Y, then its graph is the set $\{(x, y) \in X \times Y : y \in F(x)\}$. Conversely, if F is a subset of $X \times Y$ and $x \in X$, define $F(x) = \{y \in Y : (x, y) \in F\}$. Then we can assign to each subset F of $X \times Y$ a set-valued map which takes the value $F(x)$ at each point $x \in X$ and which graph is F. In this way, we identify set-valued maps with their graphs. Following [5] the term map is reserved for a set-valued map.

Notice that if $f : X \to Y$ is a single-valued function, we will use the symbol f also for the graph of f.

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Given two maps $F, G : X \to Y$, we write $G \subset F$ and say that G is contained in F if $G(x) \subset F(x)$ for every $x \in X$.

A set-valued map $F : X \to Y$ is upper semi-continuous at a point $x \in X$, if for every open set V containing $F(x)$, there exists an open neighbourhood U of x such that

$$F(U) = \bigcup \{F(u) : u \in U\} \subset V.$$

F is upper semi-continuous if it is upper semicontinuous at each point of X.

A set-valued map $F : X \to Y$ is upper quasicontinuous at a point $x \in X$ [22] if for every open set V containing $F(x)$ and every open set U containing x, there is a nonempty open set $W \subset U$ such that $F(W) \subset V$. F is upper quasicontinuous if it is upper quasicontinuous at each point of X.

Following Christensen [7] we say that a set-valued map F is usco if it is upper semi-continuous and takes nonempty compact values. Finally, a set-valued map F is said to be minimal usco [5] if it is a minimal element in the family of all usco maps (with domain X and range Y); that is if it is usco and does not contain properly any other usco map.

Densely continuous forms were introduced by Hammer and McCoy in [14]. Densely continuous forms can be considered as set-valued maps from a topological space X into a topological space Y which have a kind of minimality property found in the theory of minimal usco maps. In particular, every minimal usco map from a Baire space into a metric space is a densely continuous form. There is also a connection between differentiability properties of convex functions and densely continuous forms as expressed via the subdifferentials of convex functions, which are a kind of convexification of minimal usco maps [14].

A function $f : X \to Y$ is subcontinuous at $x \in X$ [9] if for every net (x_i) convergent to x, there is a convergent subnet of $(f(x_i))$. If f is subcontinuous at every $x \in X$, we say that f is subcontinuous.

A very useful characterization of minimal usco maps using quasicontinuous selections was given in [12] and it will be important also for our analysis.

Theorem 1.1. *Let X, Y be topological spaces and Y be a regular T_1-space. Let $F : X \to Y$ be a set-valued map. The following are equivalent:*

1. *F is a minimal usco map;*
2. *Every selection f of F is quasicontinuous, subcontinuous and $\overline{f} = F$;*
3. *There exists a quasicontinuous, subcontinuous selection f of F with $\overline{f} = F$.*

Notice that the notion of subcontinuity can be extended for so-called densely defined functions.

Let A be a dense subset of a topological space X and Y be a topological space. Let $f : A \to Y$ be a function. We say that f is densely defined.

Let A be a dense subset of a topological space X. We say that densely defined function $f : A \to Y$ is subcontinuous at $x \in X$ [17] if for every net $(x_i) \subset A$ converging to x, there is a convergent subnet of $(f(x_i))$. We say that $f : A \to Y$ is subcontinuous if it is subcontinuous at every $x \in X$.

Let A be a dense subset of a topological space X. We say that a densely defined function $f : A \to Y$ is a densely defined quasicontinuous function if $f : A \to Y$ is quasicontinuous with respect to the induced topology on A.

Let X, Y be topological spaces and $F : X \to Y$ be a map. We say that a densely defined function f is a densely defined selection of a set-valued map F, if $f(x) \in F(x)$ for every $x \in dom f$, ($dom f$ denotes the domain of f).

In [13] a characterization of minimal usco maps using densely defined quasicontinuous subcontinuous selections is given.

Theorem 1.2. *Let X, Y be topological spaces and Y be a T_1 regular space. Let $F : X \to Y$ be a map. The following are equivalent:*

(1) *F is minimal usco;*
(2) *There is a densely defined quasicontinuous subcontinuous selection f of F such that $\overline{f} = F$.*

2. Graph quasicontinuous functions and usco maps

Theorem 2.1. *Let X, Y be topological spaces and Y be a regular T_1-space. Let $f : X \to Y$ be a function such that $\overline{f}$ contains a graph of a minimal usco map. Then f is graph quasicontinuous.*

Proof. By Theorem 1.1 every selection of minimal usco map is quasicontinuous. Thus we are done. (Let $F : X \to Y$ be a minimal usco map such that $F \subset \overline{f}$. Let $g : X \to Y$ be a selection of F. Then $g \subset \overline{f}$.) □

Corollary 2.1. *Let X, Y be topological spaces and Y be a regular T_1-space. Let $f : X \to Y$ be a function such that $\overline{f}$ contains a graph of a usco map. Then f is graph quasicontinuous.*

Proof. By an easy application of Kuratowski-Zorn principle we can guarantee that every usco map from X to Y contains a minimal usco map. By Theorem 2.1 f is graph quasicontinuous. □

The following Corollary is a generalization of Corollary 1 in [11].

Corollary 2.2. *Let X, Y be topological spaces and Y be a compact Hausdorff space. Then every function $f : X \to Y$ is graph quasicontinuous.*

Proof. It is the well-known fact that every set-valued map with a closed graph and with values in a compact Hausdorff space is a usco map (see [2]). Thus $\overline{f}$ is the graph of a usco map. By Corollary 2.1 f is graph quasicontinuous. □

We say that a set-valued mapping $F : X \to Y$ is locally compact at $x \in X$ [15] if there are an open neighbourhood U of x and a compact set $K \subset Y$ such that $F(U) \subset K$. If F is locally compact at every $x \in X$, we say that F is locally compact. If f is a (single-valued) function, we have a definition of a locally compact function. Notice that if $Y = R$, the notions of local boundedness and local compactness coincide.

We have the following generalization of Corollary 2 in [11].

Corollary 2.3. *Let X, Y be topological spaces and Y be a regular T_1-space. Let $f : X \to Y$ be a locally compact function. Then f is graph quasicontinuous.*

Proof. Let $f : X \to Y$ be a locally compact function. It is very easy to verify that $\overline{f}$ is the graph of a locally compact set-valued map. It is the well-known fact that a locally compact set-valued map with a closed graph is usco (see [2]). Thus by Corollary 2.1 f is graph quasicontinuous. □

The folowing Example shows that the condition (concerning the existence of a minimal usco map) given in Theorem 2.1 is only a sufficient condition and not necessary.

Example 2.1. *Let $X = [0, 1]$ with the usual topology and let $Y = \mathbb{R}$ also with the usual topology. Let $f : X \to Y$ be a function defined as follows:*

$$f(x) = \begin{cases} n, & x \in \bigcup_{n\geq 2}(\frac{1}{2n-1}, \frac{1}{2n-2}]; \\ 0, & x \in \bigcup_{n\in N}(\frac{1}{2n}, \frac{1}{2n-1}] \cup \{0\}. \end{cases}$$

It is easy to verify that f is a quasicontinuous function, however $\overline{f}$ does not contain any graph of a usco map.

Notice that there is a graph quasicontinuous function f and quasicontinuous function g such that $g \subset \overline{f}$ but $f \cap g = \emptyset$. It is easy to define a function $f : [0,1] \to [0,1]$ such that $f(x) \neq 0$ for every $x \in [0,1]$ and with the property that $\overline{f} = [0,1] \times [0,1]$. Let $g : [0,1] \to [0,1]$ be the constant function equal to zero. Then f and g have the above property.

Theorem 2.2. *Let X, Y be topological spaces, Y be a regular T_1-space and A be a dense subspace of X. Let $f : A \to Y$ be a densely defined quasicontinuous subcontinuous function. Then the function f has a quasicontinuous extension over X.*

Proof. By Theorem 1.2 $\overline{f}$ is minimal usco. For every $x \in X \setminus A$ we choose a point $y_x \in \{y \in Y;\ (x,y) \in \overline{f}\}$. Define a function $g : X \to Y$ as follows:

$$g(x) = \begin{cases} f(x), & x \in A; \\ y_x, & x \in X \setminus A. \end{cases}$$

By Theorem 1.1 the function g is quasicontinuous function from X to Y and so g is a quasicontinuous extension of f over X. □

Theorem 2.3. *Let X, Y be topological spaces, Y be a regular T_1-space and A be a dense subspace of X. Let f be a quasicontinuous function from A to Y and for each point $x \in X \setminus A$ there is an open neighborhood $U(x)$ of x such that the set $V(x) = \overline{f(A \cap U(x))}$ is compact. Then the function f has a quasicontinuous extension $g : X \to Y$ such that $g \subset \overline{f}$.*

Proof. Denote by $\mathcal{U}(x)$ a base of open neighborhoods of $x \in X \setminus A$ such that $U \subset U(x)$ for every $U \in \mathcal{U}(x)$. Put

$$B(x) = \bigcap \{\overline{f(A \cap U(x))} :\ U \in \mathcal{U}(x)\}.$$

Since $V(x)$ is compact and $\{\overline{f(A \cap U(x))} :\ U \in \mathcal{U}(x)\}$ has the finite intersection property, $B(x)$ is nonempty for every $x \in X \setminus A$.

For every $x \in X \setminus A$ choose a point $y_x \in B(x)$. Define a function $g : X \to Y$ as follows:

$$g(x) = \begin{cases} f(x), & x \in A; \\ y_x, & x \in X \setminus A. \end{cases}$$

It is easy to verify that $g \subset \overline{f}$.

We show that g is quasicontinuous. Let $x \in A$. Let G be an open set with $g(x) \in G$ and H be an open set with $x \in H$. Let G_1 be an open set with $g(x) \in G_1$ such that $\overline{G_1} \subset G$. Since f is quasicontinuous at x there is an open set $O \subset A \cap H$ in A such that $f(O) \subset G_1$. There is an open set $O_X \subset H$ in X such that $O = O_X \cap A$. We show that $g(O_X) \subset G$. If $z \in O_X \cap A$ then $g(z) = f(z) \in G_1 \subset G$. Let $z \in O_X \setminus A$. Then $g(z) \in \overline{f(O_X \cap A)} \subset \overline{G_1} \subset G$.

Let now $x \in X \setminus A$. Let G be an open set with $g(x) \in G$ and H be an open set with $x \in H$. Since $g(x) \in B(x)$ there is a point $(z, f(z)) \in H \times G$. Then the proof can continue as above. □

Now we give a generalization of Theorem 1 in [11].

Theorem 2.4. *Let X, Y be topological spaces, Y be a regular T_1-space and $f : X \to Y$ be a function. If there is a dense subset $A \subset X$ such that the restricted function $f \restriction A$ is quasicontinuous and for each point $x \in X \setminus A$ there is an open neighborhood $U(x)$ of x such that the set $V(x) = \overline{f(A \cap U(x))}$ is compact, then the function f is graph quasicontinuous.*

Proof. By Theorem 2.3 there is a quasicontinuous function $g : X \to Y$ such that $g \subset \overline{f \restriction A} \subset \overline{f}$. Thus f is graph quasicontinuous. □

We have the following characterization of graph quasicontinuous functions with values in locally compact Hausdorff spaces.

Theorem 2.5. *Let X, Y be topological spaces and Y be a locally compact Hausdorff space. The following are equivalent:*

(1) *$f : X \to Y$ is graph quasicontinuous;*
(2) *There is a set-valued map $G : X \to Y$ such that $G \subset \overline{f}$, G is usco at every x in some open dense set $A \subset X$ and G is single-valued and upper quasicontinuous at every $x \notin A$.*

Proof. (1) $\Rightarrow$ (2) Let $g : X \to Y$ be a quasicontinuous function such that $g \subset \overline{f}$. Let τ be the topology on X. Define the following set

$$A = \{x \in X : \exists U \in \tau, x \in U, \exists \text{ compact } K \subset Y, g(U) \subset K\}.$$

It is easy to verify that the set A is open. Now we prove that A is a dense set. Let V be an open set in X and let $x \in V$. Let K be a compact set such that $g(x) \in IntK$. The quasicontinuity of g at x implies that there is a nonempty open set $H \subset X$ such that $H \subset V$ and $g(H) \subset IntK \subset K$. Thus $H \subset A \cap V$.

Let $G : X \to Y$ be the following set-valued map:

$$G(x) = \begin{cases} \{\overline{g}(x)\}, & x \in A; \\ \{g(x)\}, & x \in X \setminus A. \end{cases}$$

It is easy to verify that for every $x \in A$, G is locally compact at x and thus G is usco at every $x \in A$. Now G is single-valued for every $x \notin A$ by the definition; we prove that G is upper quasicontinuous at every $x \notin A$. Let $x \notin A$. Let U be an open set in X such that $x \in U$ and V be an open set in Y such that $G(x) \subset V$. Let O be an open set in Y such that $g(x) \in O \subset \overline{O} \subset V$ and $\overline{O}$ is compact in Y. The quasicontinuity of g at x implies that there is a nonempty open set H in X such that $g(H) \subset O$. Thus $G(H) \subset \overline{O} \subset V$; i.e. G is upper quasicontinuous at x.

(2) $\Rightarrow$ (1) Let $F : A \to Y$ be the restriction $G \restriction A$ of G to A. Then F is usco and thus by [12] there must exist a quasicontinuous selection $h : A \to Y$ of F. Define now the following function:

$$g(x) = \begin{cases} h(x), & x \in A; \\ G(x), & x \in X \setminus A. \end{cases}$$

Then $g : X \to Y$ is single-valued, $g \subset G \subset \overline{f}$. Obviously, g is quasicontinuous at every $x \in A$. The upper quasicontinuity of G at $x \notin A$, implies that g is quasicontinuous at every $x \in X$. $\square$

3. Graph quasicontinuous functions and densely continuous forms

To define a densely continuous form from X to Y [14], denote by $DC(X, Y)$ the set of all functions $f : X \to Y$ such that the set $C(f)$ of points of continuity of f is dense in X. We call such functions densely continuous.

Of course $DC(X, Y)$ contains the set $C(X, Y)$ of all continuous functions from X to Y. If $Y = \mathbb{R}$ and X is a Baire space, then all upper and lower semicontinuous functions on X belongs to $DC(X, Y)$ and if X is a Baire space and Y is a metric space then every quasicontinuous function $f : X \to Y$ has a dense G_δ-set $C(f)$ of the points of continuity of f [22]. Notice that points of continuity and quasicontinuity of functions are studied in [3].

For every $f \in DC(X, Y)$ we denote by $\overline{f \restriction C(f)}$ the closure of the graph of $f \restriction C(f)$ in $X \times Y$. We define the set $D(X, Y)$ of densely continuous forms by

$$D(X, Y) = \{\overline{f \restriction C(f)} : f \in DC(X, Y)\}.$$

Densely continuous forms from X to Y may be considered as set-valued maps, where for each $x \in X$ and $F \in D(X, Y)$, $F(x) = \{y \in Y : (x, y) \in F\}$.

Theorem 3.1. *Let X be a topological space and Y be a regular T_1 space. Let $f : X \to Y$ be a function such that $\overline{f}$ contains a graph of a densely continuous form with nonempty values. Then f is graph quasicontinuous.*

Proof. The proof follows from Proposition 3.2 in [12]. □

We have the following characterizations of elements of $D(X, Y)$ with nonempty values [12].

Theorem 3.2. *Let X be a Baire space and Y be a metric space. Let F be a set-valued map from X to Y such that $F(x) \neq \emptyset$ for every $x \in X$. The following are equivalent:*

(1) *$F \in D(X, Y)$;*
(2) *There is a quasicontinuous function $f : X \to Y$ such that $\overline{f} = F$;*
(3) *Every selection f of F is quasicontinuous and $\overline{f} = F$.*

Corollary 3.1. *Let X be a Baire space and Y be a metric space. Let $f : X \to Y$ be a function. The following are equivalent:*

(1) *f is graph quasicontinuous;*
(2) *$\overline{f}$ contains a graph of a densely continuous form with nonempty values.*

Notice that closures of graphs of quasicontinuous functions were studied also in [18].

4. Topology of uniform convergence on graph quasicontinuous functions

We say that a metric space (Y, d) is boundedly compact ([1]) if every closed bounded subset is compact. Therefore (Y, d) is a locally compact, separable metric space and d is complete. In fact, any locally compact, separable metric space has a compatible metric d such that (Y, d) is a boundedly compact space ([23]).

The following result is an improvement of Theorem 2 in [11] for boundedly compact metric spaces. Notice that we use entirely different ideas in our proof.

Theorem 4.1. *Let X, Y be topological spaces and (Y, d) be a boundedly compact metric space. Let $f : X \to Y$ be a graph quasicontinuous function. If for a function $h : X \to Y$ there is a real $M > 0$ such that $d(h(x), f(x)) \leq M$ for every $x \in X$, then h is a graph quasicontinuous function.*

Proof. Let $f : X \to Y$ be a graph quasicontinuous function. Let $g : X \to Y$ be a quasicontinuous function such that $g \subset \overline{f}$. Let $\mathcal{G}$ be a maximal family of pairwise disjoint open sets such that $\text{diam}[g(G)] < \frac{1}{2}$ for every $G \in \mathcal{G}$. Of course $\bigcup \mathcal{G}$ is dense in X.

For every $G \in \mathcal{G}$ exists a set $D_G \subset G$, dense in G such that

(*) $g \restriction G \subset \overline{f \restriction D_G}$ and $\text{diam}[\overline{f(D_G)}] \leq 1$.

Thus for every $G \in \mathcal{G}$ $\text{diam}[\overline{h(D_G)}] \leq 2M + 1$, i.e. $\overline{h(D_G)}$ is compact.

For every $G \in \mathcal{G}$ the map $\overline{h \restriction D_G} \cap (G \times Y)$ is usco. There exists a quasicontinuous selection l_G of $\overline{h \restriction D_G} \cap (G \times Y)$.

The quasicontinuity of g and the property (*) imply that

$$g \subset \overline{g \restriction \bigcup_{G \in \mathcal{G}} G} = \overline{\bigcup_{G \in \mathcal{G}} g \restriction G} \subset \overline{\bigcup_{G \in \mathcal{G}} f \restriction D_G} = \overline{f \restriction \bigcup_{G \in \mathcal{G}} D_G}.$$

Define the function $H : X \to Y$ as follows: If $x \in \bigcup_{G \in \mathcal{G}} G$, then there exists $G \in \mathcal{G}$ such that $x \in G$. Put $H(x) = l_G(x)$. Now let $x \in X \setminus \bigcup_{G \in \mathcal{G}} G$. Then $(x, g(x)) \in \overline{f \restriction \bigcup_{G \in \mathcal{G}} D_G}$; i.e. there exists a net $\{x_\sigma;\ \sigma \in \Sigma\} \subset \bigcup_{G \in \mathcal{G}} D_G$ such that $g(x) = \lim f(x_\sigma)$. Without loss of generality we can suppose that $B(g(x), M + 1)$ contains the net $\{h(x_\sigma) : \sigma \in \Sigma\}$, where $B(y, r) = \{z \in Y : d(y, z) \leq r\}$.

Thus $B(g(x), 3M + 2)$ is compact and contains $\{H(x_\sigma) :\ \sigma \in \Sigma\}$, i.e. there exists a cluster point $r(x)$ of $\{H(x_\sigma) : \sigma \in \Sigma\}$. Put $H(x) = r(x)$. We claim that H is quasicontinuous and $H \subset \overline{h}$. If $x \in G$, then $(x, H(x)) \subset \overline{h \restriction D_G} \subset \overline{h}$. If $x \notin \bigcup_{G \in \mathcal{G}} G$, $(x, H(x)) \in \overline{\bigcup_{G \in \mathcal{G}} h \restriction D_G} \subset \overline{h \restriction \bigcup_{G \in \mathcal{G}} D_G} \subset \overline{h}$. □

Let X be a topological space and (Y,d) be a metric space. Denote by $F(X,Y)$ the space of all functions from X to Y, by $G(X,Y)$ the space of all graph quasicontinuous functions from X to Y and by τ_U the topology of uniform convergence on $F(X,Y)$. We have the following Corollary of the above Theorem:

Corollary 4.1. *Let X be a topological space and (Y,d) be a boundedly compact metric space. Then $G(X,Y)$ is clopen set in $(F(X,Y),\tau_U)$.*

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